THE LOGIC OF PLAUSIBLE REASONING: AN ADVANCED REPORT

by

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Reports of the Intelligent Systems Group, ISG 86-3, UIUCDCS-F-86-951, Department of Computer Science, University of Illinois, Urbana, February 1986.

UILU-ENG-86-1775

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ISG 85-17

File No. UIUCDCS-F-86-951

Submitted to Cognitive Science

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TABLE OF CONTENTS

1. BACKGROUND FOR THE THEORY 1

2. ASSUMPTIONS UNDERLYING THE THEORY 8

3. PRIMITIVES IN THE CORE SYSTEM, 15

4. TRANSFORMS ON STATEMENTS 20

4.1 Certainty Parameters Affecting Transforms on Statements 21
4.2 Formal Representation of Transforms on Statements 23

5. OTHER INFERENCES IN THE CORE THEORY 27

6. CONCLUSION 32

7. ACKNOWLEDGEMENTS 33

8. REFERENCES 34

1. BACKGROUND FOR THE THEORY

The goal of our research on plausible reasoning is to develop a formal system based on Michalski's variable-valued logic calculus (1980, 1983) that characterizes different patterns of plausible inference humans use in reasoning about the world (Polya, 1958; Collins, 1978a). Our work attempts to formalize the plausible inferences that frequently occur in people's responses to questions for which they do not have ready answers (Carbonell & Collins, 1973; Collins, 1978a,b; Collins, Warnock, Aiello, & Miller, 1975). In this sense it is a major departure from formal logic, which represents normative theories of reasoning. Being descriptively based, it includes a variety of inference patterns that do not occur in formal logic-based theories. The central goals of the theory are to discover recurring general patterns of plausible inferences and to determine the parameters affecting the certainty of these inferences.

In order to analyze human plausible reasoning, Collins (1978b) collected a large number of people's answers to everyday questions, some from teaching dialogues and some from asking difficult questions to four subjects. These answers have the following characteristics:

1. There are usually several different inference patterns used to answer any question.
2. The same inference patterns recur in many different answers.
3. People weigh different evidence that bears on their conclusion.
4. People are more or less certain about their conclusion depending on the certainty of their information (either from some outside source or from memory), the certainty of the inference patterns and associated parameters used, and on whether different patterns lead to the same or opposite conclusions.

The analysis of the answers attempts to account for the reasoning and the conclusions drawn in terms of a taxonomy of plausible inference patterns. As will be evident, this is an inferential analysis. To use Chomsky's (1965) felicitous terms, we are trying to construct a deep structure theory from the surface structure traces of the reasoning process.

We will illustrate some of the characteristics of people's answers, as well as some of the inference patterns formulated in the theory with several transcripts. The first transcript comes from a teaching dialogue on South American geography (Carbonell & Collins, 1973) (T stands for teacher and S for student):

T. There is some jungle in here (points to Venezuela) but this breaks into a savanna around the Orinoco (points to the Llanos in Venezuela and Colombia).

S. Oh right, is that where they grow the coffee up there?

T. I don't think that the savanna is used for growing coffee. The trouble is the savanna has a rainy season and you can't count on rain in general. But I don't know. This area around Sao Paulo (in Brazil) is coffee region, and it is sort of getting into the savanna region there.

In the protocol the teacher went through the following reasoning. Initially, the teacher made a hedged "no" response to the question for two reasons. First, the teacher knew that coffee growing depends on a number of factors (e.g., rainfall, temperature, soil, and terrain), and that savannas do not have the correct value for growing coffee on at least one of those factors (i.e., reliable rainfall). In the theory this is an instance of the inference pattern called a *derivation from a mutual implication*. Second, the teacher did not know that the Llanos was used for growing coffee, which he implicitly took as evidence against its being a coffee region. The inference takes the form "I would know the Llanos produces coffee if it did, and I don't know it, so probably it does not." This is called a *lack-of-knowledge inference* (Collins et al., 1975; Gentner & Collins, 1982). This inference pattern is based on knowledge about one's own knowledge and hence is a meta-knowledge inference.

Then the teacher backed off his initial negative response, because he found positive evidence. In particular, he thought the Brazilian savanna might overlap the coffee growing region in Brazil around Sao Paulo, and therefore might produce coffee. If the Brazilian savanna produces coffee, then by functional analogy (called a *similarity transform* in our theory) the Llanos might. Hence, the teacher ended up saying "I don't know," even though his original conclusion was correct.

The teacher's answer exhibits a number of the important aspects of human plausible reasoning. In general, a number of inference patterns are used together to derive an answer. Some of these are inference chains where the premise of one inference draws on the conclusion of another inference. In other cases the inference patterns are triggered by independent sources of evidence. When there are different sources of evidence, the subject weighs them together to determine a conclusion and the strength of belief in it.

It is also apparent in this protocol how different pieces of information are found over time. What appears to happen is that the subject launches a search for relevant information (Quillian, 1968; Collins & Loftus, 1975). As relevant pieces of information are found (or are found to be missing), they trigger particular inferences. Which inference pattern is applied is determined by the relation between the information found and the question asked. For the question about growing coffee in the Llanos, if the respondent knew that savannas are in general good for growing coffee, that would trigger a deductive inference. If the respondent knew of a similar savanna somewhere that produced coffee, that would trigger an analogical inference. The search for information is such that the most accessible information is found first, as by a marker passing or spreading activation algorithm (Charniak, 1982; Quillian, 1968).

In the protocol, the more accessible information about the unreliable rainfall in savannas was found before the less accessible information about the coffee growing region in Brazil and its relation to the Brazilian savanna. The order of finding information reflects its decreasing accessibility as activation spreads through a semantic network (Quillian, 1968). Relevant information is found by autonomous search processes, and the particular information found determines what inferences are triggered.

The next protocol illustrates a plausible deduction, called a *specialization transform* in the theory (Q stands for questioner and R for respondent):

Q. Is Uruguay in the Andes Mountains?

R. I get mixed up on a lot of South American countries (pause). I'm not even sure. I forget where Uruguay is in South America. It's a good guess to say that it's in the Andes Mountains because a lot of the countries are.

The respondent knew that the Andes are in most South American countries (7 out of 9 of the Spanish speaking countries). Since Uruguay is a fairly typical South American country, he guesses that the Andes may be there too. He is wrong, but the conclusion was quite plausible. This example illustrates a *specialization transform* and two of the certainty parameters associated with it : *frequency* (he knows the Andes are in most countries), and *typicality* (Uruguay is a typical South American country).

The third protocol illustrates another kind of plausible deduction, called a *derivation from mutual implication* in the theory:

Q. Do you think they might grow rice in Florida?

R. Yeah, I guess they could, if there were an adequate fresh water supply. Certainly a nice, big, warm, flat area.

The respondent knew that whether a place can grow rice depends on a number of factors. He also knew that Florida had the correct values on at least two of these factors (warm temperatures and flat terrain). He therefore inferred that Florida could grow rice if it had the correct value on the other factor he thought of (i.e., adequate fresh water). He may or may not have been aware that rice growing also depends on fertile soil, but he did not mention it here. Florida in fact does not produce rice in any substantial amount, probably because the soil is not adequate. This protocol shows how people make plausible inferences based on their approximate knowledge about what depends on what, and how the certainty of such inferences is a function of the degree of dependency between the variable in question (rice) and the known variables (i.e. terrain, climate, water).

The fourth protocol from a teaching dialogue illustrates a functional analogy, called the *similarity transform* in the theory:

S. Is the Chaco the cattle country? I know the cattle country is down there (referring to Argentina).

T. I think it's more sheep country. It's like western Texas, so in some sense I guess it's cattle country. The cattle were originally in the Pampas, but not so much anymore.

As in the first protocol, the respondent is making a number of plausible inferences in answering this question, some of which lead to different conclusions. First, he thinks that the Chaco is used for sheep raising, but there is some uncertainty about the information retrieved, which leads to a hedged response. This supports an implicit *lack-of-knowledge inference* (a meta-knowledge inference), that takes the form "I don't know that it's cattle country, and I would know if it were (e.g., I know about sheep), so it probably is not cattle country." But then the teacher noted a similarity between the Chaco and western Texas, presumably in terms of the functional determinants of cattle raising (e.g., climate, vegetation, terrain). This led him to a very hedged affirmative response, based on a *similarity transform*. Finally the teacher alluded to the fact that the Pampas is the place in Argentina known for cattle, and the place the student most likely was thinking of. This argues against the Chaco having cattle based on another meta-knowledge inference, a *confusability inference* (Collins, 1978b): "The Chaco is confusable with the Pampas and the Pampas has cattle, so the fact that there are cattle in Argentina cannot be taken as evidence for cattle in the Chaco." In answering this question, then, two patterns of plausible inference led to a negative conclusion and one to a positive conclusion.

The fifth protocol illustrates both a *similarity* and a *dissimilarity transform*, and more importantly, the distinction between inferences based on overall similarity and those based on similarity with respect to the functional determinants of the property in question.

Q. Can a goose quack?

R. No, a goose – well, its like a duck, but its not a duck.

It can honk, but to say it can quack. No, I think its
vocal cords are built differently. They have a beak
and everything, but no, it can't quack.

The *similarity transform* shows up in the phrases, "it's like a duck" and "They have a beak and everything" as well as the initial uncertainty about the negative conclusion. It takes the form, "A duck quacks and goose is like a duck with respect to most features, so maybe a goose quacks". The certainty of the inference depends on the degree of similarity between ducks and geese.

But then two lines of negative inference led the respondent to a negative conclusion. First there is a lack-of-knowledge inference implicit in the statement "It can honk, but to say it can quack." She knew about geese honking but not about their quacking. Therefore, she thought she would know about geese quacking, if in fact they did quack.

The second line of negative inference (apparently found after she started answering) is the dissimilarity inference evident when she says, "I think its vocal cords are built differently". The dissimilarity inference takes the form "Ducks quack, geese are dissimilar to ducks with respect to vocal cords, and vocal cords determine the sound an animal makes, so probably geese do not quack". This inference was enough to lead her to a strong "no". Of course she knew nothing about the vocal cords of ducks and geese, because they don't have any. She was probably thinking of the difference in the length of their necks. Our own hypothesis is that longer necks resonate at lower frequencies and hence honking can be thought of as deep quacking.

These five examples illustrate a number of aspects of human plausible reasoning as it occurs in common discourse. They show how people bring different pieces of knowledge to bear on a question and how these pieces sometimes lead to the same conclusion and sometimes to different conclusions. Often knowledge is found after the respondent has started answering, so that the certainty of the answer seems to change in midstream. The examples also show how people's approximate functional knowledge of what depends on what often comes to play in different inferences such as deductions and analogies. Therefore these dependencies are a central part of the core theory we have developed. We will return to these examples to illustrate how the formal rules we have developed can be used to characterize different plausible inferences seen in these examples.

In our development of the theory to date we have not tried to characterize all the different types of plausible inferences that occur in the protocols. In particular we have not formalized the spatial and meta-knowledge inferences shown above. This project presents a core system centered around the plausible deductions, analogies, and inductions, seen most frequently in the protocols. In future work we plan to extend this core system to encompass the other patterns of inference, such as spatial and meta-knowledge inferences (Collins, 1978 a,b).

2. ASSUMPTIONS UNDERLYING THE THEORY

The theory assumes that a large part of human knowledge is represented in structures, we call *dynamic hierarchies*, that are interconnected by *traces*. Each hierarchy represents knowledge about a class of concepts arranged in a tree structure according to some viewpoint. Traces represent paths linking nodes in different hierarchies that record beliefs about the world. These beliefs can be recorded by our senses or derived by inference. The theory presented here shows that certain types of plausible inferences can be viewed simply as *perturbations* of traces in the knowledge structures.

The hierarchies are *dynamic* in that they are always being updated, modified or expanded. In the *core* theory described here we distinguish between two basic kinds of hierarchies, *type*– and *part*–hierarchies (Collins and Quillian, 1972). A *type*–hierarchy (also called an *abstraction* or *is–a* hierarchy) is organized by the *type* relation holding between connected nodes, or more precisely, between concepts represented by the nodes. A *part*–hierarchy is organized by the *part–of* relation holding between connected nodes. Any given node may be a member of more than one hierarchy. Each such hierarchy characterizes the node from a different viewpoint.

Nodes of a hierarchy may represent classes (e.g., flowers), individuals (e.g., a specific flower) or manifestations of individuals (e.g., a specific flower at a given moment). For the purpose of the theory, manifestations are treated just like individuals or classes.

Figure 1 shows examples of type– and part–hierarchies. In the first four examples (1a,b,c,d), the Llanos is viewed from four different perspectives. These perspectives are organizing principles of the hierarchies (Bobrow and Winograd, 1977). The type–hierarchy in figure 1a is organized according to the type of terrain. The type of terrain can be mountainous, plateau, hilly, or plain, etc. The Llanos is characterized as a type of plain, like the Chaco. The type–hierarchy in figure 1b is organized according to the geographical land type. It characterizes the Llanos as a type of savanna, which is one of the major land types that geographers divide the world into, including rain forests, deserts, steppes, Mediterranean climates, mid–latitude forests, etc. The part–hierarchy in figure 1c is organized according to regions in South America: the Andes, Amazon Jungle, Llanos, Guiana Highlands, and their subregions in different countries. The part–hierarchy in Figure 1d represents South America broken down into countries and the subregions within each.

Insert Figure 1 here

The other three examples in Figure 1 are designed to illustrate how different descriptors also are represented in hierarchies. Among colors there are green and red. Among reds there are scarlet and burgundy, and among scarlets there are bright scarlet and perhaps dull scarlet, etc. Color is a one–place descriptor applying to objects, but feeling emotion is a two place descriptor where X (a person) feels the emotion toward Y (any concept). In the emotion hierarchy there are many types of emotions, among them love and hate, and there are different kinds of love, such as romance, affection, motherly love, etc. In the weight hierarchy there are different kinds of weight, such as human weight which in turn might be divided into birth weight and adult weight. For birth weight one might think of 1 lb. as a minimum, 15 lbs as a maximum, and 7 lbs as the norm. For the purposes of the theory these can be thought of as different values of birth weight, just as red and green are different values of color. These examples are not meant to show how people represent such concepts, but to give an idea as to how the hierarchies can represent different kinds of information.

As mentioned above, traces represent recordings of information within the hierarchies. They are paths connecting the nodes of two or more hierarchies that represent *beliefs* about the world. Figure 2 shows examples of traces representing the beliefs that there are daffodils and roses in England, and that John's eyes are blue. The traces can have annotations describing their origin, their frequency of use, the certainty of belief in their correctness, and other information. The links denoting the type and part relation in generalization hierarchies can also be viewed as traces, but for the purpose of theory we will distinguish them from other traces. The knowledge organization described above includes various elements of semantic network structure (Carbonell & Collins, 1973; Collins & Quillian, 1972; Quillian, 1968) and frame structure (Bobrow & Winograd, 1977; Minsky, 1975; Schank & Abelson, 1977; Winograd, 1975).

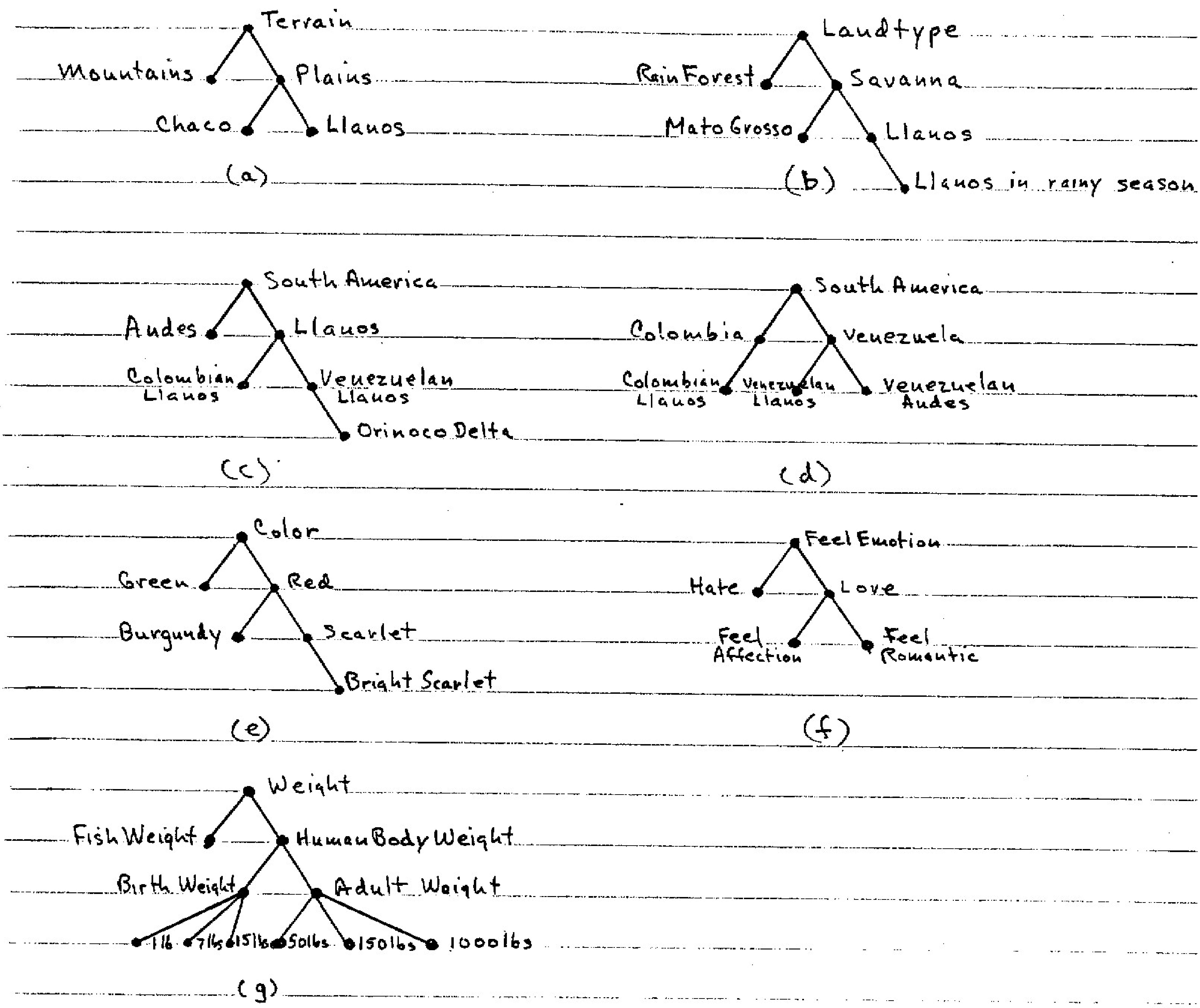


Figure 1. Examples of hierarchies.

Insert Figure 2 here

Let us explain some of the elements of annotations of a trace. By the *origin* of a trace we mean the information specifying whether the trace is a recording of a sense observation, an assertion obtained from a source of information (e.g., another person), or a statement derived through inference. Frequency of use or importance (Carborall & Collins, 1973, Collins & Quillian 1972, Collins & Loftus 1975) represents the ease of traversing a particular link, or the accessibility of one concept from another. Certainty of belief is discussed in detail in the next section.

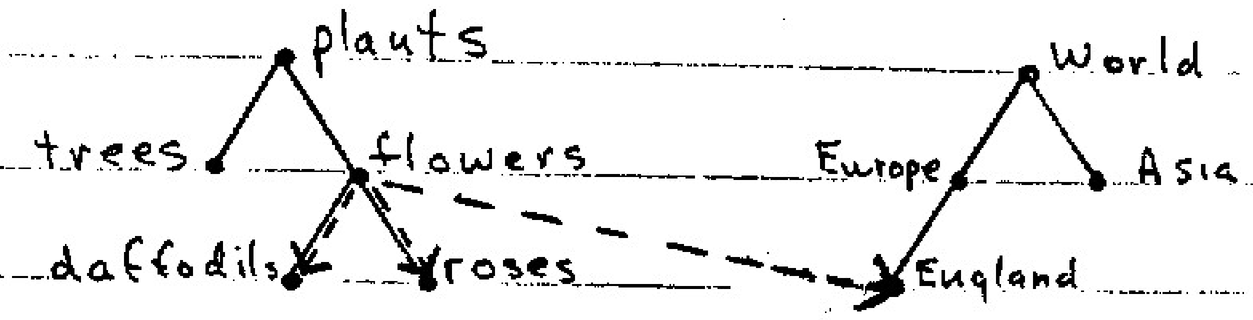
A trace may be a recording of information about one's beliefs, or denote the *applicability* relation between the nodes of different hierarchies. The applicability relation between a node A and a node B states that node A can be used as a *descriptor* of node B., i.e., that A can be used to characterize node B. We write such a relation as a term

A(B)

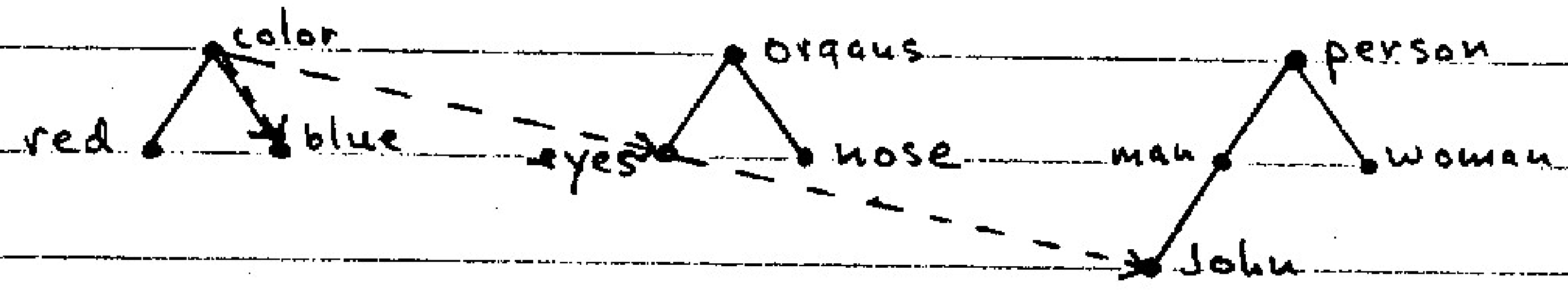
For example, the node "color" in hierarchy 1e applies as a descriptor to node "eyes" of hierarchy 1h. This is denoted as "color(eyes)." The node "eyes" can in turn be applied as descriptor to the node, say, John, in some hierarchy describing people. To express both relations we would write:

color(eyes(John))

A term A(B) can *take a value* only from the set of subnodes of A, i.e., the descendants of the node A in the hierarchy. The set of subnodes which can actually be a value of term A(B) is called the *domain* of term A(B). Applying a descriptor to an *argument* (node or a sequence of nodes) A produces a specific value characterizing the argument. This implies that only non-terminal nodes of a hierarchy can be descriptors. For example, to state that the color of the eyes of John is blue, a trace would be created that links John, color and blue as shown in Figure 2. To express this formally, we write:



flowers(England) = {daffodils, roses,..}



color (eyes(John)) = blue

Figure 2. Examples of two traces or statements

color(eyes(John))=blue

In the theory such an expression is called a *statement*.

The applicability relation observes an important property. If it has been observed that A in a type-hierarchy is *applicable* to B in a type-hierarchy, then we can infer that A is applicable to any subnode of B, and that any supernode of A is applicable to B. For example, assume that the node "eyes" applies to "person". One can infer that also "organ" applies to "person" and that "eyes" applies to "woman." Part-hierarchies, for the most part, follow the same rules as type-hierarchies with some restrictions, such as the fact that a descriptor applicable to one node may not always apply to a subnode (e.g. capital applies to states but not to cities).

It is important to mention at this point that the applicability relation is learned like any other relation. This relation does not act as a "selection restriction" assumed by some linguists. Its violation is not considered to be a semantic anomaly, but rather as a new information to be made consistent with the existing knowledge structures. For example, when one hears that "an idea is green," then usually one tries to make sense of it rather than reject it as an anomalous expression.

Figure 3 illustrates the fact that the hierarchies are partial orderings, and can be differentiated or collapsed as appropriate for the purpose of drawing plausible inferences. At a fairly early age children think of animals as coming in different types: dogs, cats, fish, birds, etc. They don't differentiate them much more than that. When they get to school they may learn there are different basic types of animals, such as fish, birds, reptiles, mammals, and amphibians, and that dogs are cats are types of mammals. Still later in biology this hierarchy might be differentiated much more finely as in Figure 3c. But the early links are never lost; they are in fact used all the time in reasoning about the world. For the purpose of the theory, therefore, any hierarchy can be collapsed or differentiated as long as the partial orderings in the hierarchy are maintained.

Insert Figure 3 here

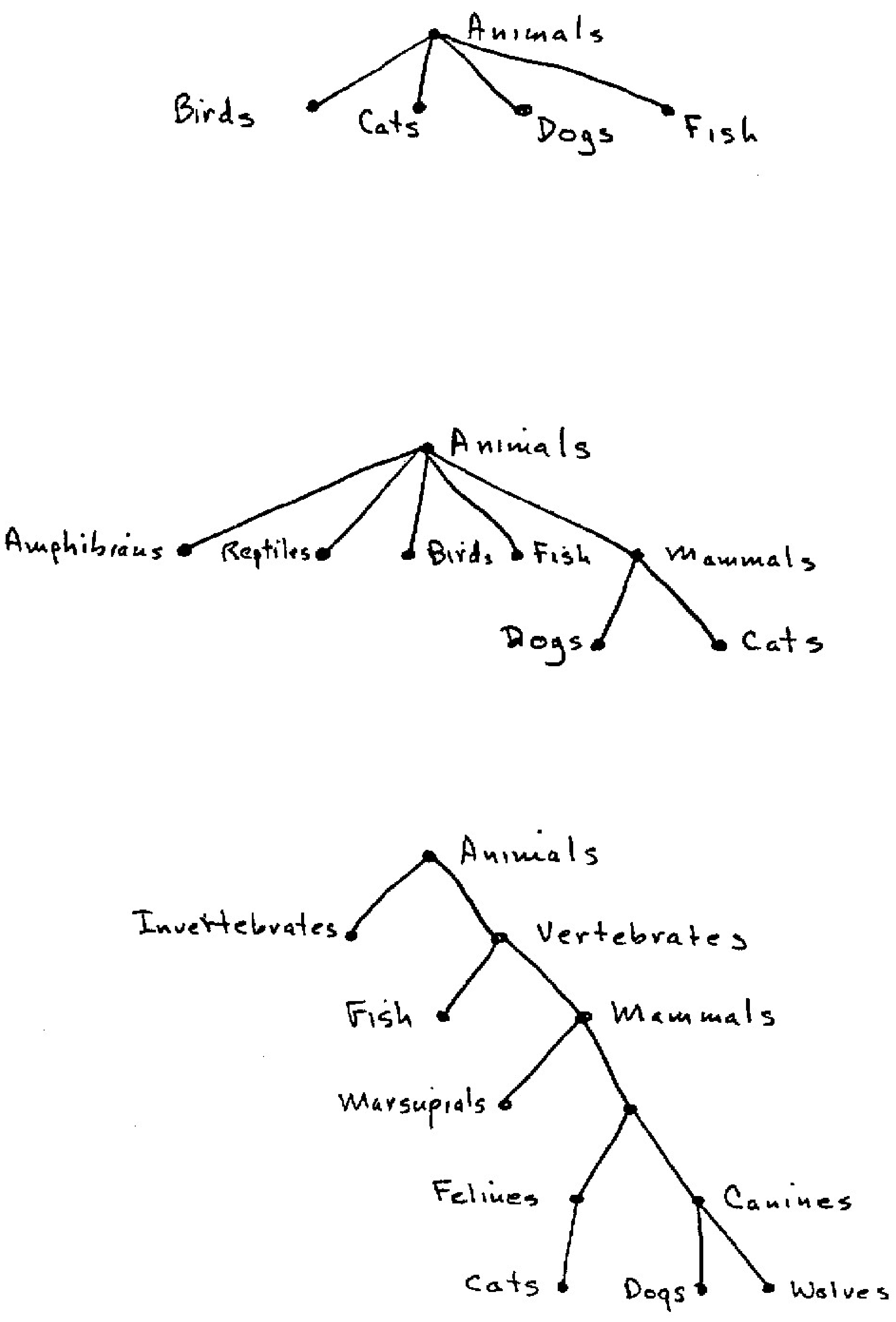


Figure 3 Differentiation of hierarchies.

Table 1 shows hypothetical frame structures for a few concepts in someone's memory (Collins & Quillian, 1972; Collins et al, 1975). These examples are not meant to provide a detailed analysis of how concepts are represented, but rather to illustrate how the statements shown in later examples can be constructed from a memory structure. In the example, type and part relations form the basis for hierarchical structures such as those shown in Figures 1 and 3. Flowers are represented as a type of plant coming in at least four varieties (i.e. roses, etc.), having various parts, various colors, and growing in all countries. Each descriptor (i.e. type/of, types, parts, color, countries) might be further specified as to how it relates to the concept flower (e.g., type/of is a biological class, colors are surface features of the petals, countries are places where flowers are grown, etc). Daffodils, which are a particular type of flower, provide further specification for each of the variables in the concept of flowers. That is, they have petals and a stem, they come in yellow and perhaps other colors, and they are grown in at least England and the United States. The frame for red is shown to illustrate how a color concept points back to various objects which it describes. Finally let us stress that we have not concerned ourselves with exactly how concepts are represented, but rather we have assumed they are represented in a structure similar to these examples.

Insert Table 1 here

Any node in a hierarchy can potentially be a *descriptor* for a node in another hierarchy. For example, if flower is in a hierarchy of things and England is in a hierarchy of places, flower-type might be a descriptor for England. This produces a *statement* of the form:

(1) flower-type (England)={daffodils, roses,....}

In (1), flower-type is a descriptor, England is an argument, flower-type (England) is a term, and daffodils and roses are references for the term. The brackets and dots indicate that daffodils and roses are not assumed to be a complete set, although the person may not know other flowers of England. Any descriptor, as a node in a hierarchy, can be further differentiated. For example, flowers can be differentiated between naturally-growing flowers vs. flowers grown in greenhouses, or between flowers sold vs. flowers grown, etc. People make finer or less fine discriminations

Table 1

Hypothetical Frames in a Person's Memory

flower

type/of =(plant)

types ={rose, daffodil, peony, bougainvillea ...}

parts ={petals, stem ...}

colors ={pink, yellow, white, red ...}

countries ={all countries}

daffodil

type/of =(flower)

parts ={petals, stem ...}

colors ={yellow ...}

countries ={England, United States ...}

red

type/of =(color)

types ={scarlet, burgundy ...}

flowers ={roses, tulips ...}

vehicles ={fire engines, London buses ...}

depending on their knowledge and purposes, and a theory of plausible reasoning must accommodate these different degrees of discrimination.

Whether a particular descriptor applies to any argument depends on what knowledge the person has. For example, it is not clear what red-type (England) might mean because one probably doesn't have knowledge in one's data base about the color of England (though one might interpret the term as the color of any part of England, such as the Union Jack and London buses).

Examples (2) to (8) below illustrate how different descriptors apply to different concepts:

(2) England-part (daffodil)={Southern England...}

(3) daffodil-part (England)={petals, stem...}

(4) country-type (daffodils)={temperate countries...}

(5) daffodil-type (England)={yellow daffodils...}

(6) England-type (daffodils)={England in the spring}

(7) love-type (John, Mary)={affection...}

(8) give-type (John, Mary, scarf)={gift-giving...}

Examples (2) and (3) illustrate statements based on part hierarchies. In (2) the descriptor selects the part of England where daffodils occur. In (3) the descriptor selects the parts of daffodils that occur in England; presumably daffodil parts in England are the same as daffodil parts anywhere in the world (though perhaps Martian daffodils are quite different). In (4) country-type applied to daffodils selects the types of countries that have daffodils (i.e., temperate countries). Statement (4) could have specified the particular countries (e.g. England, France) that have daffodils, since hierarchies can be collapsed as long as a partial order is maintained. In (5) daffodil-type applied to England selects the different daffodil types found in England, of which only one type is stored (i.e., yellow daffodils), though there may be others. In (6) we show that when you take an instance like England and look at its subtypes you get a manifestation, in this case the manifestation(s) that have daffodils. Finally, (7) and (8) illustrate multiple place predicates describing John's love of Mary, and John's giving a scarf to Mary as a gift rather than loaning it or giving it away to get rid of it. These examples show how different terms are evaluated within the theory.

These examples illustrate the most important assumptions we are making about how human memory is organized and accessed for the purposes of making plausible inferences. Further descriptions of our underlying assumptions about human memory are given in earlier papers (Carbonell & Collins, 1973; Collins & Loftus, 1975; Collins & Quillian, 1972; Collins, Warnock, Aiello & Miller, 1975).

3. PRIMITIVES IN THE CORE SYSTEM.

In the core system we have developed there is a set of primitives and a set of basic inference rules. In this section we describe the primitives in the system, consisting of basic expressions, operators, and certainty parameters.

Table 2 shows the basic elements in the core system. *Arguments* can be any node in a hierarchy, or a function of one or more nodes such as Fido's master or the flag of England. *Descriptors* apply to arguments, and together they form a *term*, such as breed (Fido). The *reference* for a term can be either a definite set of values such as collie, or brown and white, or an indefinite set of values such as brown plus other colors (or possibly no other colors).

Insert Table 2 here

Statements consist of a term on the left of an equals sign and a reference on the right, together with a set of certainty parameters. Expressions (1) through (8) above were all statements, without the certainty parameters specified. The operator statements shown below in Table 3 are a special class of statements. The certainty parameters can be thought of as approximate numbers ranging between 0 and 1, but we have represented them as verbal descriptions. In the example shown, γ refers to how certain one is the statement is true, and ϕ to the frequency that if something is a bird it can fly. These certainty parameters are all listed in Table 4, to be discussed later.

The last two types of expressions represent functional dependencies between different variables. *Dependencies between terms* represent the functional relationship between two terms, such as between the average temperature of a place and the latitude of the place. The dependency can be annotated to different degrees: it can be unmarked meaning there exists some functional relation the two, it can be marked with + or − indicating a monotonic increasing or decreasing relation, or it can be further specified to any degree (e.g., a V–shaped function with 3 values specified). For example, if one thinks that average temperature of a place in January varies between about 85° at the equator and −30° at the North Pole and + 30° at the South

Table 2

Elements of Expressions

arguments a_1, a_2, $f(a_1)$
e.g., Fido, collie, fido's master

descriptors d_1, d_2
e.g., breed, color

terms $d_1(a_1)$, $d_2(a_2)$, $d_2(d_1.(a_1))$
e.g., breed (Fido), color (collie), color (breed (Fido))

references r_1, (r_2,r_3), $\{r_2 \ldots\}$
e.g., collie, brown and white, brown plus other colors

statements $d_1(a_1)=r_1$: γ, ϕ
e.g., means of locomotion (bird)={flying...}: certain, high frequency

dependencies between terms $d_1(a_1) \textless\!\!-\!\!-\!\!-\!\!\textgreater d_2(f(a_1))$: α, β, γ
e.g., latitude (place) <----> average temperature (place): moderate, moderate, certain

implications between statements $d_1(a_1)=r_1 \Longleftarrow\!\!\!=\!\!\!\Longrightarrow d_2(f(a_1))=r_2$: α, β, γ
e.g., grain (place)={rice...} <===> rainfall (place)=heavy: high, low, certain

Pole, this relation can be represented as a V-shaped function with values $(-90^\circ, 30^\circ)$, $(0^\circ, 85^\circ)$ and $(90^\circ, -30^\circ)$, where the first coordinate is latitude and the second temperature. The α and β parameters specify the degree of constraint in the dependency from latitude to temperature and from temperature to latitude, respectively. In the latitude-temperature example the degree of constraint is moderate in both directions, as is discussed later.

Implications between statements relate particular values of functions such as the latitude-temperature function above (e.g., latitude (place) = equator <=> average temperature (place) = hot). The example shown in the table relates the grain of a place being rice to the rainfall of the place being heavy (e.g., >40 in/year). Knowing a place produces rice predicts that it will have heavy rainfall quite strongly, so that α is high (though there are exceptions like Egypt where rice is grown by irrigation). However the fact that the rainfall of a place is heavy (e.g., Oregon) only weakly predicts that rice is grown, so β is low. In general mutual implications between statements will be asymmetric in this way.

Table 3 illustrates the four operators in the core system and the kinds of statements they occur in. The generalization and specialization operators go up and down in a hierarchy, while the similarity and dissimilarity operators go between nodes at the same level in a hierarchy. Associated with the GEN and SPEC operators there is a typicality parameter τ (Rosch, 1975; Smith & Medin, 1982), and with the SIM and DIS operators there is a similarity parameter σ. There is also a dominance parameter δ associated with GEN and SPEC statements that specifies what proportion of the superset, the subset actually comprises. Finally all the statements involving operators have a certainty parameter γ associated with them.

Insert Table 3 here

Typicality and similarity are always computed in some context which is denoted by the CX variable. The first variable in the CX denotes a node in the argument hierarchy specifying the range of arguments over which typicality or similarity are computed. For GEN and SPEC this is always the superset specified in the statement (e.g., for chicken=SPEC (barnyard fowl), barnyard fowl is the superset over which

Table 3

Operators

Generalization $a' = GEN(a)$ in $CX(a,D)$: γ, τ, δ

e.g., bird=GEN(chicken) in CX (birds, physical features): certain, atypical, low dominance

Specialization $a' = SPEC(a)$ in $CX(a',D)$: γ, τ, δ

e.g., chicken=SPEC(barnyard fowl) in CX (barnyard fowl, food cost): certain, typical, moderate dominance

Similarity $a' = SIM(a)$ in $CX(A,D)$: γ, σ

e.g., ducks=SIM(geese) in CX(birds, all features): certain, highly similar

Dissimilarity $a' = DIS(a)$ in $CX(A,D)$: γ, σ

e.g., ducks=DIS(geese) in CX(birds, neck length): certain, fairly dissimilar typicality is computed, but for SIM and DIS it is the basic level category (Rosch 1975; Smith & Medin, 1982) to which the two arguments belong that is the basis for computing similarity. Hence the similarity of ducks and geese would normally be computed in the context of birds, which is their basic level category.

The second variable in the CX specifies the set of descriptors to be used in comparing the two nodes with respect to typicality or similarity. For example, one can evaluate how typical chickens are as birds with respect to their physical features, with respect to all their features, or with respect to some particular feature such as the cost of feeding them. Similarity and dissimilarity can also be computed with respect to different features. As we discussed with respect to the fifth protocol shown earlier, ducks and geese are quite similar when compared on all their features, but they are dissimilar in neck length (which is relevant to determining the sound they make). The procedure for computing typicality and similarity is described below.

Table 4 lists the certainty parameters we have identified so far that affect the certainty of different plausible inferences. We will describe each of these parameters in terms of the examples given above. The description is meant to specify our best hypothesis about how people might compute these parameters.

Insert Table 4 here

The α and β parameters can best be introduced in terms of the example: grain(place)={rice...}<===>rainfall(place)=heavy. As we said, α would be high in such case if a person thinks that most places that grow rice have heavy rainfall (say > 40 inches per year), whereas β would be low if he or she thinks there are many places with heavy rainfall, that don't produce rice. We can construct a hypothetical table that represents this view in terms of a small sample of places and the frequencies with which they have heavy rainfall and produce rice:

	Rice	No Rice	Total
Heavy Rainfall	8	8	16
No Heavy Rainfall	2	20	22
Total	10	28	38

Table 4

Certainty Parameters

α Likelihood that the right–hand side of a dependency or implication is in a particular range given that the left–hand side is in a particular range.

β Likelihood that the left–hand side of a dependency or implication is in a particular range given that the right–hand side is in a particular range.

γ Degree of certainty that a statement is true (i.e., degree of belief).

τ Degree of typicality of a subset within a set (e.g., robin is a typical bird and ostrich is an an atypical bird).

σ Degree of similarity of one set to another set.

ϕ Frequency of the reference in the domain of the descriptor (e.g., above 90% of birds fly).

δ Dominance of a subset in a set (e.g., chickens are not dominant among birds, but are dominant among barnyard fowl).

Given this table α is simply the conditional probability that a rice-producing place has heavy rainfall, in this case 8 of 10 or .8 and β is the conditional probability that a place with heavy rainfall produces rice, in this case 8 of 16 or .5. We don't think that people actually construct such tables though they may consider a small number of cases in computing rough estimates of α and β , as they do in using the availability heuristic (Tversky & Kahneman, 1973).

The α and β parameters for mutual dependencies can be constructed by an extension of the procedure for mutual implications. Suppose one considers the relationship of rainfall and grain growing as before, but instead as a mutual dependency (i.e., grain (place) <--> rainfall (place). For simplicity we can present the same hypothetical table in revised form:

	Rice	Wheat	Corn	Total
Heavy Rainfall	8	6	2	16
Light Rainfall	2	14	6	22
Total	10	20	8	38

Then α reflects the degree to which you can predict whether a place has heavy or light rainfall, given the predominant grain grown in the place, which is quite high (i.e., the prediction is correct in 28 or 38 cases or .7 assuming an optimal guessing strategy). Similarly, β reflects the degree to which you can predict whether they grow rice, wheat, or corn, given the amount of rainfall (i.e., the prediction is correct in 22 of 38 cases or .6, assuming an optimal strategy of guessing wheat for light rainfall and rice for heavy rainfall). This example makes evident the fact that the α and β parameters reflect the way the dependency partitions the known cases in the world.

The γ parameter in Table 3 reflects the certainty or subjective likelihood with which a person believes any expression is true. γ can reflect different possible sources of uncertainty. One source arises when people retrieve a fact from memory and are uncertain they may be making a memory confusion. Another basis for uncertainty arises when they doubt the source from which they got the information. Finally, if a piece of information derives from a plausible inference, there will be uncertainty as to whether the conclusion is correct, and this uncertainty will propagate to inferences dependent on it. All these sources of uncertainty are represented by the γ parameter.

Typicality (τ) and similarity (σ) can be thought of as the same parameter: in the case of typicality it is computed between a subset and its superset, and in the case of similarity it is computed between two subsets. We assume that any set (or concept) is represented as a bundle of features (Collins & Quillian, 1972), and the τ and σ parameters are computed by comparing the two concepts with respect to those features specified by the descriptor variable in the context CX. For example, "chicken" might be compared to "bird" with respect to size or with respect to all its physical features to determine its typicality. For a continuous variable like size, typicality or similarity is determined by computing how close (normalized between 0 and 1) the two values are in the distribution of sizes for the class specified by the context CX (e.g. birds). For discrete variables like "ability to fly", the two concepts either match or not (assigned either 1 or 0). Typicality or similarity are based on the average score for all the features compared, weighted for their criteriality or importance (Carbonell & Collins, 1973; Collins & Quillian, 1972).

Frequency (ϕ) and dominance (δ) reflect different ratios that affect the certainty of plausible inferences in systematic ways. Frequency reflects the proportion of members of the argument set that can be characterized by the reference specified. It reflects what "Some" or "All" reflect in logic (e.g., "Some men have arms"), but as a continuous variable between 0 and 1. For the statement "means-of-locomotion (birds)={flying...}," is the proportion of birds that fly to the total of all birds. The dominance of a subset within a set (δ) applies only to generalization and specialization statements. It reflects the proportion of members of the set that are members of the subset specified in the statement. For example, chickens constitute a high proportion of barnyard fowl, but not of birds in general.

This completes our summary of the primitives in the system. We will now describe the different plausible inference forms in the core system.

4. TRANSFORMS ON STATEMENTS

The simplest class of inferences in the core theory are called transforms on statements. If a person believes some statement, such as that the flowers growing in England include daffodils and roses [i.e., flower-type(England)={daffodils, roses...}], there are eight transforms of the statement that allow plausible conclusions to be drawn. These eight transforms can be thought of as perturbations of the statement either with respect to the argument hierarchy (starting from England) or the reference hierarchy (starting from daffodils and roses). The argument-based transforms move up (using GEN), down (using SPEC), or sideways (using SIM or DIS) in the argument hierarchy. Similarly the reference-based transforms move up, down, or sideways in the reference hierarchy. Thus each of these transforms is a perturbation in one of the two hierarchies.

Let us illustrate the eight transforms on statements in terms of hierarchies for England and roses. Figure 4 shows a part hierarchy for England and a type hierarchy for roses and daffodils that someone might have. If the person believes that, "flower-type(England)={daffodils, roses...}," then Table 5 shows eight conclusions that the person might plausibly draw.

Insert Figure 4 and Table 5 here

The first GEN inference is that Europe as a whole grows daffodils and roses. This may not be true: Daffodils and roses may be a peculiarity of England, but it is at least plausible that daffodils and roses are widespread throughout Europe. Similarly, for the SPEC relation it is a plausible inference that the county of Surrey in southern England grows roses and daffodils. There is an implicit context (CX) in GEN and SPEC transforms, that will be discussed later.

The SIM and DIS inferences are also made in some context. In the case of the argument-based transforms the context might be "countries of the world with respect to the variable climate." Holland is quite similar to England with respect to climate, while Brazil is quite dissimilar. The variables over which the comparison is made may be few or many, but people will make the comparison with respect to those variables

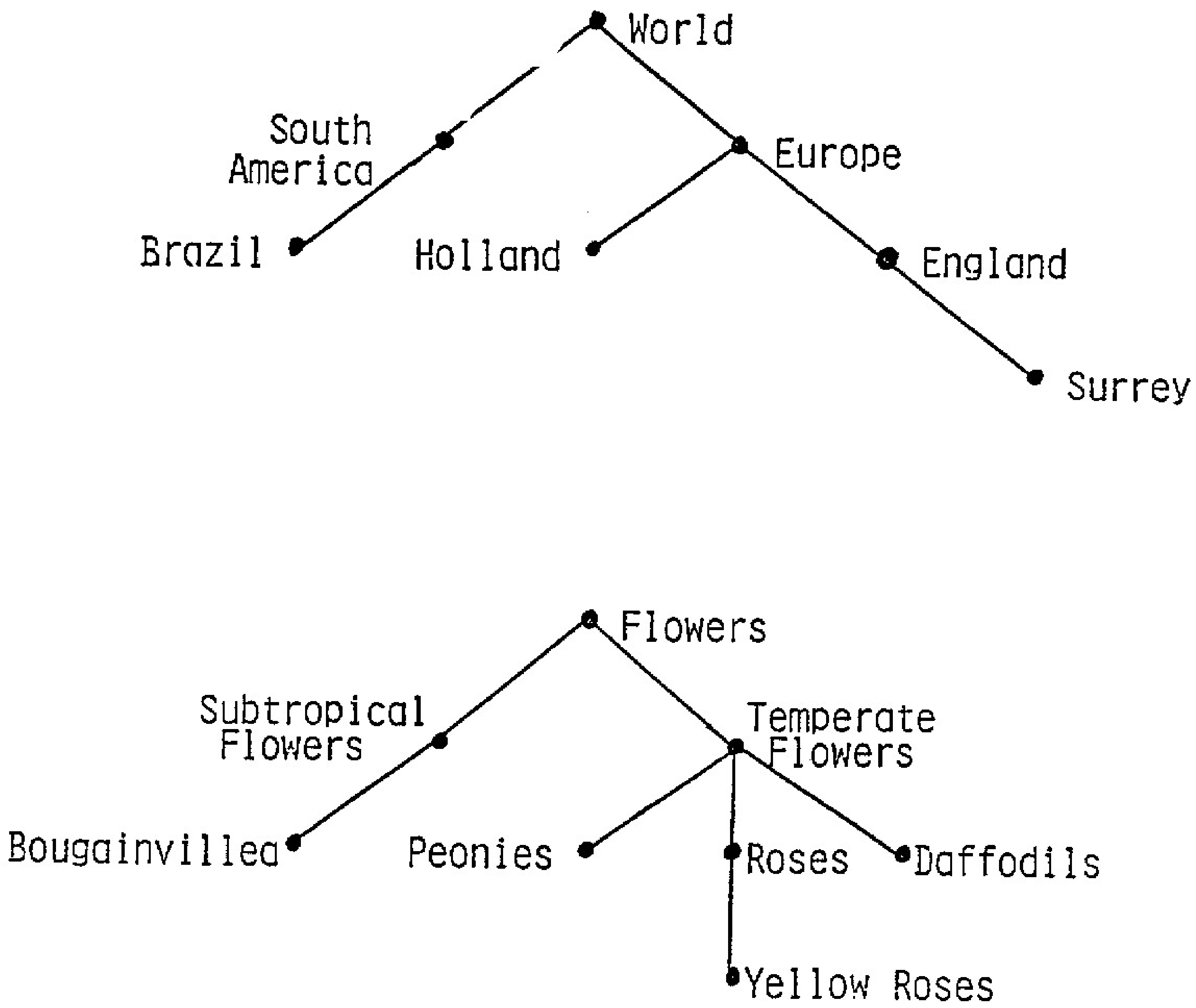


Figure 4. Part Hierarchy for England and Type Hierarchy for Roses.

Table 5

Eight Transforms on the Statement
"flower–type(England)={daffodils, roses...}"

Argument–based Transforms

(1)	GEN	flower–type(Europe)={daffodils, roses...}
(2)	SPEC	flower–type(Surrey)={daffodils, roses...}
(3)	SIM	flower–type(Holland)={daffodils, roses...}
(4)	DIS	flower–type(Brazil)≠{daffodils, roses...}

Reference–based Transforms

(5)	GEN	flower–type(England)={temperate flowers...}
(6)	SPEC	flower–type(England)={yellow roses...}
(7)	SIM	flower–type(England)={peonies...}
(8)	DIS	flower–type(England)≠{bougainvillea...}

that they think are most relevant to the question (e.g., whether they grow daffodils in Holland). That is, they base their inference on whatever mutual dependency most constrains the descriptor in question. In this case the flowers grown in a place depend highly on the climate of the place, but hardly at all on the longitude of the place. Therefore climate is a reasonable variable on which to make the comparison. We will refer to this issue later when we talk about how different parameters affect the certainty of any statement transform.

The reference transforms are perhaps easiest to understand if you substitute a fictional place like Ruritania for England, because other inferences are not invoked so easily. If one believes they grow daffodils and roses in Ruritania, then one might infer they grow temperate flowers in general there, and yellow roses in particular. It is also reasonable that they grow peonies there, since they are similar to roses and daffodils as to the climates they grow in. But bougainvillea grows in more tropical climates, so it is unlikely to grow in Ruritania (Ruritania is, after all, a small little kingdom and unlikely to encompass different climates--this is a supporting inference). These examples should give a feel for how the transforms on statements are made.

4.1 Certainty Parameters Affecting Transforms on Statements

In this section we will discuss how different certainty parameters affect the various transforms shown in Table 5.

<u>Typicality</u>. Typicality (τ) affects the certainty of any GEN or SPEC transform as shown in Table 6. In argument-based transforms the more typical the subset is of the set in the argument hierarchy, the more certain the inference. For example, in Table 5 inference (1) is more certain the more typical England is as part of Europe.

Insert Table 6 here

In making plausible inferences people compute typicality with respect to those variables, such as climate, that they think flower growing depends on. Thus, if Surrey is thought to have a typical climate for England, and climate is thought to predict the types of flowers grown in a place, then the inference is more certain.

Table 6

Effects of Different Parameters on Statement Transforms

Transforms in Table 5		Parameters τ	σ	α	ϕ	δ	Target Node
	1 GEN	+	0	+	+	+	Europe
Argument-Based	2 SPEC	+	0	+	+	+	Surrey
	3 SIM	0	+	+	+	0	Holland
	4 DIS	0	–	+	–	0	Brazil
	5 GEN	+	0	+	+	+	Tropical Plants
Reference-Based	6 SPEC	+	0	+	+	+	Yellow Roses
	7 SIM	0	+	+	+	0	Peonies
	8 DIS	0	–	+	–	0	Bougainvillea

Note: As the value of the parameter increases, a + means it has a positive effect on the certainty of the inference and a – means it has a negative effect on the certainty of the inference.

This example reveals the mutual dependency implicit in any statement transform. The mutual dependency relates the set of variables on which the typicality or similarity judgment is made (e.g., climate or all variables) to the descriptor in question (e.g., flower-type). If the typicality judgment is made considering all variables (as when we said Surrey is a typical English county), the transform will be inherently less certain because of the weak dependency between most variables and any descriptor such as flower-type. Therefore, if you know that Surrey is typical of England in general, it leads to a less certain inference than if you know Surrey is typical of England with respect to climate.

In a reference-based transform typicality works the same way, except that it is computed with respect to the subset and its superset in the reference hierarchy. In inference (5) in Table 5, the greater the typicality of daffodils and roses as temperate plants, the more certain the inference. Similarly in the inference (6), the greater the typicality of yellow roses as roses, the more certain the inference. Pink roses are more typical than yellow roses, and so they are even more likely to be found in England (or Ruritania for that matter). Again the inference is more certain if typicality is measured with respect to the climate in which the flowers are grown.

Similarity. Degree of similarity (σ) affects the certainty of any SIM or DIS inference as shown in Table 6. Like typicality, similarity can be computed over all variables or over a subset of variables (e.g., climate) that are particularly relevant.

Degree of similarity increases the certainty of SIM inferences and decreases the certainty of DIS inferences, as would be expected. In Table 5, therefore the inference (3) that Holland has daffodils and roses is more certain the more similar Holland is to England with respect to climate or whatever variables one thinks flowers are related to. The inference (4) that Brazil does not have roses and daffodils is more certain the less similar Brazil is to England. The inference (7) that England has peonies is more certain, the greater the similarity of peonies to both daffodils and roses. The inference (8) that England does not have bougainvillea is more certain, the less similar bougainvillea is to daffodils and roses. More particularly bougainvillea is dissimilar in that it tends to grow in warmer climates than daffodils and roses.

Mutual Dependency. Every transform on a statement involves an implicit mutual dependency. The inference is always more certain the greater the dependency (α) between the variables on which typicality or similarity are measured and the variable in question as shown in Table 6. If climate were the variable used for measuring typicality and similarity, the argument-based transforms would be more certain the more the climate of a place constrains the flowers grown in the place. The mutual dependency is slightly different for reference-based transforms. They would be more certain, the more the climate where flowers grow constrains the places where flowers grow.

Frequency. The frequency (ϕ) of the reference set within the domain of the argument affects the certainty of all eight inferences, as shown in Table 6. For an instance, e.g. England, frequency with respect to the argument set only makes sense if you think of England as a set of small parts (say 10 miles square) and count the frequency of parts that have daffodils and roses vs. those that do not. The more frequent daffodils and roses are in the parts of England, then all but the DIS inferences are more certain. For example, roses and daffodils are more likely to occur in Holland or Surrey if they are very frequent in England. The two DIS inferences go in the opposite direction. For example, the less frequent are daffodils and roses in England, the more likely bougainvillea will be found there (though this is a very weak inference).

Dominance. Dominance (δ) affects GEN and SPEC inferences as is shown in Table 6. In all cases, the greater the dominance of the subset, the more certain the inference. For example, for (2) if Surrey comprised most of England it would be a more certain inference that it has daffodils and roses, than if it is a very small area in England. Similarly for (6) if yellow roses were the most dominant kind of roses, they would be more likely found in England than if they are a rare type of rose.

4.2 Formal Representation of Transforms on Statements

Table 7 shows the formal representations we have developed for each of the eight transforms on statements in terms of the variable-valued notation of Michalski (1983). Most of the examples shown are from protocols we have collected (Collins, 1978b), some of which appear in the first section of this paper. We will briefly describe each of the examples.

Insert Table 7 here

We can illustrate an argument-based transform or GEN with the inference that if chickens have gizzards, then birds in general may have gizzards. The first premise, represents the belief that chickens have gizzards: presumably almost all chickens have gizzards so the frequency (ϕ) and the certainty (γ) arer high. The second premise represents the belief that chickens are birds, and that they are typical with respect to their biological characteristics. As we pointed out earlier, the subset dominance (δ) of chickens among birds is low. The third premise states that the internal organs of a bird depend highly on the biological characteristics of the bird. The conclusion that birds have gizzards is fairly certain given the high values of the critical variables.

The argument-based transform on SPEC is illustrated by an example from the beginning of the paper where the respondent inferred that the Andes might be in Uruguay. The respondent believed that the Andes are in most South American countries, so frequency (ϕ) was moderately high. With respect to the second premise, Uruguay is a typical South American country, which increases the likelihood that the Andes would be found there. But its low subset dominance (δ) in terms of the proportion of South America that Uruguay comprises makes the inference less likely. With respect to the third premise, the fact that Uruguay is typical of South American countries in general only weakly predicts that it will include the Andes mountains. Altogether, the inference is fairly uncertain given the moderate frequency and the low subset dominance of Uruguay.

We can illustrate the argument-based transform on SIM with the Chaco protocol from the beginning of the paper, where the respondent inferred that the Chaco might produce cattle given that west Texas did. In the first premise, frequency (ϕ), which reflects the degree to which different parts of west Texas have cattle, is high, which makes the inference more likely. The second premise asserts that the Chaco is a least moderately similar to west Texas in vegetation (or whatever variables the respondent had in mind). The third premise relates vegetation of a region to its livestock, which is a strong relation, given that cattle will usually be raised where the vegetation will support them. The fourth premise merely establishes the fact that west Texas and

Table 7

Formal Representations of Statement Transforms

(1) Argument–based transform on GEN

d(a)=r: γ_1, Φ
a'=GEN(a) in CX (a',D(a'): τ, γ_2, δ
D(a') <----> d(a'): α, γ_3

----

d(a')=r: $\gamma = f\ (\gamma_1, \Phi, \tau, \gamma_2, \delta, \alpha, \gamma_3)$

Internal organ (chicken) = {gizzard ...}: γ_1 =high, Φ =high
Birds = GEN (chicken) in CX (bird, biological characteristics(birds)):
τ =high, γ_2=high, δ =low
Biological characteristics (birds) <----> internal organs (birds):
α =high, γ_3=high

Internal organs (birds) = {gizzard ...}: γ=high

(2) Argument–based transform on SPEC

d(a)=r: γ_1, Φ
a'=SPEC(a) in CX (a, D(a)): τ, γ_2, δ
D(a) <----> d(a): α, γ_3

----

d(a')=r: $\gamma = f\ (\gamma_1, \Phi, \tau, \gamma_2, \delta, \alpha, \gamma_3)$

Mountains(S.A. country) = {Andes ...}: γ_1=high, Φ=high,
Uruguay=SPEC(S.A. country) in CX(S.A. country, characteristics(S.A. country)):
τ=high, γ_2 =high, δ =low
Characteristics (S.A. country) <----> mountains (S.A. country):
α =moderate, γ_3=high

Mountains (Uruguay) = {Andes ...}: γ=moderate

(3) Argument-based transform on SIM

$d(a) = r: \gamma_1, \Phi$
$a' = SIM(a)$ in $CX(A, D(A)): \sigma, \gamma_2$
$D(A) \longleftrightarrow d(A): \alpha, \gamma_3$
$a, a' = SPEC(A): \gamma_4, \gamma_5$

$d(a') = r: \gamma = f(\gamma_1, \Phi, \sigma, \gamma_2, \alpha, \gamma_3, \gamma_4, \gamma_5)$

Livestock (West Texas) = {cattle ...}: γ_1=high, ϕ =high
Chaco = SIM (West Texas) in CX (region, vegetation(region)):
σ =moderate, γ_2 =moderate
Vegetation (region) $\longleftrightarrow$ livestock (region): α=high, γ_3=high
West Texas, Chaco = SPEC (region): γ_4=high, γ_5=high

Livestock (Chaco) = {cattle ...}: γ =moderate

(4) Argument-based transform on DIS

$d(a) = r: \gamma_1, \Phi$
$a' = DIS(a)$ in $CX(A, D(A)): \sigma, \gamma_2$
$D(A) \longleftrightarrow d(A): \alpha, \gamma_3$
$a, a' = SPEC(A): \gamma_4, \gamma_5$

$d(a') \neq r: \gamma = f(\gamma_1, \phi, \sigma, \gamma_2, \alpha, \gamma_3, \gamma_4, \gamma_5)$

Sound (duck) = (quack): =high, =high
Goose = DIS (duck) in CX(bird, vocal cords (bird)):
=low, =moderate
Vocal cords (bird) $\longleftrightarrow$ sound (bird): =high, =low
Duck, goose = SPEC (bird): =high, =high

Sound (goose)$\neq$quack: γ=low

(5) Reference–based transform on GEN

d(a) = {r ...}: γ_1, ϕ
r' = GEN (r) in CX(d, D(d)): τ, γ_2, δ
D(d) <-----> A(d): α , γ_3
a = SPEC (A): γ_4

---

d(a) = {r'...}: $\gamma = f\ (\gamma_1, \phi, \tau, \gamma_2, \delta, \alpha, \gamma_3, \gamma_4)$

Agricultural product (Honduras) = {bananas ...}:
γ_1=unknown, ϕ=high,
Tropical fruits = GEN (bananas) in CX(agricultural products, climate(agricultural products)): τ=high, γ_2 =high, δ=low
Climate (agricultural products) <-----> Place (agricultural products):
α=high, γ_3=high
Honduras = SPEC (place): γ_4=high

Agricultural products (Honduras)={tropical fruits...}: γ=moderate

(6) Reference–based transform on SPEC

d(a) = {r ...}: γ_1, ϕ
r' = SPEC (r) in CX(d, D(d)): τ, γ_2, δ
D(d) <-----> A(d): α , γ_3
a = SPEC (A): γ_4

---

d(a) = {r'...}: $\gamma = f\ (\gamma_1, \phi, \tau, \gamma_2, \delta, \alpha, \gamma_3, \gamma_4)$

Minerals (South Africa) = {diamonds ...}: γ_1 = high , ϕ = high
Industrial diamonds=SPEC(diamonds) in CX(minerals, characteristics(minerals)):
τ=high, γ_2=high, δ =high
Characteristics(minerals) <-----> Place (minerals):
α=moderate, γ_3=high
South Africa = SPEC (place): γ_4=high

Minerals (South Africa) = {industrial diamonds ...}: γ =high

(7) Reference-based transform on SIM

$d(a) = \{r...\}: \ \gamma_1 , \ \phi$
$r' = SIM \ (r) \ in \ CX(d, D(d)): \sigma , \gamma_2$
$D(d) <----> A(d): \alpha , \gamma_3$
$a = SPEC \ (A): \gamma_4$

$d(a) = \{r'...\}: \ \gamma = f(\gamma_1, \phi, \sigma, \gamma_2, \alpha, \gamma_3, \gamma_4)$

Sound (wolf) = {howl...}: γ_1=high, ϕ =high,
Bark = SIM (howl) in CX(sound, means of production(sound)):
σ =high, γ_2=high
Means of production (sound) <----> animal (sound): α=high, γ_3 =high
Wolf = SPEC (animal): γ_4=high

Sound (wolf) = {bark...}: γ =moderate

(8) Reference-based transform on DIS

$d(a) = \{r...\}: \gamma_1 , \phi$
$r' = DIS \ (r) \ in \ CX(d, D(d)): \sigma , \gamma_2$
$D(d) <----> A(d): \alpha , \gamma_3$
$a = SPEC(A): \gamma_4$

$d(a) \neq \{r'...\}: \ \gamma = f(\gamma_1, \phi, \sigma, \gamma_2, \alpha, \gamma_3, \gamma_4)$

Color (Princess phones) = {white, pink, yellow...}: γ_1 =high, ϕ =high
Black = DIS (white & pink & yellow) in CX(color, hue(color)):
σ =low, γ_2=high
Hue (color) <----> object (color): α=low, γ_3=high
Princess phone = SPEC (object): γ_4 =high

Color (Princess phones)$\neq$ {black...}: γ =moderate

Chaco are regions, in support of the second and third premises. The conclusion is only moderate in certainty, given our assumption of uncertainty about how similar the Chaco and west Texas are.

To illustrate the argument-based transform on DIS, we chose the example from the protocol shown earlier as to whether a goose quacks. The first premise reflects the respondent's belief that ducks quack, which was very certain. The second premise states the belief that ducks and geese are dissimilar in their vocal cords which the respondent must have been at least a bit uncertain about (hence the low certainty assigned to the statement). The third premise relates the sound a bird makes to its vocal cords, which also must have been an uncertain belief given that it is not true. The certainty of the conclusion that geese do not quack should have been fairly low (though other inferences led to the same conclusion in the actual protocol).

We have created an example to illustrate a reference-based transform on GEN, since there are none in the protocols. The first premise asserts that Honduras produces bananas among other things. Bananas are a fairly typical tropical fruit in terms of the climates where they are grown, as the second premise states. The third premise asserts that the climate appropriate for agricultural products constrains the places where they are grown fairly strongly. The conclusion follows with moderate certainty that Honduras produces tropical fruits in general, such as mangos and coconuts.

We also created the example of a referenced-based transform on SPEC. The first premise states that South Africa produces diamonds. Industrial diamonds are a kind of low quality diamond (used in drills) and they must be fairly dominant (δ) among diamonds given their low quality, though they are not particularly typical of what we think of as diamonds. Here is a case where high dominance compensates for low typicality. The third premise is somewhat irrelevant since the typicality is low. But the inference is quite certain given the high dominance of industrial diamonds among diamonds.

The example of a reference-based transform on SIM is drawn from a protocol where the respondent, when asked whether wolves could bark, inferred they probably could (Collins, 1978b). One of his inferences derived from the fact that he knew wolves could howl, with both high frequency and certainty. He also thought that barking was similar to howling in terms of the way the sound is produced (a howl, as it were, is a sustained bark). Further the animals that make a particular sound depend on the means of production of the sound, as the third premise states. It follows then with at least moderate certainty that a wolf can bark.

The example of a reference–based transform in DIS is from a protocol where the respondent was asked if there are black princess telephones (Collins, 1978b). The respondent could remember seeing white, pink and yellow princess phones, as the first premise states. Here the frequency (ϕ) of these colors among those she had seen seemed quite high, which counts against the possibility of black princess phones. The second premise reflects the fact that black is quite dissimilar to those colors in terms of hue. The third premise states that the object associated with a particular color depends weakly (α is low) on the hue of that color (i.e., knowing the hue only somewhat constrains the object). The conclusion that princess phones are not black is uncertain given the low α in the third premise.

5. OTHER INFERENCES IN THE CORE THEORY

There are a number of other inference patterns in the core theory we have developed. In this section we will give the formal representation for each of the other inference patterns together with an example of each.

Table 8 shows that two types of *derivation from mutual implication* that occurred in the protocols shown at the beginning of the paper. The positive derivation illustrates how multiple conditions were ANDed together (i.e., a warm climate, heavy rainfall, and flat terrain) as predictors of rice growing. The belief that Florida has all three leads to a prediction that rice will be grown there. In the actual protocol the respondent was unsure about rainfall in Florida, and so concluded that rice would be grown if there was enough rain (i.e., Rainfall(Florida) = heavy <===> Product(Florida) = {rice...}). This is a slight variation on the positive derivation that can be represented as follows:

$$\begin{array}{l} d_1(a) = r_1 \wedge d_2(a) = r_2 \Longleftrightarrow d_3(a) = r_3 : \alpha, \gamma_1 \\ d_1(a') = r_1 : \phi, \gamma_2 \\ \underline{a' = \mathrm{SPEC}(a) : \gamma_3} \\ \hline d_2(a') = r_2 \Longleftrightarrow d_3(a') = r_3 : \ = f(\alpha, \gamma_1, \phi, \gamma_2, \gamma_3) \end{array}$$

Insert Table 8 here

The negative derivation illustrates the fact that if any of the variables on one side of a mutual implication that are ANDed together do not have the appropriate values, then you can conclude that the variable on the other side does not have the value assumed in the mutual implication. In the example, because the Llanos did not have reliable rainfall, the respondent concluded that the Llanos probably did not produce coffee. If variables are ORed together (e.g., either heavy rainfall or irrigation are needed for growing rice) a different pattern holds: having one or the other predicts rice is grown and having neither predicts no rice is grown.

Table 9 shows the equivalent representations for derivations from mutual

Table 8

Formal Representations of Derivations from Mutual Implication

Positive Derivation

$d_1(a) = r_1 \Longleftrightarrow d_2(a) = r_2 : \alpha, \gamma_1$
$d_1(a') = r_1 : \phi, \gamma_2$
$\underline{a' = \mathrm{SPEC}(a) : \gamma_3}$
$d_2(a') = r_2 : \gamma = f(\alpha, \gamma_1, \phi, \gamma_2, \gamma_3)$

Climate(place) = warm $\wedge$ Rainfall(place) = heavy $\wedge$ Terrain(place) = flat <==>
Product(place) = {rice...} : α = high, γ_1 = certain
Climate(Florida) = warm : ϕ_1 = moderately high, γ_2 = certain
Rainfall(Florida) = heavy : ϕ_2 = moderate, γ_3 = uncertain
Terrain(Florida) = flat : ϕ_3 = high, γ_4 = certain
<u>Florida = SPEC(place) : γ_5 = certain</u>
Product(Florida) = {rice...} : γ = uncertain

Negative Derivation

$d_1(a) = r_1 \Longleftrightarrow d_2(a) = r_2 : \alpha, \gamma_1$
$d_1(a') \neq r_1 : \phi, \gamma_2$
$\underline{a' = \mathrm{SPEC}(a) : \gamma_3}$
$d_2(a') \neq r_2 : \gamma = f(\alpha, \gamma_1, \phi, \gamma_2, \gamma_3)$

Rainfall(place) = reliable $\wedge$ climate(place) = subtropical <==>
Product(place) = {coffee...} : α = moderate, γ_1 = certain
Rainfall(Llanos) $\neq$ reliable : ϕ = high, γ_2 = fairly certain
<u>Llanos = SPEC(place) : γ_3 = certain</u>
Product(Llanos) $\neq$ {coffee...} : γ = fairly certain

dependencies. It is impossible to draw a negative conclusion from a mutual dependency, since it denotes how a whole range of values on one variable relates to a range of values on another variable. But the inference patterns are different for positive and negative dependencies, so we have separated them in the table.

Insert Table 9 here

The positive dependency represents the case where as one variable increases, the other variable also increases. In the formal analysis we have denoted the entire range of both variables by three values: high, medium, and low. When a positive dependency holds, if the values of the first variable is high, medium, or low, the value of the second variable will also be high, medium, or low, respectively. This is the weakest kind of derivation possible from a mutual dependency: In the example, if a person knows that the temperature of air predicts the water holding capacity of air, and he knows that temperature of the air outside is high, then he can infer that the air outside could hold a lot of moisture. People make this kind of weak inference very frequently in reasoning about such variables (Collins & Gentner, in press; Stevens & Collins, 1980).

The pattern for the negative dependency is reversed: if the value of one variable is high, the other is low, and vice versa. We have illustrated the derivation from a negative dependency in terms of a more precise dependency between two variables. If a person believes that the latitude of a place varies negatively (and linearly) with the temperature of the place, and also that the average temperature is near 85 degrees at the equator and 0 degrees at the poles, then he might conclude that a place like Lima, Peru, that is about 10 degrees from the equator, has an average temperature of about 75 degrees. People have both more and less precise notions of how variables interact, and we have tried to preserve flexibility within our representation for handling these different degrees of precision.

Table 10 shows two forms of a transitive inference, one based on mutual implication and the other based on mutual dependency. The example for mutual implication states that if a person believes an average temperature of 85 degrees implies a place is equatorial, and that if a place is equatorial it will tend to have high humidity, then he can infer that if the average temperature of a place is 85 degrees it

Table 9

Formal Representations of Derivations from Mutual Dependencies

Derivation from Positive Dependency

$d_1(a) \xleftrightarrow{\;+\;} d_2(a) : \alpha, \gamma_1$
$d_1(a') = \text{high, medium, low} : \phi, \gamma_2$
$a' = SPEC(a) : \gamma_3$
$d_2(a') = \text{high, medium, low} : \gamma = f(\alpha, \gamma_1, \phi, \gamma_2, \gamma_3)$

Temperature(air) $\xleftrightarrow{\;+\;}$ Water holding capacity(air) : α = high, γ_1 = certain
Temperature(air outside) = high : ϕ = high, γ_2 = certain
Air outside = SPEC(air) : γ_3 = certain
Water holding capacity(air outside) = high : γ = certain

Derivation from Negative Dependency

$d_1(a) \xleftrightarrow{\;-\;} d_2(a) : \alpha, \gamma_1$
$d_1(a') = \text{high, medium, low} : \phi, \gamma_2$
$a' = SPEC(a) : \gamma_3$
$d_2(a') = \text{low, medium, high} : \gamma = f(\alpha, \gamma_1, \phi, \gamma_2, \gamma_3)$

Abs. Val. Latitude(place) $\xleftrightarrow{\;-\;}$ Aver. Temperature(place): linear;
$0°$, $85°$; $90°$, $0°$; α = moderate, γ_1 = certain
Abs. Val. Latitude(Lima Peru) = $10°$: ϕ = high, γ_2 = fairly certain
Lima Peru = SPEC(place) : γ_3 = certain
Aver. Temperature(Lima Peru) = $75°$: γ = moderately certain

will tend to have high humidity, and vice versa. This example illustrates the way people confuse causality and diagnosticity in their understanding. If one were to write the causal links for this example, it would probably go from equatorial latitude to high temperature to high humidity. But people do not systematically make a distinction between causal and diagnostic links, nor do they store things in such a systematic order. For example, they may know that equatorial places, such as jungles, have high humidity and not link it explicitly to their high temperature. Thus, the inference in this example derives a more direct link (temperature <==> humidity) from a less direct link (latitude <==> humidity). It also should be noted that the diagnostic link in the first implication (temperature => latitude) may be more constraining than the causal link (latitude => temperature). That is, there are probably more equatorial places where the average temperature is not 85 degrees (e.g. Ecuador), than places where the temperature is 85 degrees but are not equatorial.

Insert Table 10 here

The example for a transitivity inference on mutual dependency illustrates how people reason about economics (Salter, 1983). Salter asked subjects questions, such as what is the effect of an increase in interest rates on the inflation rate of a country. People gave him chains of inferences like the one shown: if interest rates increase, then growth in the money supply will decrease, and that in turn will cause the inflation rate to decrease (the latter is a positive relation). So an increase in interest rates will lead to a decrease in the inflation rate. This kind of reasoning is a major way that people construct new mutual inplications and dependencies.

Tables 11 and 12 show a set of transforms on mutual implications that follow the same pattern as the transforms on statements in the previous section. Table 11 shows four reference transforms that parallel the last four statement transforms shown in Tables 5 and 7. (In fact there is a quite direct equivalence, because any statement can be transformed into a mutual implication in the following way: Flowers (England) = {daffodils...} goes into type(place) = England <==> flowers(place) = {daffodils...}, or more generally, d(a) =r goes into type(A)=a <==> d(A) = r.) We have represented the three positive transforms (i.e. generalization, specialization, and similarity) in the rule at the top, with the three alternatives shown (GEN, SPEC, and SIM) where they occur

Table 10

Formal Representations of Transitivity Transforms

On Mutual Implication

$d_1(a) = r_1$ <==> $d_2(a) = r_2$: α_1, β_1, γ_1

$\underline{d_2(a) = r_2}$ <==> $\underline{d_3(a) = r_3}$: α_2, β_2, γ_2

$d_1(a) = r_1$ <==> $d_3(a) = r_3$: $\alpha = f(\alpha_1, \alpha_2)$, $\beta = f(\beta_1, \beta_2)$ $\gamma = f(\gamma_1, \gamma_2)$

Aver. Temperature(place) = 85° <==> Latitude(place) = equatorial :
α_1 = high, β_1 = fairly high, γ_1 = certain
Latitude(place) = equatorial <==> Abs. humidity(place) = high :
α_2 = high, β_2 = moderate, γ_2 = certain
Aver. Temperature(place) = 85° <==> Abs. Humidity(place) = high :
α = high, β = low, γ = certain

On Mutual Dependency

$d_1(a)$ <--> $d_2(a)$: α_1, β_1, γ_1

$d_2(a)$ <--> $d_3(a)$: α_2, β_2, γ_2

$d_1(a)$ <--> $d_3(a)$: $\alpha = f(\alpha_1, \alpha_2)$, $\beta = f(\beta_1, \beta_2)$, $\gamma = f(\gamma_1, \gamma_2)$

Interest rates(country) $\overset{-}{<\!-\!->}$ Money supply growth(country):
α_1 = high, β_1 = moderate, γ_1 = certain
Money supply growth(country) $\overset{+}{<\!-\!->}$ Inflation rate(country):
α_2 = high, β_2 = high, γ_2 = certain
Interest rates(country) $\overset{-}{<\!-\!->}$ Inflation rate (country):
α = high, β = low, γ = certain

in the rule. The typicality parameter (τ) is associated with the GEN and SPEC transforms, and the similarity parameter (σ) with the SIM transform. The example omits the certainty parameters for simplicity. In English the example states the following: given the belief that if a place is subtropical, it is likely to produce oranges, this implies that if a place is subtropical, it is likely to produce citrus fruits (a generalization), or naval oranges (a specialization), or grapefruit (a similarity transform). The dissimilarity transform at the bottom follows the same pattern: if you think that subtropical places produce oranges, and apples are dissimilar to oranges with respect to their growing conditions, then probably subtropical places do not produce apples.

Insert Table 11 here

Table 12 shows the corresponding four types (i.e., GEN, SPEC, SIM, and DIS) of argument transforms. These correspond to the first four statement transforms shown in Tables 5 and 7. We illustrate the four with a demographic example: if one believes that men who live in the tropics have a short life expectancy and that farmers are typical of men in terms of their demographic characteristics, then one can plausibly infer that farmers have a short life expectancy if they live in the tropics. Similarly one can infer that people in general and women (because they are similar to men in their demographic characteristics) have short life expectancy in the tropics. Finally, one might conclude that birds do not have a short life expectancy in the tropics, if one thinks they are dissimilar to men in their demographic characteristics.

Insert Table 12 here

Table 13 shows the corresponding positive transforms for mutual dependencies. We have illustrated these with another example from economics: if one believes that the business tax rate in a state negatively impacts the amount of investment in the state, then one might generalize this relationship to any governmental unit, or particularize it to Illinois, or conclude that it would also apply to Canadian provinces. There is really no negative transform based on dissimilarity that corresponds to these three positive transforms. For example, if one believes that communist countries are quite dissimilar from states in their economics, the most one can conclude is that

Table 11

Formal Representations of Reference Transforms on Mutual Implications

Positive Transforms

$d_1(a) = r_1 <==> d_2(a) = r_2 : \alpha_1, \gamma_1$

$\{GEN\}$

$r'_2 = \{SPEC\}\ r_2 \text{ in } CX(d_2, D(d_2)) : \{{\tau \atop \sigma}\}, \gamma_2$

$\{SIM\}$

$\underline{D(d_2) <--> A(d_2) : \alpha_2, \gamma_3}$

$d_1(a) = r_1 <==> d_2(a) = r'_2 : \gamma = f(\alpha_1, \gamma_1 \{{\tau \atop \sigma}\} \gamma_2, \alpha_2, \gamma_3)$

Climate(place) = subtropical <==> Fruit(place) = {oranges...}
{Citrus fruits} {GEN }
{Naval oranges} = {SPEC} (oranges) in CX (fruit, growing conditions(fruit))
{Grapefruit} {SIM }
<u>Growing conditions(fruit) <--> Place(fruit)</u>
Climate(place) = subtropical <==> Fruit(place) = {Citrus fruit...} {Naval oranges...} {Grapefruit...}

Negative Transform

$d_1(a) = r_1 <==> d_2(a) = r_2 : \alpha_1, \gamma_1$

$r'_2 = DIS\ r_2 \text{ in } CX\ (d_2, D(d_2)) : \sigma, \gamma_2$

$\underline{D(d_2) <--> A(d_2) : \alpha_2, \gamma_3}$

$d_1(a) = r_1 <==> d_2(a) \neq r'_2 : \gamma = f(\alpha_1, \gamma_1, \sigma, \gamma_2, \alpha_2, \gamma_3)$

Climate(place) = subtropical <==> Fruit(place) = {oranges...}
Apples = DIS(oranges) in CX (fruit, growing conditions (fruit))
<u>Growing conditions(fruit) <--> Place (fruit)</u>
Climate(place) = subtropical <==> Fruit(place) $\neq$ {apple...}

Table 12

Formal Representations of Argument Transforms on Mutual Implications

Positive Transforms

$$
\begin{array}{l}
d_1(a) = \quad \Longleftrightarrow d_2(a) = r_2 : \alpha_1, \gamma_1 \\
a' = \begin{array}{l}\{GEN\ \} \\ \{SPEC\} \\ \{SIM\ \}\end{array} (a) \text{ in } CX\,(A, d_3(A)) : \left\{ \begin{array}{c}\tau \\ \sigma\end{array} \right\}, \gamma_2 \\
\underline{d_3(A) \longleftrightarrow d_2(A) : \alpha_2, \gamma_3} \\
d_1(a) = r_1 \Longleftrightarrow d_2(a) = r_2 : \gamma = f(\alpha_1, \gamma_1, \left\{ \begin{array}{c}\tau \\ \sigma\end{array} \right\} \gamma_2, \alpha_2, \gamma_3)
\end{array}
$$

Habitat(man) = tropics <==> Life expectancy (man) = short
{GEN } (farmer)
Man = {SPEC} (person) in CX(people, demographic characteristics(people))
{SIM } (woman)
<u>Demographic characteristics(people) <--> life expectancy(people)</u>

$$
\text{Habitat } \begin{array}{c}\text{(farmer)} \\ \text{(person)} \\ \text{(woman)}\end{array} = \text{tropics} \Longleftrightarrow \text{life expectancy } \begin{array}{c}\text{(farmer)} \\ \text{(person)} \\ \text{(woman)}\end{array} = \text{low}
$$

Negative Transforms

$$
\begin{array}{l}
d_1(a) = r_1 \Longleftrightarrow d_2(a) = r_2 : \alpha_1, \gamma_1 \\
a' = DIS(a) \text{ in } CX(A, d_3(A)) : \sigma, \gamma_2 \\
\underline{d_3(A) \longleftrightarrow d_2(A) : \alpha_2, \gamma_3} \\
d_1(a') = \quad \Longleftrightarrow d_2(a') = \gamma : f(\alpha_1, \gamma_1, \sigma, \gamma_2, \alpha_2, \gamma_3)
\end{array}
$$

Habitat(man) = tropics <==> life expectancy(man) = short
Man = DIS(bird) in CX(animals, demographic characteristics (animals))
<u>Demographic characteristics(animals) <--> life expectancy (animals)</u>
Habitat(birds) = tropics<==> life expectancy(birds) = low

there is no negative relation between the business tax rate (if there were one) and the amount of investment; that is to say, no conclusion can be drawn. In such a case we just omit the form from the theory, because the theory does not specify conclusions that cannot be drawn. Similarly, there can be no reference transforms on mutual dependencies, because they do not involve a reference term.

Insert Table 13 here

Table 13

Formal Representations of Argument Transforms on Mutual Dependencies

Positive Transforms

$d_1(a) \overset{\pm}{<-->} d_2(a) : \alpha_1, \gamma_1$

{GEN }
a' = {SPEC} (a) in CX (A, d_3(A)) : $\{\frac{\tau}{\sigma}\}, \gamma_2$
{SIM }

$\underline{d_3(A) <--> d_2(A) : \alpha_2, \gamma_3}$

$d_1(a') \overset{\pm}{<-->} d_2(a') : \quad = f(\alpha_1, \gamma_1, \frac{\tau}{\sigma}, \gamma_2, \alpha_2, \gamma_3)$

Business tax rate (state) $\overset{-}{<-->}$ Amount of investment (state))
{Government unit} = {GEN }
{Illinois } = {SPEC} (state) in CX(place, economics (place))
{Province } = {SIM }
<u>Economics(place) <--> Amount of investment(place)</u>

Business tax rate (government unit) (Illinois) (province) $\overset{-}{<-->}$ Amount of investment (government unit) (Illinois) (province)

6. CONCLUSION

The difficulty in constructing a theory of plausible reasoning from analyzing actual cases of human reasoning is that the theory is likely to be underconstrained. That is to say, there may be many cases where people could employ a particular reasoning pattern, but do not because of other constraints on its invocation. As it stands now, the only constraints we place on the invocation of any inference pattern is that its premises be satisfied and that its certainty parameters not drive the conclusion below some threshold level of certainty. But there may well be other factors that constrain the invocation of any inference pattern.

In order to test out the core theory, we plan to build a computer system incorporating the reasoning patterns derived from our analysis. We will then be able to see what inferences the system draws given different knowledge bases. We plan to evaluate the theory in a series of experiments comparing the system's reasoning to that of expert human reasoners. To do this we will ask expert human reasoners, working from well-specified, small knowledge bases to draw plausible conclusions from each knowledge base and to estimate the certainty of each conclusion. These experts will be asked to put aside, as best they can, other knowledge they may have about the domain.

At the same time we will run the system on each small knowledge base to see what plausible conclusions the system draws, and with how much certainty. For each knowledge base, then we will have three different classes of inference: conclusions both computer and experts draw, conclusions the computer draws but experts do not, and conclusions experts draw that the computer does not draw. The two non-overlapping lists require different kinds of refinement to the theory. Where the computer draws a conclusion experts do not, we will go to the experts to see if the conclusion seems at all plausible to them. If not, then the set of inference rules must be modified to prevent such implausible conclusions from being drawn. Where experts draw a conclusion that the computer does not, we will first have to ascertain if they are drawing upon information the computer does not have. If not, then new inference rules must be added to the system to produce the conclusions that the human experts drew. The modifications to the theory will be implemented in a new version of the system, and the whole process will recycle until a stable state is reached, where the system and expert reasoners draw the same conclusions from new knowledge bases.

7. ACKNOWLEDGEMENTS

This research was supported by the Army Research Institute under Contract No. MDA 903-85-C-0411, by the National Institute of Education under Contract No. HEW-NIE-400-80-0031 by the National Science Foundation under Grant No. DCR-84-06801, and by the Office of Naval Research under Grant No. N00014-82-K-0186, and the Defense Advanced Research Projects Agency under grant N00014-K-85-0878.

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