AQVAL/1--COMPUTER IMPLEMENTATION OF A VARIABLE-VALUED LOGIC SYSTEM VL_1 AND EXAMPLES OF ITS APPLICATION TO PATTERN RECOGNITION

by

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Proceedings of the First International Joint Conference on Pattern Recognition, pp. 3-17, Washington, DC, October 30-November 1, 1973.

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Proceedings of the

FIRST INTERNATIONAL JOINT CONFERENCE ON

PATTERN RECOGNITION

October 30 - November 1, 1973
Washington, D. C.

73 CHO 821-9C

AQVAL/1--COMPUTER IMPLEMENTATION OF A VARIABLE-VALUED LOGIC SYSTEM VL_1 AND EXAMPLES OF ITS APPLICATION TO PATTERN RECOGNITION

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Abstract

AQVAL/1 is a PL/1 program which synthesizes quasi-minimal formulas of the variable-valued logic[1] system VL_1 under cost (or complexity) functionals designed by a user from a set of available criteria. The paper presents a brief functional description of AQVAL/1 and reports current results of its application to two pattern recognition problems:

1. The synthesis of a classification rule for distinguishing between patients with cancer of the pancreas, primary cancer of the liver (primary hepatoma) and normal patients.
2. The design of spacial filters for non-uniform texture discrimination.

1. Introduction

The concept of a variable-valued logic system (VL system) was first introduced at the IFIP Working Conference on Graphic Languages in Vancouver, Canada, May 1972.[1] This concept assumes that every proposition (which conveys certain information or a description of a physical or abstract object) and all the variables in the proposition (used to represent any objects, e.g., other propositions) can assume their own number of 'truth-values' which are selected based on semantic and problem-oriented considerations. This assumption makes the system easily applicable to a variety of practical problems, in particular as a tool for describing objects or classes of objects. The assumption is supported by the observation that humans vary and adapt the 'degree of truth' or 'descriptive precision' of their statements according to the needs of the situation, and do so by changing the 'degree of truth' or 'quantization' of concepts represented by individual words or phrases in the statements. (E.g., when describing a person's height, different degrees of precision may be appropriate in various situations, so we may say that someone is 'tall', or is 'fairly tall' or is '6 feet 2 inches tall'. Each of these phrases implies the use of a different quantization of the concept 'height'.)

From the standpoint of formal logic, traditional binary logic systems and many-valued logic systems are special cases of a VL system.

Paper[1] defined and discussed a particular VL system, called VL_1. Important features of the VL_1 system are:

1. The formulas of the system (called VL_1 formulas) have very simple interpretation and can be very easily handled and evaluated (especially using parallel-sequential techniques).
2. The system is applicable for expressing and solving a variety of problems of discrete mathematics, especially problems which are intrinsically non-linear or involve variables which are measured on a nominal scale (i.e. when values of the variables are 'numerical names' of independent objects and, therefore, arithmetic relationships between these values have no meaning). The last feature makes the system applicable beyond the area of applicability of established pattern recognition methods, such as statistical techniques (parametric or nonparametric) or discriminant function methods.

The necessary condition for practical application of the system is the computational feasibility of its implementation, which means, in particular, the synthesis and minimization of VL_1 formulas.

A computer program, called* AQVAL/1, has been developed for the above tasks and the first trial applications of AQVAL/1 to various problems were quite satisfactory.

This paper gives a brief functional description of AQVAL/1 and presents current results from its application to two problems: the synthesis of classification rules for medical diagnosis, and the design of a set of spacial filters for non-uniform texture discrimination.

2. Definition of the VL_1 System

To describe AQVAL/1, knowledge of the VL_1 system has to be assumed. Therefore, to make the paper self-contained, we will review briefly the definition of VL_1 (as in paper[1] with slight modifications).

The <u>variable-valued</u> <u>logic</u> <u>system</u> VL_1 is an ordered quintuple:

$$(X, Y, S, R_F, R_I) \qquad (1)$$

where

X is a finite non-empty (f.n.) set of input variables

$$x_1, x_2, \ldots, x_n$$

whose domains, called <u>input</u> or <u>independent</u> <u>name</u> <u>sets</u>, are f.n. sets, respectively:

$$H_1, H_2, \ldots, H_n$$

where $H_i = \{0,1,2,\ldots,\not{h}_i\}$, $i=1,2,\ldots,n$, $\not{h}_i$--a natural number.

Y consists of one output variable y, whose domain is a f.n. set, called <u>output</u> or <u>dependent</u> <u>name</u> <u>set</u>:

$$H = \{0, 1, 2, \ldots, \not{h}\}, \ \not{h}\text{--a natural number.}$$

(Constants in H represent 'truth-values' which may be taken by statements (formulas) in the system.)

S is the set of 13 improper symbols:

$$= \quad \neq \quad \vee \quad \wedge \quad \neg \quad , \quad : \quad [\quad] \quad (\quad) \quad \leq \quad \geq$$

R_F is a set of <u>formation</u> or <u>syntactic</u> <u>rules</u> which define <u>well-formed</u> <u>formulas</u> (wffs) in the system (<u>VL_1 formulas</u>):

* The name AQVAL/1 was derived from 'Algorithm A^q applied for the synthesis of Variable-Valued Logic formulas.

$\gamma(e)$ is called the number of the event e. For example, the number of the event $e = (2,3,1,4)$ in the space $E(5,4,2,5)$ is: $\gamma(3) = 4 + 1\cdot 5 + 3\cdot 2\cdot 5 + 2\cdot 4\cdot 2\cdot 5 = 119$.

Assuming that the domains of variables in the formula (2) have cardinalities: $c(H_1) = 8$, $c(H_2) = 6$, $c(H_3) = 7$, $c(H_4) = 2$, $c(H_5) = 5$ and the cardinality of H, $c(H) = 5$, the formula is interpreted as an expression of a function:

$$f:\ E(8,6,7,2,5) \rightarrow \{0,1,2,3,4\} \tag{6}$$

In practice we usually deal with functions which may have an unspecified value for some events, that is, with functions:

$$f:\ E \rightarrow H \cup \{*\} \tag{7}$$

where * denotes an unspecified value ('don't care').

An incompletely specified function f can be considered equivalent to a set $\{f_i\}$ of completely specified functions f_i, each determined by a certain assignment of specified values (i.e., values from H) to events e for which $f(e) = *$. A VL_1 formula which expresses any of these functions f_i will be accepted as an expression for f.

Functions of the type (7) will be called variable-valued logic functions (VL functions).

In general, there can be a very large number of VL_1 formulas which express a given VL function (VL_1 expressions of f). Therefore, a problem arises of how to construct a formula which is minimal under an assumed cost (or complexity) functional. Depending on the application, different properties of VL_1 formulas may be desirable when expressing a given VL function, e.g., the minimal number of terms, of selectors, the minimal 'cost' of evaluation, etc. Thus, it is desirable that in a computer program which synthesizes minimal VL_1 formulas, there be available a number of cost functionals, from which the user may select the most appropriate for his type of application. For computational feasibility and convenience it is desirable to assume, however, that the available functionals be expressed in one standard form.

In the program AQVAL/1 (see Chapter 5) which synthesizes and minimizes disjunctive simple VL_1 formulas (DVL_1 formulas), a functional A measuring their minimality is assumed to be in the form:

$$A = \langle a\text{-list},\ \tau\text{-list}\rangle \tag{8}$$

where

- a-list, called attribute (or criteria) list, is a vector $a = (a_1, a_2, \ldots, a_\ell)$, where the a_i denote single- or many-valued attributes used to characterize DVL_1 formulas
- τ-list, called tolerance list, is a vector $\tau = (\tau_1, \tau_2, \ldots, \tau_\ell)$, where $0 \le \tau_i \le 1$, $i = 1,2,\ldots,\ell$, and the τ_i are called tolerances for attributes a_i.

A DVL_1 formula V is said to be a minimal DVL_1 expression for f under functional A iff:

$$A(V) \overset{\tau}{\lessdot} A(V_j) \qquad ($$

where

$A(V) = (a_1(V), a_2(V), \ldots, a_\ell(V))$

$A(V_j) = (a_1(V_j), a_2(V_j), \ldots, a_\ell(V_j))$

$a_i(V)$, $a_i(V_j)$ denote the value of the attribute a_i fo formula V and V_j, respectively. V_j, $j = 1,2,3,\ldots$ --all irredundant DVL_1 expressions for f (a DVL_1 expression is called irredundant if removing any term or selecto makes it no longer an expression for f).

$\overset{\tau}{\lessdot}$ denotes a relation, called the lexicographic order with tolerance τ, defined as:

$$A(V) \overset{\tau}{\lessdot} A(V_j) \text{ if } \begin{cases} a_1(V_j) - a_1(V) > T_1 \\ \text{or } 0 \le a_1(V_j) - a_1(V) \le T_1 \text{ and } a_2(V_j) - a_2(V) > \\ \text{or } \ldots\ldots\ldots \\ \vdots \\ \text{or } \ldots\ldots\ldots\ldots\ldots\ldots \text{ and } a_\ell(V_j) - a_\ell(V) \ge \end{cases}$$

$T_i = \tau_i \cdot (a_{imax} - a_{imin})$, $i = 1,2,\ldots,\ell$

$a_{imax} = \max_j \{a_i(V_j)\}$, $a_{imin} = \min_j \{a_i(V_j)\}$

Note that if $\tau = (0,0,\ldots,0)$ then $\overset{\tau}{\lessdot}$ denotes the lexicographic order in the usual sense. In this case, A will be specified just as $A = \langle a\text{-list}\rangle$.

To specify a functional A one selects a set of attributes, puts them in the desirable order in the a-list, and sets values for tolerances in the τ-list.

4. Synthesis of VL_1 Formulas

4.1 Basic Concepts from Covering Theory

The synthesis of a minimal VL_1 formula for expressing a given function f under a functional A is based on the results of covering theory[2,3,4,5,6]. This theory deals with the problems of expressing sets as unions of certain standard subsets ('building blocks') called complexes. One of the basic concepts is that of a cover of a set against another set.

Let E^1 and E^0 be disjoint event sets in a universe E and sets

$$L = \bigcap_{i \in I} X_i^{A_i} \tag{10}$$

where
$$A_i \subseteq H_i,\ I \subseteq \{1,2,\ldots,n\} \tag{11}$$

$$X_i^{A_i} = \{e = (\dot{x}_1, \dot{x}_2, \ldots, \dot{x}_i, \ldots, \dot{x}_n) \mid \dot{x}_i \in A_i\} \tag{12}$$

'building blocks' which are used to express event sets. Event sets L are called cartesian complexes and sets $X_i^{A_i}$ --cartesian literals.

A cartesian cover $C(E^1|E^0)$ of set E^1 against E^0 is a set $\{L_i\}$ of cartesian complexes L_i such that

$$E^1 \subseteq \bigcup L_i \subseteq E \setminus E^0 \tag{13}$$

If sets A_i are restricted to sequences of consecutive numbers from H_i, then the complexes L are called determining the value of variable x_i, and I is the set of indices of those variables (among $x_1, x_2, \ldots, x_n$, specified in the input data) which actually appear in the output VL_1 formula,

4. $g(V)$ - the '<u>degree of generalization</u>' defined as:

$$g(V) = \frac{1}{g_0} \sum_{k=1}^{\not{k}} \sum_{\ell=1}^{s_k} g(L_{k\ell}) \tag{19}$$

where

$$g(L_{k\ell}) = \frac{c(L_{k\ell})}{c(L_{k\ell} \cap F^k)}$$

$L_{k\ell}$--the set of events which satisfy* the ℓ-th term in the sequence of terms in V with the constant k,

s_k --number of terms with the constant k

g_0 --total number of terms in V

$F^k = \{e | f(e) = k\}$

5. $o(V)$ - the ordering criterion (not single-valued attribute). This criterion can be used when events in each set F^k, $k = \not{k}, \not{k}-1, \ldots, 0$ are assigned weight $w(e)$ (which represents, e.g., the frequency of event occurrence or 'importance' of the event) and a VL_1 formula is desirable, in which the first term T_1^k of each sequence of terms $(T_1^k, T_2^k, \ldots)$ with constant k, $k = \not{k}, \not{k}-1, \ldots, 1$ will have maximal possible weight $w(T_1^k)$, then the next term T_2^k will have the maximal possible weight, etc., where:

$$w(T_i^k) = \sum_{e \in E_i^k} w(e) \tag{20}$$

and E_i^k--the set of events from F^k, which satisfy T_i^k, but do not satisfy $T_{i-1}^k, T_{i-2}^k, \ldots, T_1^k$. Formally, $o(V) = \langle -w(T_1), -w(T_2), \ldots \rangle$ and minimum $o(V)$ means that

$$o(V) \prec o(V_i) \tag{21}$$

where V_i--all other VL_1 formulas expressing f

$\prec$ the lexicographic order

6. $\ell(V)$ - the total length of references:

$$\ell(V) = \sum_{j=1}^{t} \ell(T_j) \tag{22}$$

where

$$\ell(T_j) = \sum_{i \in I_j} \ell(x_i)$$

I_j--the index set specifying variables occurring in selectors of the term T_j

$\ell(x_i)$--the number of constants in the compressed reference of the variable x_i, $i \in I$

t--the number of terms in V

7. $r(V)$ - relative scope of references:

$$r(V) = \sum_{j=1}^{t} r(T_j) \tag{23}$$

where

$$r(T_j) = \sum_{i \in I_j} \left(\frac{c_{max} - c_{min} + 1}{\ell_e(x_i)} \right)^2$$

c_{min}, c_{max}--minimal and maximal value in the reference of the variable x_i,

$\ell_e(x_i)$--the number of constants in the extended form of reference of variable x_i,

t, I_j--defined as previously.

5.3 <u>Type of Variables</u>

A simple selector $[x_i \# c]$, $\# \in \{=, \neq\}$, whose reference c is $c_1 : c_2$ or c, is called <u>cyclic selector</u>. A cyclic selector in which '#' is just '=' is called an <u>interval selector</u>.

If we want the VL_1 formula to be synthesized to include only cyclic (interval) selectors for a given variable x_i, then x_i is called (in the program) a <u>cyclic (interval) variable</u>, otherwise a <u>cartesian variable</u>.

AQVAL/1 permits a user to specify the <u>type</u> (cyclic, interval, or cartesian) of each input variable independently. Some guidelines for the specification of the type of the input variables follow.

Interval (and secondarily, cyclic) selectors are usually the simplest for evaluation (depending, in general, on the way the selectors are represented and on how the formula is evaluated--by a computer program or by a specialized machine). Also, the synthesis of minimal VL_1 formulas usually takes considerably less computational time and memory if variables are specified as interval (or secondarily, cyclic) rather than as cartesian variables. On the other hand, VL_1 formulas with interval (or cyclic) selectors will usually have considerably more terms and selectors than formulas obtained when no restrictions on the selectors were assumed.

If a variable takes values which have natural linear order (e.g., represent temperature, height, length, grey-level of a picture element, etc.) then it is usually advisable to specify it as an interval (or cyclic) variable. If this is not the case (e.g., the variable represents a set of independent objects, relations, properties, etc, i.e. is measured on a nominal scale), then the variable should be specified as a cartesian variable.

5.4 <u>Restriction Parameters <ms,cs></u>

Parameters <ms,cs>, called <u>max-star</u> and <u>cut-star</u>, are 'tree pruning' parameters which are used to control

* By <u>set of events which satisfy the term</u> T is meant the set of all events which satisfy every selector in T.

Admitting Information

x_3	- Age (years)	$(h_3=7)$

Blood Biochemistry & Hematology

x_{24}	- Sodium (meq/ℓ)	$(h_{24}=4)$
x_{25}	- Potassium (meq/ℓ)	$(h_{25}=5)$
x_{26}	- Chlorides (meq/ℓ)	$(h_{26}=4)$
x_{28}	- Fasting Glucose (mg/100 mℓ)	$(h_{28}=7)$
x_{29}	- Blood Urea Nitrogen (mg/100 mℓ)	$(h_{29}=8)$
x_{30}	- Serum Glutamic Oxaloacetic Transaminase (SGOT) (units)	$(h_{30}=7)$
x_{31}	- Alkaline Phosphatase (King-Armstrong units)	$(h_{31}=6)$
x_{34}	- Bilirubin: Total (mg/100 mℓ)	$(h_{34}=6)$
x_{36}	- White Cell Count (# per mm^3)	$(h_{36}=6)$
x_{38}	- Hematocrit (%)	$(h_{38}=6)$
x_{39}	- Neutrophiles (%)	$(h_{39}=5)$
x_{40}	- Eosinophiles (%)	$(h_{40}=4)$
x_{41}	- Basophiles (%)	$(h_{41}=5)$
x_{42}	- Lymphocytes (%)	$(h_{42}=5)$
x_{43}	- Monocytes (%)	$(h_{43}=5)$

Urinalysis

x_{45}	- Casts (count)	$(h_{45}=4)$
x_{46}	- WBC/hpf (count)	$(h_{46}=5)$
x_{47}	- RBC/hpf (count)	$(h_{47}=5)$

The ranges of variability of each of these variables were quantized into discrete units whose numbers (h_i) are shown on the right-hand side of the above list (the quantization was done according to the thresholds described in Loew[9], case '8'). Thus, the data characterizing each patient (i.e. each case) can be considered an event in the space $E = E(7,4,5,4,7,8,7,6,6,6,6,5,4,5,5,5,4,5,5)$. All variables were assumed to be interval.

The set of 87 events characterizing patients with either case of cancer ('cancer events') was partitioned into two sets:

F^c - learning set of 40 events,

T^c - testing set of 47 events.

The set of 63 events characterizing normal patients ('normal events') was partitioned into two sets:

F^n - learning set of 40 events,

T^n - testing set of 23 events.

The learning sets consisted only of completely specified events; the testing sets included completely and also incompletely specified events.

AQVAL/1 was applied to find two formulas: V(c/n) ('cancer versus normal') and V(n/c) ('normal versus cancer'), DVL_1 expressions of a function

$$f: \; E \rightarrow \{0,1\} \qquad (24)$$

which satisfies the conditions:

$$\{e/f(e) = 1\} = F^c \text{ and } \{e/f(e) = 0\} = F^n \text{ - for } V(c/n)$$

$$\{e/f(e) = 1\} = F^n \text{ and } \{e/f(e) = 0\} = F^c \text{ - for } V(n/c)$$

Such a pair of formulas, $\{V(c/n), V(n/c)\}$, is called a <u>conjugate pair of formulas</u> and is denoted by $V(c*n)$.

Assuming the cost functional $A = <(t,s,z)>$ (where $z(x_i)$, costs associated with variables x_i, were all assumed to be 1) and restriction parameters $<ms,cs> = <15,5>$, AQVAL/1 produced the following formulas:

1. $$V(c/n) = [x_{38}=2:4][x_{42}=0][x_{43}\neq 0] \vee [x_3=3:5][x_{26}\neq 3][x_{30}\geq 2][x_{38}\geq 3][x_{42}=1:3][x_{47}=0] \vee [x_3\geq 5][x_{30}=1][x_{38}=3,4] \vee [x_3=2,3][x_{26}=3] \qquad (25)$$

The formula comprises 4 terms, which consecutively cover (or are satisfied by) (28,28), (7,7), (6,3) and (2,2) events in the set F^c, respectively (the first number in each pair denotes the total number of events covered by a given term, and the second--the number of events covered by a given term and not covered by any of the previous terms). The formula depends in toto on only seven variables out of 19. The execution time on the IBM 360/75 was ≈ 39 seconds.

2. $$V(n/c) = [x_{25}=1:3][x_{29}\leq 3][x_{30}\leq 4][x_{31}=1,2] \wedge [x_{36}=2,3][x_{45}=0] \qquad (26)$$

The formula comprises only one term (which covers all 40 events) and depends in toto on only six variables (out of 19). (Note that with one exception all variables in this formula are different from those used in the formula V(c/n), which is quite surprising.) The execution time was ≈ 22 seconds.

The fact that V(n/c) has only one term and is considerably simpler than V(c/n) (4 terms) seems to confirm the common intuition that cases of 'normal health' are more or less similar, i.e. all parameter values fall simultaneously into certain 'normal' ranges, unlike cancer cases which may be quite different because they may represent different types of cancer. (Note that in an interval DVL_1 formula each term corresponds to exactly one multidimensional interval in a subspace of E.)

Translating the VL_1 values of variables back into real values of tests (expressed in units given in the list of variables), the formula V(n/c) can be paraphrased:

$$[3 < x_{25} \leq 6][x_{29} \leq 25][x_{30} \leq 90][5 < x_{31} \leq 25] \wedge [5000 < x_{36} \leq 11000][x_{45} \leq 1] \qquad (27)$$

(In the book[12] 'normal' ranges of the variables used in this formula are listed as: $4 \leq x_{25} \leq 5$; $9 \leq x_{29} \leq 20$; $10 \leq x_{30} \leq 40$; $4 \leq x_{31} \leq 13$; $4000 \leq x_{36} \leq 11000$; x_{45}--'a few casts may be present normally', page 164.)

2. <u>Testing phase</u>

The formulas V(c/n) and V(n/c) were applied individually and jointly (as a conjugate pair) to classify events in testing sets T^c and T^n.

In individual testing of the formulas we assumed that: V(c/n) produces a correct result when a 'cancer event' $e \in T^c$, is covered (i.e. satisfies the formula), or a 'normal event' $e \in T^n$, is not covered by the formula; V(n/c) produces a correct result when the opposite is true, i.e. an $e \in T^n$ is covered or an $e \in T^c$ is not covered.

2. $V(p/c) = [x_1 = 0][x_{31} \le 2][x_{36} = 2:4][x_{44} \ge 3]$

$$[x_{24} = 2:4][x_{26} = 3][x_{28} \ge 2][x_{38} = 3]$$

$$[x_{28} = 2:3][x_{30} = 1:3][x_{34} = 1:2] \qquad (29)$$

The formula comprises 3 terms, which consecutively cover (9,9), (5,5) and (5,4) events, respectively, in set F^p. The formula depends on 10 variables, 4 of which (x_{24}, x_{26}, x_{28}, x_{34}) were used in the formula $V(\ell/p)$. The execution time was 56 seconds.

2. <u>Testing phase</u>

The formulas $V(\ell/p)$ and $V(p/\ell)$ were applied individually and jointly to classify events in testing sets T^ℓ and T^p.

The performance of the formulas in both cases was summarized in table 3. Since the purpose of the formulas was to discriminate between two forms of cancer, the performance categories in joint testing were chosen slightly differently, as shown in the right half of table 1 (discrimination categories).

As can be observed in table 3, the performance of the formulas in individual and, as well, in joint testing was completely unsatisfactory (with the exception of classification of pancreas events by formula $V(\ell/p)$, which was 95% correct). Note also that the joint testing gave a very high percentage of indecisions (33%).

There can be few potential reasons explaining the results:

(1) the size of learning sets was too small for the program to make a proper generalization,

(2) the 22 variables used in the experiment, which are all general medical tests, may not have included any tests which are relevant in a sufficient degree to the distinction between cancer of the pancreas and cancer of the liver, thus ruling out the possibility of obtaining positive results in this experiment,

(3) the quantization of variables was not fine enough,

(4) the type of generalizations of input data, which AQVAL/1 is able to produce, is not adequate for the particular problem,

(5) the medical data which we used had errors (e.g., some cases listed in our data as cancer of the pancreas cases might have been in fact cases of cancer of the liver, or conversely).

1. With regard to reason 1 (learning sets too small), recall that the learning sets included only 16 (set F^ℓ) and 18 (set F^p) events, which makes it quite probable that they were of insufficient size for making adequate generalization. (To more clearly imagine the problem, note that the program was given only 16 + 18 = 34 specified events out of $\approx 8.5 \cdot 10^{15}$ events constituting the event space.)

2. In view of the present poor results as compared to the previous quite satisfactory results (100% of cases of cancer (47 cases) were detected), reason 2 (irrelevant variables) seems to be more probable. Namely, in the cancer detection experiment, AQVAL/1 was able to produce good rules also based on relatively small learning sets -- 40 + 40 = 80 specified events out of $\approx 2.10^{14}$ (though $\approx$ 100 times larger, relative to the size of the event space, than in the cancer discrimination experiment). Thus, it seems to imply that the variables used in the experiments were sufficiently relevant for detecting the cancer but were <u>not</u> sufficiently relevant for discriminating between the two cancers.

3. It is natural to expect that differences in general tests, which were used in the experiment, will be much more subtle in distinguishing between two types of cancer than in distinguishing between cancer (either type) and 'normal health'. Therefore, for the cancer discrimination experiment a much finer quantization of variables might have been required. Since essentially the same quantization was used in the cancer detection and the cancer discrimination experiments, reason 3 (quantization of variables not fine enough) is perhaps a very significant factor here (new experiments using a finer quantization of variables are desirable).

4. Reason 4 (inadequate type of generalization) seems unlikely in view of the good results of the first experiment.

5. Reason 5 (incorrect data), of course, would undoubtedly cause bad results, if true.

Finally, it should be noted that the problem of cancer discrimination using general medical tests is not of very significant interest for medicine because there exists a specific test ('alpha-feto protein test') which can easily discriminate between these two cancers (once the presence of cancer has been detected). This imparts a greater value to the experiments on cancer detection than to those on cancer discrimination, when only general medical tests are used.

6.2 <u>Texture Discrimination</u>

This experiment deals with a problem of describing and discriminating non-uniform textures. AQVAL/1 together with some auxiliary programs has been applied to a solution of this problem. For the lack of space, we will give here only a very brief description of the general idea of the method applied and then illustrate it with an example (various aspects of earlier versions of this method were described in papers[1,13,14]).

Suppose there is given a set of texture classes $\{\mathbf{T}^k\}$, $k=0,1,2,\ldots,\not{k}$, and the problem is to find an operator t, which applied to any texture $T^k \in \mathbf{T}^k$ produces the value $t(T^k) = k$. We will assume here that the $T^k \in \mathbf{T}^k$, $k=0,1,\ldots,\not{k}$, represent digitized textures, namely $\alpha \times \beta$ matrices with entries, called <u>picture elements</u>, being positive integers between 0 and some number g, inclusively. We will also assume that the scale and position of the pictures of the real textures are 'appropriately' adjusted before the scanning and digitizing operations are applied (for learning and, as well, testing phases).

The operator t (a 'texture descriptor') is assumed here to be a composite operator:

$$t = (\Phi_1 \circ \Phi_2 \circ \ldots \circ \Phi_\mu) \circ d \qquad (30)$$

$\hat{E}^1$ - of events which are in E^1 but not in E^0,

$\hat{E}^\Phi$ - of events which are in both E^0 and E^1.

Columns $c(\hat{E}^1)$, $c(\hat{E}^0)$ and $c(\hat{E}^\Phi)$ give the length of these sequences. In constructing the filtering operators Φ_i, a cutting operator (considered as a part of the operator p) was applied to the sequences E^0 and E^1 which:

1. removed from E^1 and E^0 all vectors which occur in these sequences less than r times (r - occurrence threshold),

2. computed for the rest of vectors the parameter λ defined as:

$$\lambda(e) = \frac{e(E^1)}{e(E^1) + e(E^0)} \tag{32}$$

   where $e(E^1)$, $e(E^0)$ - number of times the event e occurs in E^1 and E^0, respectively.

3. removed from E^0 all events for which $\lambda > lb$, and from E^1 - all events for which $\lambda < ub$, where lb and ub are parameters called lambda lower bound and lambda upper bound, respectively, $0 < lb \le ub < 1$.

4. produced sets F^0 and F^1 consisting of all distinct events remaining in sequences E^0 and E^1, respectively (note that due to the operation 3, F^0 and F^1 are disjoint). Columns $c(F^0)$ and $c(F^1)$ give the numbers of events in sets F^0 and F^1.

The experiment consisted of six computer runs (jobs):

1,2 - in which operators v were constructed as VL_1 formulas $V(F^0/F^1)$, which assume value 1, if an event $e \in F^0$ and value 0, if $e \in F^1$.

3,4,5,6 - in which operators v were constructed as formulas $V(F^1/F^0)$ which assume value 1, if $e \in F^1$, and value 0, if $e \in F^0$.

In table 4c by $V_{12}(F^0/F^1)$ is meant $V_1(F^0/F^1) \circ V_2(F^0/F^1)$, i.e., first is applied operator $V_1(F^0/F^1)$ and then $V_2(F^0/F^1)$; ($V_{12}(F^1/F^0)$, $V_{123}(F^1/F^0)$, etc., have analogous meaning.)

Parameters cl and ct (table 4c) called correctness ratio for learning area and correctness ratio for testing area, respectively, are defined:

$$cl = \frac{CL}{TL} \qquad ct = \frac{CT}{TT}$$

where CL and CT are total numbers of events which VL_1 formula classified correctly (i.e., produced for them value 1, if $e \in E^1$ and value 0 if $e \in E^0$), in learning and testing areas, respectively. TL and TT are total number of entries in learning and testing areas, respectively.

Figures 3b,c and Figures 4b,c show the transformed textures of 1st and 2nd iteration, i.e., results of applying operators $\Phi_i = a \circ \ell \circ p \circ v$, i=1,2, to pictures in figure 15 and figure 16 where a, ℓ, p and v are defined in table 1 (jobs 1 and 2). As we can see, the densities of '0s' in figure 3c and '1s' in figure 4c are very large, both in learning and testing areas (they are measured by cl and ct). Thus, the operator d will give the correct recognition decisions even if the samples taken from pictures in figures 3c and 4c are relatively very small (recall that the decision will be correct if: the number of '0s' is greater than the number of '1s' in a sample taken from the testing area in figure 3c or, if the number of '1s' is greater than the number of '0s' in a sample taken from the testing area in figure 4c).

7. Concluding Remarks

Let us briefly summarize the most important properties of the VL_1 system and its AQVAL/1 implementation, from the application viewpoint:

1. AQVAL/1 logically infers quasi-minimal VL_1 formulas (under an assumed cost-functional) from sets of events representing 'facts' or 'true' statements about any objects or classes of objects. This process is problem-independent; it is based entirely on logical properties of the VL_1 system.

2. The VL_1 formulas inferred by AQVAL/1 are expressions of a function

$$f: H_1 \times H_2 \times \ldots \times H_n \to H$$

   where H_i, i=1,2,...,n and H are f.n. sets of non-negative integers.

   An important property of the VL_1 system is that every such function can be expressed by a VL_1 formula. Since a large number of problems from different fields, in particular various pattern recognition problems, can be formulated as problems of finding an expression for such a function, AQVAL/1 has a potential for broad application.

3. The system can be especially useful for solving problems which are intrinsically non-linear or involve variables measured on nominal scale (i.e. values of variables denote independent objects and arithmetic relationships between the values have no meaning; such variables are specified in the program as cartesian variables). This is a very unique feature of the system.

4. The VL_1 formulas are very simple for interpretation and evaluation. (When only interval variables are used, they correspond to decision rules involving only hiperplanes perpendicular to individual variable axes.)

5. AQVAL/1 automatically reduces redundant variables.

Acknowledgment

The texture experiments reported in the paper were supported by contract AT(11-1)-2118 with the U.S. Atomic Energy Commission. The author gratefully acknowledges the programming help he received from the following people at the various stages of the development of AQVAL/1 and its earlier versions: Val Tareski (AQ-1,2,3), Peter Raulefs (AQ-4,5,6), Celso Frazer and Kurt Hirchert (AQVAL/1 --v7). The author expresses sincere thanks to his research assistants: Mr. Lee Chapman for help in editing the paper and in the experiments with medical data, and Mr. Tze-Wah Wong for help in the experiments concerning the method of texture discrimination. Thanks go also to Mr. S. N. Jayaramamurthy, who provided the event extraction program used in the experiments with textures.

		W(r/p)			W(p/r)		
		Correct	Incorrect	Total	Correct	Incorrect	Total
Cancer	# cases	35	7	42	41	5	46
	Percentage	83%	17%	100%	89%	11%	100%
Normal	# cases	16	2	18	21	1	22
	Percentage	89%	11%	100%	96%	4%	100%
Total	# cases	51	9	60	62	6	68
	Percentage	85%	15%	100%	91%	9%	100%

Individual testing of formulas W(r/p) and W(p/r).

a.

W(r/p) + (W(r/p), W(p/r))								
Correct				Incorrect				Total
Strongly	Moderately	Weakly	Total	Weakly	Moderately	Strongly	Total	
29	6	12	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]
63%	13%	26%	[illegible]		[illegible]	[illegible]	[illegible]	100%
15	5		20	2	1	[illegible]	3	23
65%	22%		[illegible]	9%	5%	[illegible]	13%	100%
44	11	12	[illegible]	2	1	[illegible]	3	[illegible]
63%	16%	17%	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	100%

Joint testing of formulas W(r/p) and W(p/r).

b.

CANCER DETECTION

Table 2.

		W(r/p)			W(p/r)		
		Correct	Incorrect	Total	Correct	Incorrect	Total
Lung	# cases	[illegible]	5	11	[illegible]	11	21
	Percentage	45%	55%	100%	[illegible]	[illegible]	100%
Pancreas	# cases	19	1	20	[illegible]	[illegible]	[illegible]
	Percentage	[illegible]	[illegible]	100%	[illegible]	[illegible]	100%
Total	# cases	[illegible]	[illegible]	31	[illegible]	[illegible]	[illegible]
	Percentage	[illegible]	[illegible]	100%	[illegible]	[illegible]	100%

Individual testing of formulas W(r/p) and W(p/r).

a.

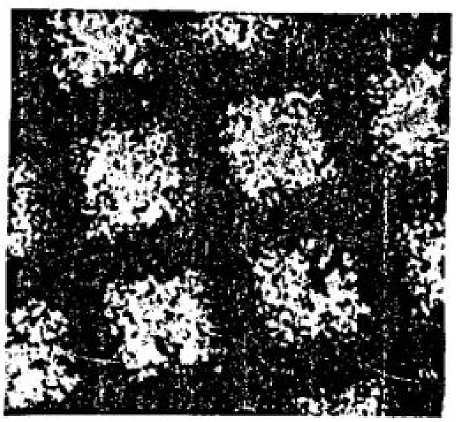
W(r/p) + (W(r/p), W(p/r))							
Correct			Indecisive	Incorrect			Total
Strongly	Moderately	Total		Moderately	Strongly	Total	
1	[illegible]	11	[illegible]	5	[illegible]	9	[illegible]
[illegible]	30%	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	100%
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	16	[illegible]	[illegible]	1	[illegible]	11	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	100%

Joint testing of formulas W(r/p) and W(p/r).

b.

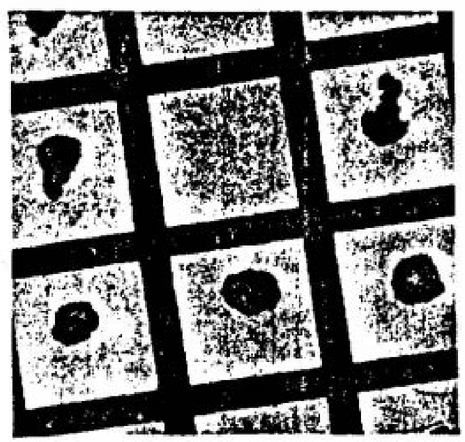
CANCER DISCRIMINATION

Table 3.



Picture No. 15

Figure 1



Picture No. 16

Figure 2

Job Number	Job Description	Event Extraction Step												
		Pictures	a	ℓ	p	$c(\hat{E}^0)$	$c(\hat{E}^1)$	$c(\hat{E}^\phi)$	Cutting Operator			$c(F^0)$	$c(F^1)$	Execution Time (sec)
									r	lb	ub			
1	1 iteration for F^0/F^1	Pic. 15 '1'-- (grid with noise)	$a_{3,3}$	ℓ_2	p_{12}	169	129	66	2	<0.25	>0.75	81	81	48
2	2 iteration for F^0/F^1	Pic. 16 '0'-- (grid)	$a_{1,1}$	ℓ_2	p_{12}	111	159	2	2	0.25	0.75	111	159	40
3	1 iteration for F^1/F^0		$a_{3,3}$	ℓ_2	p_{12}	225	152	88	3	0.5	0.5	75	92	45
4	2 iteration for F^1/F^0		$a_{1,1}$	ℓ_2	p_{12}	288	296	15	2	0.25	0.75	101	90	41
5	3 iteration for F^1/F^0		$a_{1,1}$	ℓ_2	p_{12}	79	170	3	2	0.25	0.75	26	54	38
6	4 iteration for F^1/F^0		$a_{1,1}$	ℓ_2	p_{12}	17	93	0	2	0.25	0.75	15	37	38

a - averaging operator
ℓ - grey-level operator
p - event-shape operator

r - occurrence threshold
lb - lamba lower bound
ub - lamba upper bound

a.

Job Number	AQVAL/1 Step							
	Event Space and h	Restriction <ms ,cs>	Cost-Functional A	VL_1 Formula	# of Terms	# of Selectors	# of Variables	Execution Time (sec)
1	$E(\overbrace{2,2,2,2,2,2,\ldots 2}^{12\ \text{times}})$ 2	<1,1>	c-list:(2,3,5) t-list:(0.1,0.1,0.1)	$V_1(F^0/F^1)$	22	121	12	132
2	"	<1,1>	"	$V_2(F^0/F^1)$	12	68	12	126
3	"	<1,1>	c-list:(1,2,3) t-list:(0.1,0.1,0.1)	$V_1(F^1/F^0)$	23	117	12	138
4	"	<1,1>	c-list:(2,3,5) t-list:(0.1,0.1,0.1)	$V_2(F^1/F^0)$	8	32	12	67
5	"	<1,1>	"	$V_3(F^1/F^0)$	2	4	4	9
6	"	<1,1>	"	$V_4(F^1/F^0)$	2	4	4	6

b.

Job Number	Filtering and Testing Step (application of the operator: a•ℓ•p•v)					
	Operator v	Picture 15		Picture 16		Execution Time (sec)
		Correctness Ratio for Learning Area (cl)	Correctness Ratio Testing Area (ct)	Correctness Ratio for Learning Area (cl)	Correctness Ratio for Testing Area (ct)	
1	$V_1(F^0/F^1)$	0.784	0.835	0.824	0.785	42
2	$V_{12}(F^0/F^1)$	0.989	0.998	1.0	0.911	33
3	$V_1(F^1/F^0)$	0.833	0.870	0.918	0.735	40
4	$V_2(F^1/F^0)$	0.958	0.974	0.946	0.921	31
5	$V_{23}(F^1/F^0)$	1.0	0.997	1.0	1.0	25
6	$V_{34}(F^1/F^0)$	1.0	1.0	1.0	1.0	26

c.

Summary of the experiment on texture discrimination

Table 4.