

DETECTION OF CONCEPTUAL PATTERNS THROUGH INDUCTIVE INFERENCE

by

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DETECTION OF CONCEPTUAL PATTERNS THROUGH INDUCTIVE INFERENCE

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Abstract: The determination of pattern recognition rules is viewed as a problem of inductive inference, guided by generalization rules - which control the generalization process, and problem knowledge rules - which represent the underlying semantics relevant to the recognition problem under consideration. The paper formulates the theoretical framework and a method for inferring general and optimal (according to certain criteria) descriptions of object classes from examples of classification or partial descriptions. The language for expressing the class descriptions and the guidance rules is an extension of the first order predicate calculus, called variable-valued logic calculus $VL_{\mathcal{V}}$. $VL_{\mathcal{V}}$ involves typed variables and contains several new operators, especially suited for conducting inductive inference, such as *selector*, *interval designation*, *interval conjunction*, *exception*, *generalization*.

Important aspects of the theory include:

- 1) a formalization of several kinds of generalization rules
- 2) an ability to uniformly and adequately handle descriptors (i.e., variables, functions and predicates) of different type (nominal, linear and structured) and of different arity (i.e., different number of arguments)
- 3) an ability to generate new descriptors, which are derived from the initial descriptors through a rule-based system (i.e., an ability to conduct the so called *constructive induction*)
- 4) an ability to use the semantics underlying the problem under consideration.

An experimental computer implementation of the method is briefly described and illustrated by an example.

Index Terms

pattern recognition techniques, knowledge acquisition, generalization methods, inductive inference, learning from examples, many-valued logic, plausible inference, computer consulting systems, theory formation

1. INTRODUCTION

A pattern recognition rule can be viewed as a rule

DESCRIPTION $\Rightarrow$ RECOGNITION CLASS

(1)

which assigns a situation (an object, a process, etc.) to the RECOGNITION

CLASS, when the situation satisfies the DESCRIPTION. In the decision theoretic approach the DESCRIPTION is an analytical expression involving a set of numerical variables selected a priori. Variables spanning the decision space are treated uniformly, are usually assumed to be measured on at least an interval scale, and are desired to be relevant and independent characteristics of the objects. When the variables are strongly interconnected and/or the relevant object characteristics are not numerical variables but various relations among other variables, or among parts or subparts of objects, then the decision theoretic approach becomes inadequate. In such situations the structural approach can be useful.

In the structural (or syntactic) approach, the DESCRIPTION is a formal grammar (usually a phase-structure grammar) in which terminals are certain elementary parts of objects, called 'primitives'. The types of relationships which can be expressed "naturally" in terms of a formal grammar are, however, quite limited. If the relevant characteristics include, for example, some numerical measurements in addition to relations and symbolic concepts, then grammars involving them are very cumbersome or inadequate. This is a strong limitation, because in many problems an adequate class description requires both numerical characterizations of objects and a specification of various relationships among properties of objects and/or of object parts, i.e., involve *descriptors* of mixed arity* and measured on different scales.

*'arity' - the number of arguments of a descriptor. Unary descriptors are called attributes, or, generally, variables. Two or more argument descriptors with the value set (TRUE, FALSE) are called predicates.

To overcome this limitation attributed Grammars were proposed [1]. Both the decision theoretic approach and the syntactic approach make a little use of the underlying semantics of the problem under consideration, and therefore the scope of patterns they are able to discover is limited. Also they tend to produce descriptions which are not easily comprehensible by humans, as they do not directly correspond to the 'natural language type' descriptions which human experts would develop observing the same data, and which they would normally like to use. Although in many applications 'human comprehensibility' may not be important, in other applications (e.g., in expert computer consulting systems) it is crucial.

This paper presents results, still early and limited, of an attempt to develop a uniform conceptual framework and an implementation method which appropriately handles descriptors of different type, is able to use the semantics of the problem and satisfies the requirement of human comprehensibility. Another aspect of this method is that the final descriptions which it produces may involve new descriptors (variables or relations) which were not included in the initial characterization of objects. This is achieved through the application of 'metrules' which represent the underlying knowledge of the problem at hand and of the properties of descriptors used in formulating the descriptions of exemplary data. The presented theory uses as a language for expressing the class descriptions and rules an extension of the first order predicate calculus, called variable-valued logic system VL_{21} , and is most closely related to the body of work termed *computer induction*. The ability to develop new descriptors (variables, predicates, functions) in addition to those given a priori, places this work in the category of what we call 'constructive induction', as opposed to 'non-constructive induction', in which the final descriptions relate only descriptors initially provided.

*The author thanks Larry Travls of the University of Wisconsin for suggesting this name.

2. RELATED RESEARCH

It would be a very difficult task, requiring more space than provided, to characterize adequately various important contributions to computer induction. Therefore, we will make here only a very limited and certainly not adequate review of some more recent works.

A dissatisfaction with the early work on general methods of induction (in the early sixties) led some workers to concentrate on inductive tasks within a specific problem domain. For example, programs collectively called METAENDRAL [2] use a model-directed heuristic search to determine rules that describe the molecular structure of an unknown chemical compound from mass spectrometry data. In [3] Winston describes a method for determining a graph description of simple block structures from examples.

A program developed by Lenat [4] generates concepts (represented as collections of a priori defined properties) of elementary mathematics, under the guidance of a large body of heuristic rules. Soloway and Riseman [5] describe a method for creating multi-level descriptions of a part of a baseball game, starting with 'snapshots' of the game, and using rules representing general knowledge of the game.

The program, such as those mentioned above usually incorporate a large body of task-specific knowledge and tend to perform quite well on tasks they were designed for. They demonstrate again that high performance requires specialized solutions. On the other hand, it is usually not easy to determine the general ideas they contribute to the understanding of inductive processes. Also, it is difficult to apply such methods directly to other problem areas.

A significant part of research has been concerned with determining patterns in sequences of symbols (e.g., Simon [6], Waterman [7]). Simon [6] found that descriptions of such patterns consistently incorporate only a few basic relations: 'some' and 'next' between symbols, iterations between subpatterns, and hierarchic phrase structure. Gaines [8] developed a method for generating finite-state automata, which approximate a given symbol string, and represent different trade-offs between the complexity and poorness-of-fit. Shaw, Swartout and Green [9] developed a program for inferring LISP code from a set of examples of LISP statements. Also, Jouannaud Gullio and Treull [10] have developed an interactive system which can infer a class of LISP linear recursive functions from a set of examples.

The above works are related to the general subject of grammatical inference (i.e., inference of a grammar which may have produced a given set of strings). Early work in this area was concerned with the inference of a phrase structure grammar (e.g., Feldman et al. [11]). More recent work moves into inferring 'multi-dimensional' grammars (e.g., work by Brayer and Fu [12]).

In recent years there has been a new trend toward the development of general methods of induction.

In previous papers the author and his collaborators [e.g., 13,14, 15] have described a methodology and computer programs for learning optimal discriminant descriptions of object classes from examples (in the framework of an extended propositional calculus with many-valued variables called VL_1). Examples are presented as sequences of attribute-value pairs. Each attribute has an associated value set and type. Work in a similar spirit, although more limited in scope, was reported by Stoffel [16] (the elementary statements used there are restricted to the 'variable-value' forms, i.e., to *elementary selectors as described in Section 4*).

An early work which recognizes the need for logic style descriptions for pattern recognition was done by Kanerji [17]. A more recent continuation

of this work is in Kanerji [18], and Cohen [19], who developed a logic-based description language CODE utilizing LISP-like notation.

An important problem of induction, that of *empirical prediction*, was studied by Zagoruike [25], who developed a general method of "strengthening hypotheses" by narrowing the uncertainty ranges of values of output variables. Hendrick [26] developed a method of learning of production systems describing symbol series using a semantic net of predefined concepts. Many authors use a restricted form (usually quantifier-free) of the first-order predicate calculus (FOPC) or some equivalent notation as the formal framework for expressing descriptions and hypotheses. Morgan [20] describes a formal method of hypothesis generation, called *L-resolution*, which stems from deductive resolution principles. Various theoretical issues of induction in FOPC were considered by Ploeklin [21].

Fikes, Hart and Nilsson [22] describe an algorithm for generalizing robot plans. Hayes-Roth and McDermott (e.g., [23]), also Vere [24], describe methods and computer programs for generating conjunctive descriptions of least generality (which they call 'maximal abstractions'), of a set of objects represented by products of *n*-ary predicates. The rules of generalization which they use can be characterized as 'dropping a condition' and 'turning constants into variables' (see section 5.3).

This paper presents a theoretical framework for generalizing and optimizing descriptions of object classes in the form of decision rules. The decision rules can involve descriptors of three different types: nominal, linear and structured, employ some new syntactic forms, and use problem knowledge for guiding induction and generating new descriptors. The formal notation is a modification and extension for FOPC, called variable-valued logic system VL_{21} . This formalism is claimed to be more adequate than the traditional FOPC as a conceptual framework for describing the inductive processes under consideration. The paper is an extension and modification of the report [27], and stresses the conceptual principles

of the induction method, rather than specific algorithms and implementation details. Most of the latter are described in [28, 29, 30].

3. PROBLEM STATEMENT

A VL transformation rule is defined as a rule

$$\text{DESCRIPTION}_1 \Rightarrow \text{DESCRIPTION}_2 \quad (2)$$

where DESCRIPTION₁ and DESCRIPTION₂ are expressions in VL_{q1} system (section 4),

$\Rightarrow$ stands for various transformation operators which define the meaning of the rule.

A DESCRIPTION may look like:

$2P_1, P_2 \text{ on-top}(P_1, P_2) [\text{size}(P_1)=3..5] [\text{color}(P_2)=\text{blue}, \text{yellow}, \text{red}] \wedge$

$[\text{length}(P_1) \cdot \text{length}(P_2)=\text{small}])$

where

.. the range operator

• (after the equality sign) denotes the *internal disjunction*

• denotes the *internal conjunction*.

(For explanation of notation see section 4).

We will consider here the following transformation operators:

- (1) $::>$ the operator defines a *decision rule*. DESCRIPTION₂ specifies a decision (or a sequence of decisions) which is assigned to a situation which satisfies DESCRIPTION₁.

(In the application to pattern recognition, DESCRIPTION₂ defines the recognition class.)

If a situation does not satisfy the DESCRIPTION₁, the rule assigns to it a NULL decision.

- (11) $\rightsquigarrow$ the operator defines an *inference rule*. If a situation satisfies DESCRIPTION₁, the rule assigns the 'truth-status' 'TRUE' to DESCRIPTION₂, otherwise the truth-status of DESCRIPTION₂

is '1'. (In an inference rule DESCRIPTION₁ is called the *condition* and DESCRIPTION₂ is called the *consequence*.)

- (111) $\models$ the operator denotes a *generalization rule*, which states.

that the DESCRIPTION₂ is more general than DESCRIPTION₁.

i.e., the set of situations which satisfy DESCRIPTION₂

is a *superset* of the set of situations satisfying DESCRIPTION₁.

- (1v) $\models$ the operator denotes an *equivalence preserving rule* (when the above mentioned sets are equal).

The rule is a special case of a generalization rule.

The problem considered in this paper is defined as follows.

- Given are:

- (a) a set of VL decision rules, called *data rules*, which specify initial knowledge, $\{C_{ij}\}$, about some situations (objects, processes, ...) and the recognition class, K_i , associated with them:

$$\begin{array}{llll} C_{11} ::> K_1, & C_{12} ::> K_1 & \dots & C_{1n} ::> K_1 \\ C_{21} ::> K_2, & C_{22} ::> K_2 & \dots & C_{2n} ::> K_2 \\ \vdots & \vdots & \ddots & \vdots \\ C_{m1} ::> K_m, & C_{m2} ::> K_m & \dots & C_{mn} ::> K_m \end{array}$$

(3)

- (b) The *problem knowledge rules* which represent the background knowledge about the recognition problem under consideration. This knowledge includes the type of each descriptor used in the data rules, its value set, the problem constraints, the relationship among descriptors that reflect the semantics of the problem and various *constractive generalization rules* (sec. 5.3).
- (c) a *preference criterion*, which for any two 'comparable' sets of decision rules specifies which one is more preferable, or states that they are equally preferable.
- The problem is to determine, through an application of *generalization rules* and *problem knowledge rules*, a new set of decision rules called *output rules* or *hypotheses* :

$$\begin{aligned} C_{11}^i &::> K_1, & C_{11}^i &::> K_1, & \dots & C_{1r_1}^i &::> K_1 \\ C_{21}^i &::> K_2, & C_{21}^i &::> K_2, & \dots & C_{2r_2}^i &::> K_2 \\ & \cdot & & & & & \\ & \cdot & & & & & \\ C_{m1}^i &::> K_m, & C_{m2}^i &::> K_m, & \dots & C_{mr_m}^i &::> K_m \end{aligned} \quad (4)$$

which is most preferable among all sets of rules that with regard to the input rules are *consistent* and *complete*.

The output rules are *consistent* with regard to input rules, if for any situation to which the input rules assign a non-NULL class, the output rules assign to it the same class, or the NULL class.

The output rules are *complete* with regard to input rules, if for any situation to which the input rules assign a non-NULL class, the output rules also assign to it a non-NULL class.

It is easy to see that if the output rules are consistent and complete with regard to the input rules then they are semantically equivalent (i.e., assign the same decision to the same situation) or more general than the input rules (i.e., they may assign a non-NULL class to situations to which the input rules assign a NULL class).

From a given set of data rules it is usually possible to derive many different sets of output rules which are consistent and complete and which satisfy the problem constraints. The role of the preference criterion is to select one (or a few alternative sets of rules) which is most desirable in the given problem domain. The preference criterion may refer to, e.g.:

- the computational simplicity (or complexity) of the rules
- the cost of measuring the information needed for rule evaluation
- the degree-of-fit to the data.

In this paper we accept the restriction that the *DESCRIPTIONS*, C_{ij} and C_{ij}^i , are disjunctive simple V_{L1} expressions (section 4). Such expressions have a very simple linguistic interpretation, and seem to be of interest to many applications.

4. VL EXPRESSIONS AS DESCRIPTIONS

4.1 Definition of VL_{21}

Data rules, hypotheses, problem knowledge rules and generalization rules are all expressed using the same formalism, that of variable-valued logic calculus VL_{21} . * VL_{21} is an extension of predicate calculus designed to facilitate a compact and uniform expression of descriptions of different degrees and different types of generalization. The formalism also provides a simple linguistic interpretation of descriptions without losing the precision of the conventional predicate calculus. To make the paper self-contained, we will provide here a brief description of VL_{21} .

There are three major differences between VL_{21} and the first order predicate calculus (FOPC):

1. In place of predicates, it uses *selectors* (or *relational statements*) as basic operands. A selector, in the most general form, specifies a relationship between one or more atomic functions and other atomic functions or constants. A common form of a selector is a test to ascertain whether the value of an atomic function is a specific constant or is a member of a set of constants. The selectors represent compactly certain types of logical relationships which cannot be directly represented in FOPC but which are common in human descriptions. They are particularly useful for representing changes in the degree of generality of descriptions and for syntactically uniform treatment of descriptors of different types.

* VL_{21} is a subset of a more complete system VL_2 under development.

2. Each atomic function (a variable, a predicate, a function)

is assigned a value set (domain), from which it draws values,

and its type, which defines the structure of the value set (see p.5.2).

This feature facilitates a representation of the semantics

of the problem and the application of generalization rules appropriate to the type of descriptors.

3. An expression in VL_{21} can have a truth status: TRUE, FALSE or ? (UNKNOWN).

The truth-status '?' provides an interpretation of a VL_{21} description in the situation, when, e.g., outcomes of some measurements are not known.

Definition 1: An *atomic function* is a variable, or a function symbol followed by a pair of parentheses which enclose a sequence of atomic functions and/or constants. Atomic functions which have a defined interpretation in the problem under consideration are called *descriptors*.

A *constant* differs from a variable or a function symbol in that its value set is empty. If confusion is possible, a constant is typed in quotes.

Examples

Constants 2 * red

Atomic forms: x_1 color(box) on-top(pl,p2) f(x₁,g(x₂))

Exemplary

value sets: $D(x_1) = \{0, 1, \dots, 10\}$

$D(\text{color}) = \{\text{red, blue}, \dots\}$

$D(\text{on-top}) = \{\text{true, false}\}$

$D(f) = \{0, 1, \dots, 20\}$

Definition 2: A *selector* is a form

[L # R]

(5)

where L - called *reference*, is an atomic function, or a sequence of atomic functions separated by '...' (The operator '#' is called the *internal conjunction*.)

$\neg$	?	?
	F	T
	T	F

$\wedge$	?	?	F	T
	?	?	F	?
	F	F	F	F
	T	?	F	T

$\vee$	?	F	T
	?	?	T
	F	?	T
	T	T	T

$\rightarrow$	?	F	T
	?	?	T
	F	T	T
	T	?	F

DEFINITION OF CONNECTIVES

 $\neg, \wedge, \vee$ AND $\rightarrow$

Figure 1.

Thus the truth-status of

 $\exists x(V)$ is

TRUE (FALSE) if there exists { does not exist } a value of x in the given situation for which the truth-status of V is TRUE

if it is not known whether there exists . . .

 $\forall x(V)$ is

TRUE (FALSE) if for every value of x in the situation the truth-status of V is { is not } TRUE

if it is not known whether for every . . .

A constant * ('irrelevant') is introduced to substitute for R , in a selector $[L = R]$, when R is the sequence of all possible values the L can take.

A VL expression in the form

$$QF_1, QF_2, \dots (P_1 \vee P_2 \vee \dots \vee P_1) \quad (7)$$

where QF_i is a quantifier form $\exists x_1, x_2, \dots$ or $\forall x_1, x_2, \dots$ and P_i is a conjunction of selectors (a term), is called a *disjunctive simple* VL expression (a DVL expression).

5. INFERENCE AND GENERALIZATION RULES

5.1 Interpretation of Inference Rules

An inference rule

$$\text{DESCRIPTION}_1 \rightarrow \text{DESCRIPTION}_2 \quad (8)$$

is used by applying it to situations. A situation is, in general, a source of information about values of variables and atomic functions in DESCRIPTION_1 (the condition part of the rule). A situation can, e.g., be a data base storing values of variables and procedures for evaluating atomic functions, or it can be an object on which various tests are performed to obtain these values.

A decision rule is viewed as a special case of an inference rule, when DESCRIPTION_2 (the consequence or decision part of the rule) is a constant, an elementary selector, or a product of elementary selectors involving decision variables (i.e., the DESCRIPTION_2 uniquely defines a decision or a sequence of decisions). The truth status of the condition and decision part of a rule, before applying it to a situation, is assumed to be UNKNOWN.

Let Q denote the set of all possible situations under consideration.

To characterize situations in Q , one determines a set S , called the *descriptor set*, which consists of variables, predicates and atomic functions (called, generally, *descriptors*), whose specific values can adequately characterize (for the problem at hand) any specific situation. We will assume here that the arguments of atomic functions are single variables, rather than other atomic functions. A situation is characterized by an event which is a sequence of assignments ($L := v$), where L is a variable or an atomic function with specific values of arguments, and v is a value of the variable or atomic function which characterizes the situation. It is assumed that each descriptor has defined a value s_v (domain) which contains all possible values the

descriptors can take for any situation in Q . Certain descriptors may not be applicable to some situations and therefore it is assumed that a descriptor in such cases takes value NA , which stands for *not applicable*. Thus, the domains of all descriptors always include by default the value NA . The set of all possible events for the given descriptor set S is called the *event space*, and denoted $\mathcal{E}(S)$. The domain of quantified variables are assumed to be determined by a given situation or object. E.g., if the quantified variable is a part, then its values are assumed to be individual parts of the object. In an event describing such an object, there will be a sequence of pairs $(L := V_l, l = 1, 2, \dots)$, where L is a quantified variable, and V_l stands for different values this variable takes in the object.

An event $e \in \mathcal{E}(S)$ is said to *satisfy* a selector $\{f(x_1, \dots, x_k) \# R\}$

iff the value of function f for values of $x_i, i = 1, 2, \dots, k$, as specified in the event e , is related to R by $\#$. For example, the event

$$e: (\dots x_5 := a_1, x_6 := a_2, f_{20}(a_1, a_2) := 5, \dots)$$

satisfies the selector:

$$[f_{20}(x_5, x_6) = 1, 3, 5]$$

A satisfied selector is assigned truth-status TRUE. If an event does not satisfy a selector then the selector is assigned truth-status FALSE. If an event does not have enough information in order to establish whether a selector is satisfied or not then the selector has UNKNOWN truth-status with regard to this event.

Let us assume first that the condition part of an inference rule is a quantifier-free formula. Interpreting the connectives $\neg, \wedge, \vee$, as described in figure 1, one can determine from the truth status of selectors the truth-status of the whole formula. An event is said to *satisfy* a rule, iff an application of the condition part of the rule to the event gives the formula truth-status TRUE. Otherwise, the event is said to *not satisfy* the rule.

Suppose now that the condition formula is in the form

$$\exists x(V)$$

An application of this formula to an event assigns status TRUE to the formula iff there exists in e a value assigned to x such that V achieves status TRUE. For example, the formula

$$\exists \text{part} [\text{color}(\text{part}) = \text{red}]$$

is satisfied by the event:

$$e = (\dots \text{part} := P1, \text{color}(P1) := \text{blue}, \text{part} := P2, \text{color}(P2) := \text{yellow}, \text{part} := P3, \text{color}(P3) := \text{red} \dots)$$

If the condition part is a form

$$\forall x(V)$$

then it is assigned status TRUE if every value of x in the event applied to it satisfies V .

If the condition part assumes truth-status TRUE then the decision part is assigned status TRUE. When the decision part reaches status TRUE then variables and functions which occur in it are assumed to have values which make this formula TRUE. These values may not, in general, be unique.

For example, suppose that V is a decision part with status TRUE:

$$V: [p(x_1, x_2) = 2][x_3 = 2..5][x_5 = 7]$$

V is interpreted as a description of a situation in which p has value 2 (if a specification of $p(x_1, x_2)$ is known, then from it we can infer what values of x_1 and x_2 might be), x_3 has a value between 2 and 5, inclusively, and x_5 has value 7. (Note that the formula does not give precise information about the value of x_3 .) After applying a formula to an event, the truth status of the condition and decision part returns to UNKNOWN. The role of an inference rule can then be described as follows: the rule is applied to an event, and if the event satisfies the condition part, then an assignment of values to variables

and functions is made as defined by the decision part. This assignment defines a new event (or a set of events which satisfy the decision part). Another inference rule can now be applied to this event (or set of events), and if satisfied by it (or by all of them), a new assignment of values to some variables and functions can be made.

Examples of VL inference rules:

$$\begin{aligned} [p(x_1, x_2) = 3] [q(x_2) = 2, 5] [x_7 \neq 0] &\Rightarrow [d(y_1) = 7] [p(y_1, y_2) = 2] \\ \exists x_3 ([p(x_1, x_3) = 2, \dots, 3] [q(x_7, x_3) \geq 2]) \vee [t(x_1) = 1] &\Rightarrow [d(y_1) = 7] \\ \text{TRUE} \Rightarrow [p(x_2, x_7) = 2] [x_7 = 2, 3, 5] \end{aligned}$$

5.2 Specification of the problem environment in the form of inference rules

Types of descriptors

The process of generalizing a description depends on the type of descriptors used in the description. The type of a descriptor depends on the structure of the value set of the descriptor. We distinguish here among three different structures of a value set:

1. Unordered

Elements of the domain are considered to be independent entities, no structure is assumed to relate them. A variable or function symbol with this domain is called *nominal* (e.g., blood-type).

2. Linearly Ordered

The domain is a linearly ordered set. A variable or function symbol with this domain is called *linear* (e.g., military rank, temperature, weight).

3. Tree Ordered

Elements of the domain are ordered into a tree structure. A superior node in the tree represents a concept which is more general than the concepts represented by the subordinate nodes (e.g., the superior of nodes 'triangle, rectangle, pentagon, etc. may be a 'polygon'). A variable or function symbol with such a domain is called *structured*.

Each descriptor (a variable or function symbol) is assigned its type in the specification of the problem. In the case of structured descriptors, the structure of the value set is defined by inference rules (e.g., see eqs. (13), (14), (15)).

In addition to assigning to each variable and function symbol a domain, one defines properties of variables and atomic functions characteristic for the given problem. They are represented in the form of inference rules. Here are a few examples of such properties.

1. Restrictions on Variables

Suppose that we want to represent a restriction on the event space saying that if a value of variable x_1 is 0 ('a person does not smoke'), then the variable x_3 is 'not applicable' (x_3 - kind of cigarettes the person smokes). This is represented by a rule:

$$[x_1 = 0] \Rightarrow [x_3 = NA]$$

NA = not applicable

2. Relationships Between Atomic Functions

For example, suppose that for any situation in a given problem, the atomic function $f(x_1, x_2)$ is always greater than the atomic function $g(x_1, x_2)$. We represent this:

$$T \Rightarrow \forall x_1, x_2 [f(x_1, x_2) > g(x_1, x_2)]$$

3. Properties of Predicate Functions

For example, suppose that a predicate function is transitive. We represent this:

$$\forall x_1, x_2, x_3 ([\text{left}(x_1, x_2)] [\text{left}(x_2, x_3)]) \Rightarrow [\text{left}(x_1, x_3)]$$

Other types of relationships characteristic for the problem environment can be represented similarly.

5.3. Generalization rules

The transformation of data rules (3) into hypotheses (4) can be viewed as a process of applying certain *generalization rules* to data rules. A generalization rule transforms one or more decision rules associated with the same *generalization class* (which, in our case, is the same as recognition class), into a new decision rule, which is equivalent to or more general than the initial rules.

A decision rule

$$V ::= K$$

(9)

is equivalent to a set of decision rules

$$\{V_i ::= K, i = 1, 2, \dots\}$$

(10)

If any event which satisfies at least one of the V_i , $i = 1, 2, \dots$, satisfies also V , and conversely. If the converse is not required, the rule (9) is said to be *more general than* (10).

The generalization rules are applied to data rules under the condition of preserving consistency and completeness, and achieving optimality according to the preference criterion. A basic property of a generalization transformation is that the resulting rule has UNKNOWN

truth-status (is a *hypothesis*): its truth-status has to be tested on new data.

Below is a list of a few basic generalization rules (K denotes a generalization class).

Non-constructive rules:

(1) Dropping Condition Rule

If a description is a logical product of conditions which must be satisfied, then one way to generalize it is to drop one or more of these conditions. For example:

one or
more
rules

[size (box)=small][color (box)=blue] ::= K | K [size (box)=small] ::= K
This reads: the description 'small and blue box' can be generalized to 'small box'. ($|$ is the generalization operator.)

In general this rule can be expressed:

$$W [L=R] ::= K \mid W ::= K$$

where W - an arbitrary description. This rule is generally applicable (the type of L does not matter).

(11) Turning Constants to Variables Rule

When we have two or more descriptions, each referring to a single object in a class, and the descriptions differ in having different constants in the same predicate, then they can be generalized into one description with an existentially quantified variable in the place of the constants:

$$\left\{ \begin{array}{l} W[p(a,y)] ::= K \\ W[p(b,y)] ::= K \\ \vdots \\ W[p(i,y)] ::= K \end{array} \right\} \mid \exists x W[p(x,y)] ::= K$$

where p - is a predicate

x - stands for one or more arguments of the predicate p

For example:

$$\left\{ \begin{array}{l} [\text{INSIDE (ball, box)}] ::= K \\ [\text{INSIDE (cup, box)}] ::= K \end{array} \right\} \mid \exists x [\text{INSIDE (x, box)}] ::= K$$

The generalization (on the right of $|$) states that if an object is a BOX which has something inside, then it belongs to class K .

This rule together with the dropping condition rule are two basic generalization rules used in the literature on

computer induction. Both these rules can, however, be viewed as special cases of the following rule.

(iii) *Generalization by Internal Disjunction*
(The Extending Reference Rule)

A description can be generalized by extending the set of values that a descriptor (a variable, predicate or a function) is allowed to take on in order than an object satisfies the description. This extension is expressed by the *internal disjunction* (def. 2) (i.e., logical OR involving values of the same variable). For example:

$W[\text{color}(\text{wall}) = \text{blue}] ::= K \mid K \mid W[\text{color}(\text{wall}) = \text{blue}, \text{red}, \text{green}, \dots] ::= K$
 $W[\text{color}(\text{wall}) = \text{red}] ::= K \mid K \mid W[\text{color}(\text{wall}) = \text{blue}, \text{red}, \text{green}, \dots] ::= K$
 (The "*" denotes internal disjunction)

In general we have:

$W[L = R_1] ::= K \mid K \mid W[L = R_2] ::= K$

where L - is an atomic function

R_1, R_2 - references, i.e., subsets of values from the domain of L expressed as internal disjunction and $R_1 \subseteq R_2$.

Although the internal disjunction seems at first glance to be just a notational abbreviation, this operation is one of fundamental operations people use in generalizing descriptions. In addition to the previous two rules, there are two more important special cases of this rule. First, when the descriptor involved takes on values which are linearly ordered (a linear descriptor) and the second when the descriptor takes on values which are natural language concepts representing different levels of generality (a structured descriptor).
 In the case of a linear descriptor we have:

(iv) *Closing Internal Rule*

For example, suppose two objects of the same class have all the same characteristics except that they have different sizes, a and b . Then, it is plausible to hypothesize that all objects which share these characteristics but which have sizes between a and b are also in this class.

$W[\text{size}(x) = a] ::= K \mid K \mid W[\text{size}(x) = a..b] ::= K$
 $W[\text{size}(x) = b] ::= K \mid K \mid W[\text{size}(x) = a..b] ::= K$

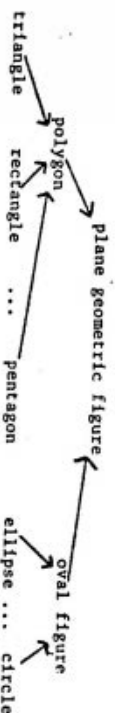
In general:

$W[L = a] ::= K \mid K \mid W[L = a..b] ::= K$
 $W[L = b] ::= K \mid K \mid W[L = a..b] ::= K$

This rule is applicable only when L is a linear descriptor. In the case of structured descriptors we have:

(v) *Climbing Generalization Tree Rule*

Suppose the value set of the shape descriptor is the tree (in general it could be a partially ordered set):



With this tree structure, values such as triangle and rectangle can be generalized (by climbing the generalization tree) into a polygon:

$[\text{shape}(x) = \text{rectangle}] ::= K \mid K \mid [\text{shape}(x) = \text{polygon}] ::= K$
 $[\text{shape}(x) = \text{triangle}] ::= K \mid K \mid [\text{shape}(x) = \text{polygon}] ::= K$

A general rule is:

one or more rules

$$\left\{ \begin{array}{l} W[L = a] ::= K \\ W[L = b] ::= K \\ \vdots \\ W[L = i] ::= K \end{array} \right\} \mid K \mid [L = a] ::= K$$

where L is a structured descriptor

a - represents the superior node (a concept at the next 'level of generality') of nodes a,b,...and L, in the tree domain of L.

The rule is applicable only to selectors involving structured descriptors. This rule has been used, e.g., in [3], [4], [26].

(v) Extension Against Rule

This rule applies when a description is being generalized in the presence of another description, representing the 'negative examples' of the given recognition class. The latter description provides an obvious limit for the generalization of the given description, since these two descriptions should not intersect in order to avoid inconsistency. For example:

$$\begin{aligned} & \text{XP}_1, \text{P}_2([\text{on-top}(\text{P}_1, \text{P}_2)][\text{color}(\text{P}_1)=\text{red})] \Rightarrow K \\ & \text{XP}_1, \text{P}_2([\text{left-of}(\text{P}_1, \text{P}_2)][\text{color}(\text{P}_1)=\text{green})] \Rightarrow K' \\ & \text{XP}_1[\text{color}(\text{P}_1) \neq \text{green}] \Rightarrow K \end{aligned}$$

The description produced by the rule: 'There exists P₁ whose color is not green' is the most general statement which satisfies both premises on the left of K'.

In general the rule states:

$$\begin{aligned} & \text{W}_1[\text{L} = \text{R}_1] \Rightarrow K \\ & \text{W}_2[\text{L} = \text{R}_2] \Rightarrow K' \\ & \text{where } \text{R}_1 \cap \text{R}_2 = \emptyset \\ & \text{W}_1 \text{ and } \text{W}_2 - \text{arbitrary descriptions.} \end{aligned}$$

This rule is very useful in generating the *discriminant descriptions* of object classes (see next section). It is one of the basic rules used in the inductive program AQUA/L [14], whose version is used as a basic procedure in program INDUCE 1.1 described in section 6.2.

Constructive Rules:

Constructive rules generate generalized descriptions of the data rules in terms of certain new descriptors (*metadescriptors* or *derived descriptors*). There can be very many such rules. We will restrict ourselves here to two examples. Some constructive rules are encoded as specialized procedures.

(v1) the *counting* rule

$$W[\text{attribute}_1(\text{P}_1)=A] \dots [\text{attribute}_1(\text{P}_k)=A][\text{attribute}_1(\text{P}_{k+1}) \neq A] \dots$$

$$\dots [\text{attribute}_1(\text{P}_r) \neq A] \Rightarrow K \quad K' [\#P_attribute_A=k] \Rightarrow K$$

where P₁, P₂, ..., P_k, ..., P_r - are constants denoting, e.g., parts of an object

attribute₁ - stands for a certain attribute of P₁'s, e.g., color, size, texture, etc.

#P_attribute₁A - denotes a new descriptor interpreted as the 'number of P₁'s (e.g., parts) with attribute equal A'.

Example:

$$\begin{aligned} & W[\text{color}(\text{P}_1)=\text{RED}][\text{color}(\text{P}_2)=\text{RED}][\text{color}(\text{P}_3)=\text{BLUE}] \Rightarrow K \\ & K' [\#P_color_red=2] \Rightarrow K \end{aligned}$$

(The above is a formalization rule, because a set of objects with any two red parts is a superset of a set of objects with two parts which are red and one part which is blue)

(viii) the *generating chain properties* rule

If the arguments of different occurrences of the same relation in an event are linearly ordered by the relation (e.g., are objects ordered linearly by a relation ABOVE, LEFT-OF, NEXT-TO, CONTAINS, etc.), that is form a *chain*, the rule generates descriptors which characterize various objects in the chain, for example

LAST-object : the 'least object', i.e., the object at the beginning of the chain (e.g., the bottom object in the case of relation ABOVE)

MSI-object : the 'most object', i.e., the object at the end of the chain

MDI-object : the 'middle' object

lth-object : the lth object in the chain

or characterize the chain itself, for example the *chain-length*.

5.4 The preference criterion

The preference criterion defines what is the desired solution to the problem, i.e., what kind of hypotheses are being sought. The question of what should be the preference criterion is a broad subject beyond the scope of the paper. We will, therefore, discuss here only the underlying ideas behind the presented approach. First, we disagree with many authors who seem to be searching for one universal criterion which should guide induction. Our position is that there are many dimensions, independent and interdependent, on which a hypothesis can be evaluated. The weight given to each dimension depends on the ultimate use of the hypothesis. Among these dimensions are various forms of simplicity of a hypothesis (e.g., the number of operators in it, the quantity of information required to encode a hypothesis using operators from an a priori defined set [31], etc.), the scope of the hypothesis, which relates the events predicted by a hypothesis to the events actually

observed (e.g., the 'degree of generalization' [13], the 'precision' [31]), the cost of measuring the descriptors in the hypothesis, etc. Therefore, instead of defining a specific criterion, we specify only a general form of the criterion. The form permits a user to define various specific criteria to the inductive program, which are appropriate to the application. The form, called a 'lexicographic functional' consists of an ordered list of criteria (of dimensions of hypothesis quality) and a list of 'tolerances' for these criteria [13, 14].

An important and somewhat surprising property of such an approach is that by properly defining the preference criterion, the same computer program can produce either the *characteristic* or *discriminant* descriptions of object classes. The *characteristic* description specifies the common properties shared by the objects of the same class (most work on induction considers only this type of descriptions, e.g., [3], [6], [22]), while the *discriminant* description specifies only the properties necessary for distinguishing the given class from all the other classes (Michalski [13, 32], Larson [28]).

5.5 Arithmetic descriptors

In addition to initial linear descriptors used in the data rules, new linear descriptors can be formulated as arithmetic functions of the original ones. These descriptors are formulated by a human expert as suggestions to the program.

6. OUTLINE OF ALGORITHM AND OF COMPUTER IMPLEMENTATION

In this section we outline the top level algorithm for rule induction and its implementation in the computer program INDUCE-1.1 ([26][29][30]). The algorithm is illustrated by an example.

INDUCE-1.1 is considered to be only an *aid* to rule induction. Its successful application to practical problems requires a cooperation between the program and an expert, whose role is to determine the initial set of descriptors, to formulate data rules and the problem knowledge rules, to define the preference criterion and other parameters, evaluate the obtained rules, repeat the process if desired, etc.

6.1 Computer representation of VL decision rules

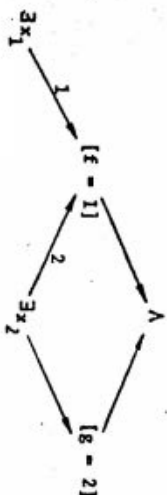
Decision rules are represented as graphs with labeled nodes and labeled directed arcs. A label on a node can be:

- a selector with a descriptor without the argument list,
- a logical operation,
- a quantifier form $\exists x$ or $\forall x$.

Arcs link arguments with selectors or descriptors, and are labeled by 0,1,2,... to specify the position of an argument in the descriptor indicated at the head of the arc (0 indicates that the order of arguments is not important).

Several different types of relations may be represented by an arc. The type of relation is determined by the label on the node at each end of the arc. The types of relations are: 1) functional dependence, 2) logical dependence, 3) implicit variable dependence, 4) scope of variables.

Figure 2 gives a graph representing a VL_{21} expression. The two arcs connected to the logical operation (A) represent the logical dependence of the value of the formula on the values of the two selectors. The other arcs in the figure represent the functional dependence of f on x_1 and x_2 , and g on x_2 .



VL Graph Structure: $\exists x_1 x_2 ([f(x_1, x_2) = 1][g(x_2) = 2])$

6.2. Outline of the Top Level Algorithm

The implementation of the inductive process in the program INDUCE-1.1 was based on ideas and algorithms adopted from the earlier research on the generalization of VL_1 expressions (Michalski [13,32], and some new ideas and algorithms developed by Larson [28,29]).

The top level algorithm (in somewhat simplified form) can be described as follows:

- At the first step, the data rules (whose condition parts are in the disjunctive simple forms) are transformed to a new set of rules, in which condition parts are in the form of *c-expressions*. A *c-expression* (a *conjunctive expression*) is a product of selectors accompanied by one or more quantifier forms, i.e., forms $QFx_1, x_2, \dots$, where QF denotes a quantifier. (Note, that due to the use of the internal disjunction and quantifiers, a *c-expression* represents a more general concept than a conjunction of predicates (used, e.g., in [23][24]).

- A decision class is selected, say K_1 , and all *c-expressions* associated with this class are put into a set $F1$, and all remaining *c-expressions* are put into a set $F0$ (the set $F1$ represents events to be covered, and set $F0$ represents constraints, i.e., events not to be covered).

- By application of inference rules (describing the problem environment), constructive generalization rules, and rules generating arithmetic descriptors (sec.5.5), new selectors are generated. The 'most promising' selectors (according to a certain criterion) are added to the *c-expressions* in $F1$ and $F0$.

- A *c-expression* is selected from $F1$, and a set of consistent generalizations (a *restricted star*) of this expression is obtained. This is done by starting with single selectors (called 'seeds'), selected from this *c-expression* as the 'most promising' ones (according to the preference criterion). In each

subsequent step, a new selector is added to the c-expression obtained in the previous step (initially the seeds), until a specified number (parameter NCONSIST) of consistent generalizations is determined. Consistency is achieved when a c-expression has NULL intersection with the set F0. This 'rule growing' process is illustrated in Fig. 3.

5. The obtained c-expressions, and c-expressions in F0, are transformed to two sets E1 and E0, respectively, of V_L events (i.e., sequences of values of certain discrete variables).

A procedure for generalizing V_L descriptions is then applied to obtain the 'best cover' (according to a user defined criterion) of set E1 against E0 (the procedure is a version of AQVAL/1 program [13]).

During this process, the extension against, the closing the interval and the climbing generalization tree rules are applied.

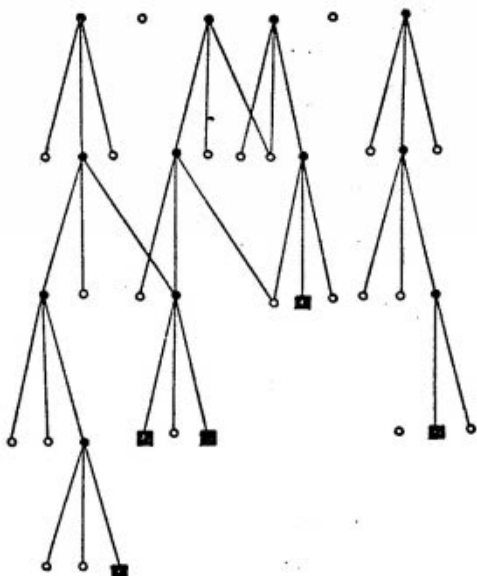
The result is transformed to a new set of c-expressions (a restricted star) in which selectors have now appropriately generalized references.

6. The 'best' c-expression is selected from the restricted star.

7. If the c-expression completely covers F1, then the process repeats for another decision class. Otherwise, the set F1 is reduced to contain only the uncovered c-expressions, and steps 4 to 7 are repeated.

The implementation of the inductive process in INDUCE-1.1 consists of a large collection of specialized algorithms, each accomplishing a certain task. Among the most important tasks are:

1. the implementation of the 'rule growing process'
2. testing whether one c-expression is a generalization of ('covers') another c-expression. This is done by testing for subgraph isomorphism.



- - a discarded c-rule
- ◻ - an active c-rule
- - a terminal node denoting a consistent c-rule

Each arc represents an operation of adding a new selector to a c-rule

The branching factor is determined by parameter ALTER. The number of active rules (which are maintained for the next step of the rule growing process) is specified by parameter MAXSTAR. The number of terminal nodes (consistent generalizations) which program attempts to generate is specified by parameter NCONSIST.

Illustration of the rule growing process
(an application of the Dropping Condition Rule in reverse order)

Figure 3

3. generalization of a c-expression by extending the selector references and forming irredundant c-expressions (includes application of AQUA/L1 procedure).

4. Generation of new descriptors and new selectors.

Program INDUCE 1.1 has been implemented in PASCAL (for Cyber 175 and DEC 10); its complete description is given in [30].

6.3. Example

We will present now an example illustrating some of the features of INDUCE-1.1. Suppose given are two sets of trains, Eastbound and Westbound, as shown in Fig. 4. The problem is to determine a concise (logically sufficient) description of each set of trains, which distinguishes one set from the other (i.e., a discriminant description which contains only necessary conditions for distinguishing between the two sets).

As the first step, an initial set of descriptors is determined (by a user) for describing the trains. Eleven descriptors are selected in total.

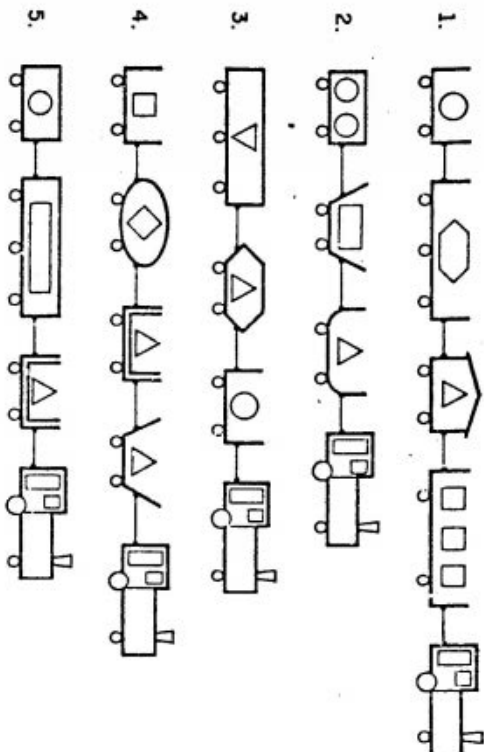
Among them:

- $\text{infront}(\text{car}_i, \text{car}_j)$ - car_i is in front of car_j (a nominal descriptor)
- $\text{length}(\text{car}_i)$ - the length of car_i (a linear descriptor)
- $\text{car_shape}(\text{car}_i)$ - the shape of car_i (a structured descriptor with 12 nodes in the generalization tree; see eqs. (13) and (14))
- $\text{cont_load}(\text{car}_i, \text{load}_j)$ - car_i contains load_j (a nominal descriptor)
- $\text{load_shape}(\text{load}_i)$ - the shape of load_i (a structured descriptor)

The value set:

- circle
- hexagon
- triangle
- polygon
- rectangle

1. EASTBOUND TRAINS



2. WESTBOUND TRAINS

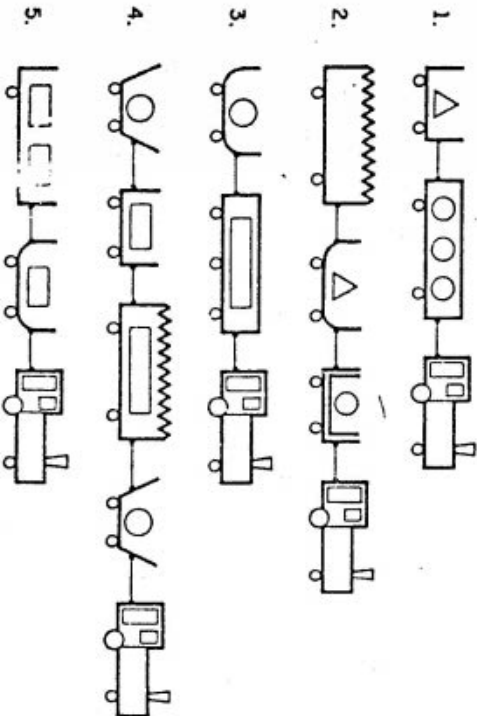


Figure 4

- $\text{nrpts-load}(\text{car}_1)$ - the number of parts in the load of car_1
(a linear descriptor)
- $\text{nrwheels}(\text{car}_1)$ - number of wheels in car_1
(a linear descriptor)

The data rules consist of descriptions of the individual

trains in terms of the selected descriptors, together with the

specification of the train set they belong to. For example, the data

rule describing the second eastbound train is:

$\text{Zcar}_1, \text{car}_2, \text{car}_3, \text{car}_4, \text{load}_1, \text{load}_2, \dots$

$[\text{infront}(\text{car}_1, \text{car}_2)] [\text{infront}(\text{car}_2, \text{car}_3)] \dots [\text{length}(\text{car}_1) = \text{long}] \wedge$

$[\text{car-shape}(\text{car}_1) = \text{engine}] [\text{car-shape}(\text{car}_2) = \text{U-shaped}] [\text{cont-load}(\text{car}_2, \text{load}_1)] \wedge$ (12)

$[\text{load-shape}(\text{load}_1) = \text{triangle}] \dots [\text{nrwheels}(\text{car}_3) = 2] \dots :: \{ \text{class} = \text{Eastbound} \}$

Rules describing the problem environment in this case are only

rules defining structures of structured descriptors (arguments of descriptors are omitted):

$[\text{car-shape} = \text{open rectngl}, \text{open trapezoid}, \text{U-shaped}, \text{dbl open rectngl}] \rightarrow$ (13)

$[\text{car-shape} = \text{open top}]$

$[\text{car-shape} = \text{ellipse}, \text{closed rectngl}, \text{jagged top}, \text{sloping top}] = [\text{car-shape} = \text{closed top}]$ (14)

$[\text{load-shape} = \text{hexagon}, \text{triangle}, \text{rectangle}] \rightarrow [\text{load-shape} = \text{polygon}]$ (15)

The criterion of preferences was to minimize the number of rules

(c-expressions) in describing each class, and, with secondary priority,

to minimize the number of selectors in each rule.

Rules of constructive generalization included in the program are able to construct, among other descriptors, such descriptors as the length of a chain, properties of elements of a chain, number of objects satisfying a certain relation, etc. For example, from the data rule (12), the constructive generalization rules can produce new selectors such as:

* At this moment, before proceeding further, the reader is advised to look at the pictures and to try to solve this problem on his/her own.

$[\text{nr cars} = 4]$ - the number of cars in the train is 4
(the length of chain defined by relation infront)

$[\text{nr cars-length} = \text{long} = 1]$ - the number of long cars is 1 (the engine)

$[\text{nr pts-load}(\text{last-car}) = 2]$ - the number of parts in the load of the last car is 2

$[\text{position}(\text{car}_1) = 1]$ - the position of car_1 is 1

Suppose that eastbound trains are considered first. The set F1 contains then all c-expressions describing eastbound trains, and F0, all c-expressions describing westbound trains. The description

e is selected from F1 (suppose it is the above description of the second eastbound train), and supplemented by 'most promising' metadescriptors generated by problem environment rules and constructive generalization

rules. In this case, the metaselector $[\text{shape}(\text{last-car}) = \text{rectangle}]$ is added to e. Next, a set G (a restricted star) of certain number (NCONSIST) of

consistent generalizations of e is determined.

This is done by forming a sequence of partial stars (a partial star may include inconsistent generalizations of e). If an element of a partial star is consistent, it is placed into the set G. The initial partial star (P_1) contains the set of all selectors of e_1 . This partial star and each subsequent partial star is reduced according to a user specified preference criterion to the 'best' subset, before a new partial star is formed. The size of the subset is controlled by a parameter called MAXSTAR. A new partial star P_{i+1} is formed from an existing partial star P_i in the following way: for each c-expression in P_i , a set of c-expressions is placed into P_{i+1} , each new c-expression containing the selectors of the original c-expression plus one new selector from e, which is not in the original c-expression. Once a sufficient number of consistent generalizations have been formed, a version of the AQUA/1. program (Michalski [13]) is

applied to extend the references of all selectors in each consistent generalization. As the result, some selectors may be removed and some may have more general references.

In the example, the best subset of selectors of e (i.e., the

reduced partial star (P_1) was:

$zcar_1[car_shape(car_1)=U-shaped]$ (16)

$zcar_1[car_shape(car_1)=open-trapezoid]$ (17)

$zcar_1[car_shape(car_1)=rectangle]$ (18)

$[car_shape(last-car)=rectangle]$ (19)

The last c-expression is consistent (has empty intersection with c-expressions in P_0) and, therefore, is placed in G . From the remaining,

a new partial star is determined. This new partial star contains a

consistent generalization:

$zcar_1[car_shape(car_1)=rectangle][length(car_1)=short]$ (20)

which is added to G . Suppose G is restricted to have only two elements (NCONSIST=2). Now, the program AQVAL/1 is applied to generalize references of the selectors in c-expressions of G , if it leads to an improvement (according to the preference criterion).

In this case, a generalization of (20) produces a consistent and

complete generalization:

$zcar_1[car_shape(car_1)=closed-top][length(car_1)=short]$ (21)

(the generalization of (19), $[car_shape(last-car)=polygon]$, is not complete; it does not cover all P_1).

In this example, only 2 partial stars were formed, and two consistent generalizations were created. In general, a set of consistent generalizations is created through the formation of several partial stars. The size of each partial star and the number of alternative generalizations are controlled by user supplied parameters.

Assuming a larger value of NCONSIST, and applying the above

procedure to both decision classes, the program INDOCE-1.1 produced the following alternative descriptions of each set of trains:

(The selectors or references underlined by a dotted line were generated by application of constructive generalization rules or problem environment rules).

Eastbound trains:

$zcar_1[length(car_1)=short][car_shape(car_1)=closed-top]:::[class=Eastbound]$ (22)

(the same as (21)). It can be interpreted:

If a train contains a car which is short and has a closed top, then it is an eastbound train.

Alternatively,

$zcar_1[car_1,load_1,load_2][infront(car_1,car_2)][count-load(car_1,load_1)]$

$A[count_load(car_2,load_2)][load_shape(load_1)=triangle]$

$A[load_shape(load_2)=polygon]:::[class=Eastbound]$ (23)

It can be interpreted:

If a train contains a car whose load is a triangle, and the load of the car behind is polygon, then the train is eastbound.

Westbound trains:

$[incars=3] V zcar_1[car_shape(car_1)=jagged-top]:::[class=Westbound]$ (24)

$zcar_1[incars, length, long=2][position(car_1)=3][shape(car_1)=open-top,jagged-top]$ (25)

$:::[class=Westbound]$

It is interesting to note that the example was constructed with rules (23) and (24) in mind. The rule (22) found by the program as an alternative was rather surprising because it seems to be conceptually simpler than rule (23). This shows that the combinatorial part of an induction process can be successfully handled by a computer program, and, therefore, programs like the above have a potential to serve as an aid to induction processes in various applied sciences.

7. SUMMARY

We have presented an outline of a theory and an implementation method which views pattern recognition as a rule-guided inductive inference. The initial data rules (examples) are transformed to general recognition rules by an application of *generalization rules* and *problem knowledge rules*, under the control of a *preference criterion*. The implemented method (in the form of computer program INDUCE 1.1):

- applies different generalization rules according to the type of descriptors in the data (nominal, linear, structured)
- takes into consideration the properties of the interrelationships of descriptors characteristic to the recognition problem
- permits the specification by a user of a preference criterion, which evaluates the usefulness of the rules from the viewpoint of the given application
- can generate certain new descriptors ('metadescriptors') and blend them with the initial ones to provide a basis from which the final description chooses its most appropriate descriptors
- uses the same representation language (VL_{21}) to describe the learning events as well as problem knowledge rules, which simplifies for a user the task of the data preparation for the program
- permits a user to suggest to the program various arithmetic transformations of the original (linear) variables which seem promising as relevant characterization of object classes.

The implemented method has many limitations. Among major limitations is a restricted form in which program can express the recognition rules (i.e., in the form of a disjunctive simple VL_{21} expression with limited use of quantifiers), and a restricted number of operators and mechanisms which the program uses in constructing a generalized description. Also, the method does not take into consideration any probabilistic information.

Among the advantages is the significant generality of the approach and an ability to use the semantics underlying the recognition problem. An important property of the method is the simplicity of conceptual interpreta-

tion of the pattern recognition rules. The strength of the method was illustrated by a testing example where program was able to discover a pattern unknown to the authors. On the practical side, an earlier program (AQ11) was able to determine from examples the rules for diagnosis of soybean diseases which gave better performance than the rules obtained by representing an expert's knowledge [33].

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