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PRESHAPE JACOBIANS FOR MINIMUM MOMENTUM GRASPING

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Abstract

This paper deals with the planning of dexterous grasps for multifingered robot hands. We derive a real-time, joint space finger path planning algorithm for the *enclose* phase of grasping motion. The algorithm minimizes the impact *momentum* of the hand. It uses a *Preshape Jacobian* matrix to map task-level hand preshape requirements into kinematic constraints. A master-slave scheme avoids inter-finger collisions and reduces the dimensionality of the planning problem.

1. INTRODUCTION

The work described in this paper is motivated by applications that involve dexterous manipulation by autonomous or teleoperated robots in unstructured, uncertain environments. Examples may include equipment maintenance and repair operations in space, under the sea, in a nuclear power plant, or in a chemically or otherwise contaminated area.

Robot manipulators have traditionally used a gripper (capable of opening and closing motions) attached to their wrist to achieve a rather modest level of mechanical dexterity. This has been adequate for simple manufacturing applications in which the environment may be conveniently structured. The need for a higher level of dexterity, more versatility and more adaptability in end-effectors has become increasingly apparent as the application of automation has grown into areas where the environment is unstructured, and tasks have become more complex. Multifingered hands hold a great deal of promise because of their ability to impart precise localized forces and velocities to objects, and because of their ability to provide stable grasps. Unfortunately, complications arise, as finger coordination, finger trajectory planning, and task planning are not well defined.

The motion of a robot hand is subdivided into five phases [6]: (i) the reach phase during which the hand moves to the vicinity of some object, (ii) the preshaping phase which defines an approach volume between the fingers, (iii) the enclose phase where the object reaches the focus of the approach volume and finger/object contacts occur, (iv) the grip phase during which fingers apply forces to the object, (v) the manipulation phase which deals with the transfer of the available degrees of freedom to the object. Discrete preshape classes such as lateral pinch, cylindrical grasp, hook grasp, fingertip grasp have been defined on the basis of qualitative task requirements (e.g. power vs precision) in dexterous manipulation [1]. A topological model of task-oriented preshaping is described in [9].

In this paper, we assume that the reach and preshape phases have been completed, and focus our attention on the derivation of a master-slave finger path planning algorithm during the enclose phase. Inputs to the algorithm include the desired Cartesian position of the master fingertip, as well as the desired (task dependent) hand preshape. We develop a *minimum momentum* strategy that generates a sequence of knot points (in joint space) through which the fingers must pass. We base this strategy on two goals: (i) minimizing the impact momentum of the master finger, (ii) preserving the hand preshape constraints during the enclose phase. We accomplish the first goal through a local Newton iteration applied to the master finger. This problem is first formulated in section 2, then analyzed mathematically in section 3 by developing a dyadic formalism of grasping motion. To accomplish the second goal we derive a *Preshape Jacobian* matrix that injects global (hand level) preshape constraints into the local (finger level) Newton scheme. We derive in section 4, Preshape Jacobians for three types of preshapes: *unilateral pinch*, *pinch*, and *hook*, for the special case of planar grasping. The extension of our method to 3D motion is finally discussed in section 5.

2. FINGER PATH PLANNING

In this section, we develop an on-line path planning algorithm for a single finger. Given any initial state of the finger in joint space, a sequence of knot points to the desired final state (contact) is found such that the finger momentum at that final state is minimized.

2.1 Finger momentum

When a wrench vector is applied to the ith finger, it causes changes in its momentum vector G_i. Let the vectors r_i, v_i, and Δr_i denote the position, velocity, and a finite displacement of the master fingertip, all in Cartesian coordinates relative to a base (palmar) frame. Similarly, let the vectors θ_i, ω_i, and $\Delta\theta_i$ denote the joint space position, velocity, and a finite displacement of the master finger. We assume, without loss of generality, that the desired position of the finger is given by $r_i^d = 0$ and $\theta_i^d = 0$. Any other values can be accommodated by a simple translation of the coordinate systems. Similarly, we assume that $v_i^d = 0$ and $\omega_i^d = 0$.

The *momentum* of the ith finger is:

$$G_i = r_i \times m_i v_i + m_i v_i$$

where $r_i \times m_i v_i$ and $m_i v_i$ are the angular and linear momentum of the finger. m_i is the mass of the finger, assumed (as a worst case) to be concentrated at the fingertip.

2.2 Finger path planning

The differential mapping of $\Delta \mathbf{r}_i$ into momentum changes $\Delta \mathbf{G}_i$ is: $\Delta \mathbf{G}_i = \mathbf{J}_{r_i} \Delta \mathbf{r}_i$ where

$$\mathbf{J}_{r_i} = \begin{bmatrix} \frac{\partial G_{i_x}}{dx} & \frac{\partial G_{i_x}}{dy} & \frac{\partial G_{i_x}}{dz} \\ \frac{\partial G_{i_y}}{dx} & \frac{\partial G_{i_y}}{dy} & \frac{\partial G_{i_y}}{dz} \\ \frac{\partial G_{i_z}}{dx} & \frac{\partial G_{i_z}}{dy} & \frac{\partial G_{i_z}}{dz} \end{bmatrix}$$

Let $\mathbf{J}_{f_i}$ be the finger Jacobian, i.e. $\Delta \mathbf{r}_i = \mathbf{J}_{f_i} \Delta \theta_i$

Then, $\Delta G_i = \mathbf{J}_{r_i} \Delta \mathbf{r}_i = \mathbf{J}_{r_i} \mathbf{J}_{f_i} \Delta \theta_i = \mathbf{J}_{\theta_i} \Delta \theta_i$

Expanding the momentum function about the final state of the finger, a sequence of knot points is generated by the Newton iteration:

$$\theta_i^{k+1} = \theta_i^k - \mathbf{J}_{\theta_i}^{-1} \mathbf{G}_i(\theta_i^k)$$

that converges when $\mathbf{G}_i(\theta_i^k)$ tends to zero.

2.3 Dyadic expansion of the momentum

Since the grasp momentum is a vector function, the derivation of the matrix $\mathbf{J}_{\theta_i}$ (in section 4) requires the expansion of the differential momentum in its *dyadic* form [10], i.e.:

$$\Delta \mathbf{G}_i = (\Delta \mathbf{r}_i \cdot \nabla) \mathbf{G}_i$$

$$= \frac{1}{2}[\nabla \times (\mathbf{G}_i \times \Delta \mathbf{r}_i) + \nabla(\mathbf{G}_i \cdot \Delta \mathbf{r}_i) + \mathbf{G}_i(\nabla \cdot \Delta \mathbf{r}_i)$$

$$+ \Delta \mathbf{r}_i(\nabla \cdot \mathbf{G}_i) - \mathbf{G}_i \times (\nabla \times \Delta \mathbf{r}_i) - \Delta \mathbf{r}_i \times (\nabla \times \mathbf{G}_i)]$$

The differential momentum includes three groups of terms:

gradient	$\nabla(\mathbf{G}_i \cdot \Delta \mathbf{r}_i)$		
divergence	$\mathbf{G}_i(\nabla \cdot \Delta \mathbf{r}_i)$	$\Delta \mathbf{r}_i(\nabla \cdot \mathbf{G}_i)$	
curl	$\nabla \times (\mathbf{G}_i \times \Delta \mathbf{r}_i)$	$\mathbf{G}_i \times (\nabla \times \Delta \mathbf{r}_i)$	$\Delta \mathbf{r}_i \times (\nabla \times \mathbf{G}_i)$

For planar grasping, the angular momentum does not lie in the plane of motion. The linear and angular components of the momentum are therefore not additive. Instead, we use the *planar discrepancy* between angular and linear momentum. Assuming, for simplicity, a unit mass, we get:

$$\mathbf{G}_{2_i} = (\mathbf{v}_i \times \mathbf{r}_i) \times \mathbf{v}_i$$

The expansion of the dyadic $\mathbf{G}_{2_i}$ yields:

$$(\Delta \mathbf{r}_i \cdot \nabla) \mathbf{G}_{2_i} = (\Delta \mathbf{r}_i \cdot \nabla)(\mathbf{v}_i \times \mathbf{r}_i) \times \mathbf{v}_i$$

$$= \frac{1}{2}\{ [(\mathbf{v}_i \cdot \Delta \mathbf{r}_i)\mathbf{r}_i - (\mathbf{r}_i \cdot \mathbf{v}_i)](\nabla \cdot \mathbf{v}_i)$$

$$+ [(\mathbf{v}_i \times \mathbf{r}_i) \cdot \Delta \mathbf{r}_i](\nabla \times \mathbf{v}_i)$$

$$+ (\mathbf{r}_i \cdot \mathbf{v}_i)[\mathbf{v}_i \times (\nabla \times \Delta \mathbf{r}_i) + \Delta \mathbf{r}_i \times (\nabla \times \mathbf{v}_i)]$$

$$- (\mathbf{v}_i \cdot \mathbf{v}_i)[\Delta \mathbf{r}_i \times (\nabla \times \mathbf{r}_i) + \mathbf{r}_i \times (\nabla \times \Delta \mathbf{r}_i)]$$

$$+ \nabla[(\mathbf{v}_i \cdot \mathbf{v}_i)(\mathbf{r}_i \cdot \Delta \mathbf{r}_i) - (\mathbf{r}_i \cdot \mathbf{v}_i)(\mathbf{v}_i \cdot \Delta \mathbf{r}_i)]$$

$$- (\mathbf{v}_i \cdot \Delta \mathbf{r}_i)(\mathbf{J}_{V_i} \mathbf{r}_i - \mathbf{J}_{R_i} \mathbf{v}_i + 2\mathbf{v}_i) + 2(\mathbf{v}_i \cdot \mathbf{v}_i)\Delta \mathbf{r}_i \}$$

where

$$\mathbf{J}_{V_i} = \begin{bmatrix} \frac{\partial v_{i_x}}{dx} & \frac{\partial v_{i_x}}{dy} \\ \frac{\partial v_{i_y}}{dx} & \frac{\partial v_{i_y}}{dy} \end{bmatrix}, \quad \mathbf{J}_{R_i} = \begin{bmatrix} \frac{\partial r_{i_x}}{dx} & \frac{\partial r_{i_x}}{dy} \\ \frac{\partial r_{i_y}}{dx} & \frac{\partial r_{i_y}}{dy} \end{bmatrix}$$

The terms of this dyadic expansion can be interpreted by using the concepts of flux and curl from vector field theory [10].

3. HAND FLUX AND CURL

We postulate that the grasping can be analyzed in terms of two motion characteristics:the *divergence* of the fingertips, andthe *curling* of each finger. Fig.1 shows a schematic representation of a three fingered hand. The hand encloses an arbitrary *volume* bounded by the fingers. We consider two types of slices through this volume: *link slices*, and *joint slices*. Let M be the number of fingers, and N the number of joints per finger. The ith (i = 1,...,M) link slice LS_i is any planar closed curve which is tangent to all the links of the ith finger. The jth (j = 1,...,N) joint slice DS_i is a planar surface that passes through the jth link of each finger.

3.1 Hand flux

Let **n** be the unit normal vector of the joint slice DS_i (Fig.1a), and let F be a vector function. The *flux* of F is the surface integral of the normal component of F, denoted by $\int\int_S \mathbf{F} \cdot \mathbf{n}\, ds$. The limit of the ratio of this integral to the volume sliced by the surface DS_i, as the volume shrinks to zero, about a point D_0 is the *divergence*:

$$\nabla \cdot \mathbf{F} = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \int\int \mathbf{F} \cdot \mathbf{n}\, ds$$

It represents the extent to which F *diverges* from D_0, and measures the flux flowing through a volume shrinking to that point. The *divergence theorem* relates surface integrals to volume integrals:

$$\int\int_S \mathbf{F} \cdot \mathbf{n}\, ds = \int\int\int_V \nabla \cdot \mathbf{F}\, dV$$

Fig.1a shows the *capping surface* of the hand volume (the fingertip joint slice), with a unit normal vector **n**. We consider the *flow* of fingertips on this capping surface. The total mass crossing an infinitesimal surface Δs in a time interval Δt is:

$$\Delta t\, \Delta s \sum_i^M \mathbf{v}_i \cos\theta_i .$$

The total mass through the slice DS_i is equal to the rate of flow through the capping surface. From the divergence theorem,

$$\sum_i \int\int_S \mathbf{v}_i \cdot \mathbf{n}\, ds = \sum_i \int\int\int_V \nabla \cdot \mathbf{v}_i\, dV = \int\int\int_V \sum_i \nabla \cdot \mathbf{v}_i\, dV$$

The grasping divergence is a measure of the extent to which the moving mass *diverges* from a point, and is therefore equal to the sum of the divergences of individual fingertip velocities. We derive the flux of the hand as the sum of the flux of its fingers moving over a shrinking preshape volume:

$$(\nabla \cdot \mathbf{v})^H = \sum_i (\nabla \cdot \mathbf{v}_i)$$

3.2 Hand curl

Consider a path integral around a closed curve C (on the link slice LS_i in fig.1b) of the tangential component of a vector function: $\oint \mathbf{F} \cdot \mathbf{t}\, dc$, where dc is an infinitesimal curve segment, **F** is a vector function defined everywhere on curve C, and **t** is a unit vector tangent to C. This path integral is called a *circulation*. The *curl* is defined as the limit of the circulation to area ratio as the area tends to zero. It is a vector function normal to the surface enclosed by the curve. Let $\oint_{C_a} \mathbf{F} \cdot \mathbf{t}\, ds$ be the circulation of **F** about some path enclosing a surface with normal vector **n**. Then

$$\nabla \times \mathbf{F} = \lim_{\Delta S \to 0} \frac{1}{\Delta S} \oint_{C_a} \mathbf{F} \cdot \mathbf{t}\, ds$$

The curl of a line integral around a closed path expresses rotation, or *curling around*. We model the curling around of a finger by the circulation of the fingertip along a curve contained in its link slice surface. As the fingers close during grasping, the fingertip moves along a path enclosing a surface that shrinks. The curl $\nabla \times \mathbf{r}_i$ relates the angular velocity of the entire finger (due to changes of the fingertip position $\mathbf{r}_i$) to the joint angles forming the curvature. Similarly, the curl $\nabla \times \mathbf{v}_i$ relates the angular velocity of the finger to the joint velocities shaping its curvature. Each component of the curl vectors defines the directional curling of the finger with respect to its own base. The *curl vectors* of an N-fingered hand are defined as:

$$(\nabla \times \mathbf{r})^H = \sum_{i=1}^{N} \nabla \times \mathbf{r}_i \text{ and } (\nabla \times \mathbf{v})^H = \sum_{i=1}^{N} \nabla \times \mathbf{v}_i$$

The other curl terms in the dyadic expansion are:

$$\mathbf{v}_i \times (\nabla \times \Delta \mathbf{r})^H, \qquad \Delta \mathbf{r}_i \times (\nabla \times \mathbf{v})^H$$

$$\Delta \mathbf{r}_i \times (\nabla \times \mathbf{r})^H, \qquad \mathbf{r}_i \times (\nabla \times \Delta \mathbf{r})^H$$

The first two account for *curving while translating* behavior of the hand, while the last two are measures of positional discrepancy.

4. 2D PRESHAPE JACOBIANS

In this section, we derive a *Preshape Jacobian* matrix for different types of 2D preshapes for planar grasping motion. Our goal is to map high-level task specifications into joint angles and velocities.

4.1 The Preshape Jacobian

A sequence of knot points for the ith finger was generated in section 2 by using a Newton iteration in joint space:

$$\theta_i^{k+1} = \theta_i^k - \mathbf{J}_{\theta_i}^{-1}\, \mathbf{G}_i(\theta_i^k)$$

We now modify this iteration so that global (hand level) preshape constraints, embedded in the Preshape Jacobian, are accounted for. Applying the iteration to the master finger (i = m),

$$\theta_m^{k+1} = \theta_m^k - \mathbf{J}_\pi^{-1}\, \mathbf{G}_m(\theta_m^k)$$

$\mathbf{J}_\pi$ is the *Preshape Jacobian* matrix, defined by:

$$\mathbf{J}_\pi \Delta\theta_m = \Delta \mathbf{G}_\pi$$

For 2D motion, the derivation of the *preshape momentum differential* yields [3]:

$$\begin{aligned}
\Delta \mathbf{G}_{2_\pi} = \frac{1}{2}\{ & [(\mathbf{v}_m \cdot \Delta \mathbf{r}_m)\mathbf{r}_m - (\mathbf{r}_m \cdot \mathbf{v}_m)](\nabla \cdot \mathbf{v})^H \\
& + [(\mathbf{v}_m \times \mathbf{r}_m) \cdot \Delta \mathbf{r}_m](\nabla \times \mathbf{v})^H \\
& + (\mathbf{r}_m \cdot \mathbf{v}_m)[\mathbf{v}_m \times (\nabla \times \Delta \mathbf{r})^H + \Delta \mathbf{r}_m \times (\nabla \times \mathbf{v})^H] \\
& - (\mathbf{v}_m \cdot \mathbf{v}_m)[\Delta \mathbf{r}_m \times (\nabla \times \mathbf{r})^H + \mathbf{r}_m \times (\nabla \times \Delta \mathbf{r})^H] \\
& + \nabla[(\mathbf{v}_m \cdot \mathbf{v}_m)(\mathbf{r}_m \cdot \Delta \mathbf{r}_m) - (\mathbf{r}_m \cdot \mathbf{v}_m)(\mathbf{v}_m \cdot \Delta \mathbf{r}_m)] \\
& - (\mathbf{v}_m \cdot \Delta \mathbf{r}_m)(\mathbf{V}_m \mathbf{r}_m - \mathbf{R}_m \mathbf{v}_m + 2\mathbf{v}_m) + 2(\mathbf{v}_m \cdot \mathbf{v}_m)\Delta \mathbf{r}_m \}
\end{aligned}$$

$\Delta \mathbf{G}_{2_\pi}$ is obtained from $\Delta \mathbf{G}_i$ by replacing the flux and curl terms for a single finger, i.e. $\nabla \cdot \mathbf{v}_i$, $\nabla \times \mathbf{v}_i$, and $\nabla \times \mathbf{r}_i$, by the flux and curl expressions for the whole hand, i.e.:

$$(\nabla \cdot \mathbf{v})^H = \sum_{i=1}^{N} \sigma_i^{\bullet v}\, \nabla \cdot \mathbf{v}_i$$

$$(\nabla \times \mathbf{v})^H = \sum_{i=1}^{N} \sigma^{\times v}\, \nabla \times \mathbf{v}_i$$

$$(\nabla \times \mathbf{r})^H = \sum_{i=1}^{N} \sigma_i^{\times r}\, \nabla \times \mathbf{r}_i$$

where the coefficients: $-1 \le \sigma^{\bullet v}, \sigma^{\times v}, \sigma^{\times r} \le 1$ depend on the specific hand preshape constraint. Two examples are given below.

4.2 The Pinch Jacobian

In the *pinch grasp* (fig.2), we assume that the two fingers are preshaped symmetrically, and that they remain symmetric during the reach motion. We use the following set of constraints to model the pinch preshape:

$$(\nabla \times \mathbf{v})^H = 0, \quad (\nabla \times \mathbf{r})^H = 0, \quad (\nabla \cdot \mathbf{v})^H = \nabla \cdot v_1 + \nabla \cdot v_2$$

The results from the derivation of the Preshape Jacobian for a pinch grasp are listed in Appendix A.

4.3 The Hook Jacobian

We form the hook grasp (fig.2) by coupling the slave fingertip 2 with a joint (j) of the master finger 1. The constraints for this model are:

$$\mathbf{r}_{2_x} = \mathbf{r}_{j_x} \qquad \mathbf{v}_{2_x} = \mathbf{v}_{j_x}$$

$$\mathbf{r}_{2_y} = -\mathbf{r}_{j_y} \qquad \mathbf{v}_{2_y} = -\mathbf{v}_{j_y}$$

and lead to:

$$(\nabla \times \mathbf{r})^H = \nabla \times \mathbf{r}_2 - \nabla \times \mathbf{r}_1$$

$$(\nabla \times \mathbf{v})^H = \nabla \times \mathbf{v}_2 - \nabla \times \mathbf{v}_1$$

$$(\nabla \cdot \mathbf{v})^H = \nabla \cdot \mathbf{v}_2 + \nabla \cdot \mathbf{v}_1$$

The results from the derivation of the Preshape Jacobian for a hook grasp are listed in Appendix B.

5. 3D PRESHAPE JACOBIANS

For 3D grasping, the ith finger momentum (per unit mass) is:
$G_{3_i} = v_i \times r_i + v_i$

From the dyadic expansion of the differential momentum,

$$
\begin{aligned}
(\Delta r_i \cdot \nabla) G_{3_i} &= (\Delta r_i \cdot \nabla)(v_i \times r_i + v_i) \\
&= \frac{1}{2}[(v_i \cdot \Delta r_i)(\nabla \times r_i) - (r_i \cdot \Delta r_i)(\nabla \times v_i) \\
&+ \Delta r_i (r_i \cdot \nabla \times v_i - v_i \cdot \nabla \times r_i) - v_i \times (\nabla \times \Delta r_i) \\
&+ \Delta r_i \times (\nabla \times v_i) - \nabla(v_i \cdot \Delta r_i) - (\nabla \cdot v_i)(\Delta r_i \times r_i) \\
&+ J_{\Delta_i}(v_i \times r_i - v_i) + J_{V_i} r_i \times \Delta r_i + J_{V_i} \Delta r_i \\
&+ J_{R_i} v_i \times \Delta r_i + 3\Delta r_i \times v_i + 3 v_i]
\end{aligned}
$$

where J_{V_i}, J_{Δ_i}, J_{R_i} are respectively equal to:

$$
\begin{bmatrix}
\frac{\partial v_{i_x}}{dx} & \frac{\partial v_{i_x}}{dy} & \frac{\partial v_{i_x}}{dz} \\
\frac{\partial v_{i_y}}{dx} & \frac{\partial v_{i_y}}{dy} & \frac{\partial v_{i_y}}{dz} \\
\frac{\partial v_{i_z}}{dx} & \frac{\partial v_{i_z}}{dy} & \frac{\partial v_{i_z}}{dz}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial h_{i_x}}{dx} & \frac{\partial h_{i_x}}{dy} & \frac{\partial h_{i_x}}{dz} \\
\frac{\partial h_{i_y}}{dx} & \frac{\partial h_{i_y}}{dy} & \frac{\partial h_{i_y}}{dz} \\
\frac{\partial h_{i_z}}{dx} & \frac{\partial h_{i_z}}{dy} & \frac{\partial h_{i_z}}{dz}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial r_{i_x}}{dx} & \frac{\partial r_{i_x}}{dy} & \frac{\partial r_{i_x}}{dz} \\
\frac{\partial r_{i_y}}{dx} & \frac{\partial r_{i_y}}{dy} & \frac{\partial r_{i_y}}{dz} \\
\frac{\partial r_{i_z}}{dx} & \frac{\partial r_{i_z}}{dy} & \frac{\partial r_{i_z}}{dz}
\end{bmatrix}
$$

The preshape differential momentum is:

$$
\begin{aligned}
\Delta G_{3_x} &= \frac{1}{2}[(v_m \cdot \Delta r_m)(\nabla \times r)^H - (r_m \cdot \Delta r_m)(\nabla \times v)^H \\
&+ (\Delta r_m (r_m \cdot (\nabla \times v)^H - v_m \cdot (\nabla \times r)^H) \\
&- v_m \times (\nabla \times \Delta r)^H + \Delta r_m \times (\nabla \times v)^H \\
&- (\nabla \cdot v)^H (\Delta r_m \times r_m) + J_\Delta^H (v_m \times r_m - v_m) \\
&+ J_V^H (r_m \times \Delta r_m + \Delta r_m) + J_R^H (v_m \times \Delta r_m) \\
&- \nabla(v_m \cdot \Delta r_m) + 3\Delta r_m \times v_m + 3 v_m]
\end{aligned}
$$

where

$$
J_V^H = \sum_{i=1}^{3} J_{V_i}, \quad J_\Delta^H = \sum_{i=1}^{3} J_{\Delta_i}, \quad J_R^H = \sum_{i=1}^{3} J_{R_i}
$$

5.1 Limit grasps

In this subsection, we describe *limit* grasps that occur when one or more joint angles reach their limit.

The *fist* occurs in the case of pure curling motion (i.e. finite curl vector and zero divergence). The fingers curl onto themselves, leading to a *fist* characterized by maximum curl.

The *stretched finger pinch* occurs when both the divergence and the curl vectors tend to zero. Zero divergence implies zero translational motion, while zero curl corresponds to extended fingers with joint angles at their extreme values. This is the case of a hand closing to a *stretched finger pinch* grasp with each joint slice reduced to a point.

The *palmar* grasp occurs for maximum divergence and zero curl, i.e. the hand is opening from a preshape to a *palmar* grasp.

Any preshape that does not correspond to a limit grasp is characterized by finite curl and finite divergence. There is a continuum of such grasp types, corresponding to intermediate values of finger curl and divergence. We now consider four special cases.

5.2 Fingertip Grasp

The *fingertip* grasp has a relatively small number of constraints. We derive the curl vector for this preshape from the system of curl vectors shown in fig.1b, leading to the following constraints:

$$
\begin{aligned}
\nabla \cdot v &= \nabla \cdot v_1 + \nabla \cdot v_2 + \nabla \cdot v_3 \\
\nabla \times r &= \nabla \times r_1 + \nabla \times r_2 - \nabla \times r_3 \\
\nabla \times v &= \nabla \times v_1 + \nabla \times v_2 - \nabla \times v_3
\end{aligned}
$$

The sign of the curl vector is determined by the right hand rule. The divergence is due to the component of the fingertip Cartesian velocity that is normal to the capping surface. The projection of the velocity on the joint slice is a constraint on J_V.

Let v'_i denote the velocity component parallel to the capping surface. Fig.1 shows the constraints on the velocities, $v'_1 \cdot v'_2 = |v'_1||v'_2|\cos\alpha$, $v'_1 \cdot v'_3 = |v'_1||v'_3|\cos\beta$

Finger 1 is the master, while fingers 2 and 3 are slaves. These constraints lead to the matrix:

$$
J_V^H = \begin{bmatrix}
\frac{\partial \phi_1}{dx} & \frac{\partial \phi_1}{dy} & \frac{\partial \phi_1}{dz} \\
\frac{\partial \phi_2}{dx} & \frac{\partial \phi_2}{dy} & \frac{\partial \phi_2}{dz} \\
\frac{\partial \phi_3}{dx} & \frac{\partial \phi_3}{dy} & \frac{\partial \phi_3}{dz}
\end{bmatrix}
$$

where

$$
\begin{aligned}
\phi_1 &= v_{x1} + v_{x2}\cos\alpha + v_{x3}\cos\beta \\
\phi_2 &= v_{y1} + v_{y2}\cos\alpha + v_{y3}\cos\beta \\
\phi_3 &= v_{z1} + v_{z2} + v_{z3}
\end{aligned}
$$

This expression is for the case when the slice coordinate frame is aligned with the palm base frame. However if a different orientation or tilt of the slice is to be preserved during the motion, then these vectors are premultiplied by the rotation matrices $R_{z_i,\gamma}$ and $R_{y_i,\delta}$ to adjust the orientation γ and tilt δ of the capping surface with respect to the palm base frame.

5.3 Cylindrical grasp

All finger curl vectors are aligned (Fig.3). The hand preshape curl vectors are obtained by adding the finger curl vectors:

$$
\begin{aligned}
\nabla \cdot v &= \nabla \cdot v_1 + \nabla \cdot v_2 + \nabla \cdot v_3 \\
\nabla \times v &= \nabla \times v_3 + \nabla \times v_2 - \nabla \times v_1 \\
\nabla \times r &= \nabla \times r_3 + \nabla \times r_2 - \nabla \times r_1
\end{aligned}
$$

Fig.3b shows the geometric velocity constraints $v_1 // v_2 // v_3$ for the cylindrical grasp, i.e.:

$J_V^H = J_{V_1} + J_{V_2} + J_{V_3}$

5.4 Hook grasp

The *hook* grasp is a special case of the cylindrical grasp. The major difference between the two grasps is the coupling that occurs on the fingertip distal slice. For a hook grasp, we couple, on this slice, the fingertips of two fingers with the ith joint of the third finger. Fig.3 shows a hook grasp with the distal joint slice coupling fingertips 2 and 3 with the $(N-1)^{st}$ joint of finger 1.

For the hook grasp, the computation of the total preshape curl vectors, $\nabla \times \mathbf{v}$ and $\nabla \times \mathbf{r}$, and that of total divergence $\nabla \cdot \mathbf{v}$ are similar to the cylindrical grasp case. However the position and velocity variables used for slave fingers 2 and 3 are different. We obtain the position vectors $\mathbf{r}_i$ of these fingers as functions of the position of the $(N-1)^{st}$ joint of the master finger.

6. DISCUSSION

The minimum momentum grasp planning algorithm described in this paper was motivated by applications such as a space station *retriever* that is designed to grasp loose objects tumbling freely in space. One finger is designated as the master, and its path is planned so as to minimize the impact momentum. A Preshape Jacobian is derived to map task-dependent preshape constraints into kinematic constraints, and thus provide the necessary coupling with the slave fingers. Planning the paths of the slave fingers follows directly from these constraints.

We are currently conducting computer simulations for 2D and 3D grasps. The concept of Julia sets [2] is used to graphically study the convergence of the grasping process in various regions of the state space. We are also in the process of deriving expressions of four (fingertip, lateral pinch, cylindrical, and hook) 3D Preshape Jacobians for the Utah/MIT hand.

Grasp taxonomies [1,4] map different high level task and object characteristics to preshape requirements. Our parameterization of the grasping motion in terms of hand flux and hand curl provides a mechanism for mapping such high level grasp requirements (e.g. power and precision) to low level kinematic attributes.

7. ACKNOWLEDGEMENTS

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APPENDIX A
PREHSAPE JACOBIAN FOR A PINCH GRASP

$$\mathbf{J}_x = \mathbf{A}_{pinch}\,\mathbf{J}_f + \mathbf{B}_{pinch}\,\mathbf{J}_\delta$$

$$\mathbf{J}_f = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial x}{\partial \theta_1} & \dfrac{\partial x}{\partial \theta_2} \\ \dfrac{\partial y}{\partial \theta_1} & \dfrac{\partial y}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} s_{12}+s_1 & s_{12} \\ -c_{12}-c_1 & -c_{12} \end{bmatrix}$$

$$\mathbf{J}_\delta \Delta\theta = \begin{bmatrix} \dfrac{\partial \Delta x}{\partial y} & \dfrac{\partial \Delta y}{\partial x} \end{bmatrix}^T$$

$$\mathbf{J}_\delta = -\begin{bmatrix} 2+\left(\dfrac{J_{11}}{J_{21}}\right)^2+\left(\dfrac{J_{12}}{J_{22}}\right)^2 & \dfrac{J_{22}}{J_{21}}+\dfrac{J_{11}J_{12}}{J_{21}^2}+\left(\dfrac{J_{12}}{J_{22}}\right)^2-1 \\ -2-\left(\dfrac{J_{21}}{J_{11}}\right)^2-\left(\dfrac{J_{22}}{J_{12}}\right)^2 & -\dfrac{J_{12}}{J_{11}}-\dfrac{J_{21}J_{22}}{J_{11}^2}-\left(\dfrac{J_{22}}{J_{12}}\right)^2-1 \end{bmatrix}$$

$$\mathbf{A}_{pinch} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}$$

$$\alpha_{11} = 2v_x^2 + 3v_y^2 + \left(2\frac{\partial x}{\partial y} - \frac{\partial y}{\partial x}\right)v_x v_y - (2xv_x + 3yv_y)v_{xx} - 2yv_x v_{xy} + (2xv_y - yv_x)v_{yx}$$

$$\alpha_{12} = \frac{\partial y}{\partial x}v_x^2 + 2\frac{\partial x}{\partial y}v_y^2 - v_x v_y + (2yv_x - xv_y)v_{xx} - 2yv_y v_{xy} - xv_x v_{yx}$$

$$\alpha_{21} = \frac{\partial x}{\partial y}v_y^2 - v_x v_y + 2yv_x v_{xx} - yv_y v_{xy} + (2xv_y - yv_x)v_{yy}$$

$$\alpha_{22} = 3v_x^2 + 2v_y^2 - \frac{\partial x}{\partial y}v_x v_y - 2xv_x v_{xx} + (2yv_x - xv_y)v_{xy} - xv_x v_{yy}$$

$$\mathbf{B}_{pinch} = \begin{bmatrix} yv_x^2 - xv_x v_y & 0 \\ 0 & xv_y^2 - yv_x v_y \end{bmatrix}$$

$$v_x = -J_{11}\dot\theta_1 - J_{12}\dot\theta_2,\; v_y = -J_{21}\dot\theta_1 - J_{22}\dot\theta_2$$

$$v_{xx} = \frac{\partial v_x}{\partial x} = -\left(\frac{J_{21}}{J_{11}} + \frac{J_{22}}{J_{12}}\right)\dot\theta_1 - \left(\frac{J_{22}}{J_{11}} + \frac{J_{22}}{J_{12}}\right)\dot\theta_2$$

$$v_{xy} = \frac{\partial v_x}{\partial y} = -2\dot\theta_1 + \frac{2c_{12} - 2s_{12} + c_1}{J_{21}}\dot\theta_2$$

$$v_{yx} = \frac{\partial v_y}{\partial x} = 2\dot\theta_1 + \frac{2s_{12} + s_1}{J_{11}}\dot\theta_2$$

$$v_{yy} = \frac{\partial v_y}{\partial y} = \left(\frac{J_{11}}{J_{21}} + \frac{J_{12}}{J_{22}}\right)\dot{\theta}_1 + \left(\frac{J_{12}}{J_{21}} + \frac{J_{12}}{J_{22}}\right)\dot{\theta}_2$$

$$\frac{\partial y}{\partial x} = \frac{J_{21}}{J_{11}} + \frac{J_{22}}{J_{12}} \quad \frac{\partial x}{\partial y} = \frac{J_{11}}{J_{21}} + \frac{J_{12}}{J_{22}}$$

APPENDIX B
PREHSAPE JACOBIAN FOR A HOOK GRASP

$$\mathbf{J}_x = \mathbf{A}_{hook}\,\mathbf{J}_f + \mathbf{B}_{hook}\,\mathbf{J}_\delta + \mathbf{C}_{hook}$$

$$A_{hook} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}$$

$$\alpha_{11} = -\left(x\frac{c_1}{s_1} + y\right)\dot{\theta}_1 v_x - y\frac{c_1^2 - s_1^2}{c_1 s_1}\dot{\theta}_1 v_y$$

$$+ 2v_x^2 + \left(\frac{s_1}{c_1} + \frac{\partial x}{\partial y} - \frac{\partial y}{\partial x}\right)v_x v_y + 3v_y^2$$

$$-(xv_x + 2yv_y)v_{xx} - yv_x v_{xy} + (2xv_y - yv_x)v_{yx} - yv_y v_{yy}$$

$$\alpha_{12} = -2x\dot{\theta}_1 v_x - \left(-y + \frac{s_1}{c_1}x\right)\dot{\theta}_1 v_y$$

$$+ \left(\frac{c_1^2 - s_1^2}{c_1 s_1} + \frac{\partial x}{\partial y}\right)v_x^2 - \left(\frac{c_1}{s_1} - \frac{\partial y}{\partial x} + 2\frac{\partial x}{\partial y}\right)v_y^2 - v_x v_y$$

$$+ (2yv_x - xv_y)v_{xx} - xv_x v_{xy} - (2xv_x + yv_y)v_{yx} + xv_y v_{yy}$$

$$\alpha_{21} = \left(-x + y\frac{c_1}{s_1}\right)\dot{\theta}_1 v_x + 2y\dot{\theta}_1 v_y$$

$$+ \left(\frac{s_1}{c_1} - \frac{\partial x}{\partial y} + 2\frac{\partial y}{\partial x}\right)v_x^2 + \left(\frac{-c_1^2 + s_1^2}{c_1 s_1} + \frac{\partial y}{\partial x}\right)v_y^2 - v_x v_y$$

$$+ yv_x v_{xx} - (xv_x + 2yv_y)v_{xy} + yv_y v_{yx} + xv_y v_{yy}$$

$$\alpha_{22} = -\left(x\frac{c_1^2 - s_1^2}{c_1 s_1}\right)\dot{\theta}_1 v_x + \left(x + \frac{ys_1}{c_1}\right)\dot{\theta}_1 v_y$$

$$+ 3v_x^2 + 2v_y^2 + \left(-\frac{\partial x}{\partial y} + \frac{\partial y}{\partial x}\right)v_x v_y$$

$$-xv_x v_{xx} + (2yv_x - xv_y)v_{xy} - xv_y v_{yx} - (2xv_x + yv_y)v_{yy}$$

$$B_{hook} = \begin{bmatrix} -xv_x v_y + yv_x^2 & 0 \\ 0 & xv_y^2 - yv_x v_y \end{bmatrix}$$

$$C_{hook} = \begin{bmatrix} yv_x^2 - xv_x v_y & xv_x v_y - yv_x^2 \\ yv_x v_y - xv_y^2 & xv_y^2 - yv_x v_y \end{bmatrix} \begin{bmatrix} -\frac{1}{c_1^2} & 0 \\ \frac{1}{s_1^2} & 0 \end{bmatrix}$$

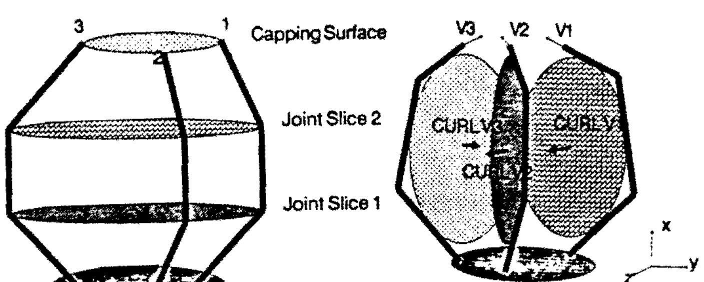
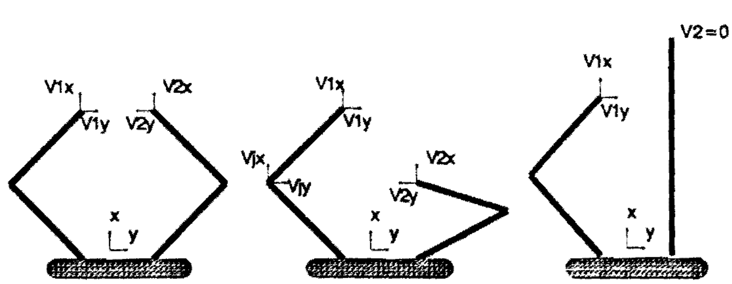


Fig. 1a Joint slices

Fig. 1b Link slices

Fig. 1 Volumetric hand model

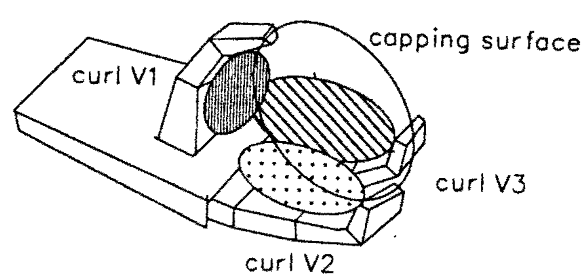


(a) Pinch

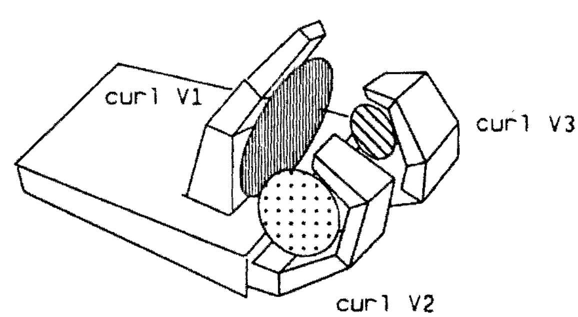
(b) Hook

(c) Unilateral pinch

Fig. 2 2D Preshapes



Cylindrical grasp



Hook grasp

Fig. 3 3D Preshapes