

**VALIDATION, VERIFICATION, AND
EXPERIMENTATION WITH
ABACUS.2**

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BY

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PROJECT

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ABSTRACT

ABACUS.2, a quantitative discovery system, was ported from the Symbolics Zeta LISP to Vax and Sun Common LISP. ABACUS.2 was then validated and verified. During the validation and verification process, many errors and omissions were found, both in the documentation and in the source code. These were corrected and/or documented. Overall, the system performed as stated in the documentation, and was able to discover both linear and non-linear terms from sets of quantitative data. A second phase of this study is now underway, and the results will be reported at an upcoming workshop and conference on quantitative discovery.

1. INTRODUCTION

This paper reports the results of a project than began in November 1988 with the receipt of the ABACUS.2 quantitative discovery system [ABACUS.2, 1989]. ABACUS.2 was written as part of a master's thesis, and incorporates earlier work on ABACUS and other quantitative discovery systems [Greene, 1988]. For additional readings on quantitative and qualitative discovery systems, see [Langley, Simon, Bradshaw, and Zytkow, 1987].

The original purpose of this study was to replace the quantitative discovery module of EMERALD [EMERALD, 1988], which is based on ABACUS [Falkenhainer and Michalski, 1986], with Greene's ABACUS.2. EMERALD is a system that contains five modules, with each module demonstrating a specific area of machine learning. The second part of the project was to integrate EMERALD's existing examples of quantitative discovery with new examples, and to provide a uniform user interface to these examples via the SHARE

functions, a set of graphics and windowing primitives built from MIT's X Windows package.

For many reasons, as described in the following paragraphs, the direction of this study deviated from the original goals. ABACUS.2 had to be ported to the Vax or Sun because there was no way to transfer the contents of the DC600 cartridge containing ABACUS.2 to the Symbolics. The porting of ABACUS.2 resulted in many modifications to the code, and about three months to complete. This was due mostly to the learning curve associated with becoming familiar with the source code and the difference between Zeta, Vax, and Sun Common LISP.

Moreover, after porting the system, ABACUS.2 needed to be validated and verified. In addition, it was necessary to test whether the ported versions of the program still behaved correctly with the modifications made during the porting process. ABACUS.2 could not be integrated with EMERALD until the new version of ABACUS.2 was beta tested (i.e., to find bugs, to receive feedback from users, and so on).

There are many unresolved issues related to developing a user interface for ABACUS.2. The main issue is whether the system is to be used as a tool for experimentation by researchers who are familiar with the topic of quantitative discovery, or by people who have a limited knowledge of quantitative discovery, LISP, and so on. The former will require the development of a separate interface from that used in EMERALD. Another issue is how to adequately demonstrate to un-sophisticated users, the power of ABACUS.2. Part of the power of ABACUS.2 is the capability provided to the user to modify parameters that are used to guide the discovery process. These and other issues are being studied.

In addition, Drs. Zytkow and Kodratof requested working copies of the ABACUS.2 system for use in their research, as well as their students' research. This required the

debugging, baselining, and documentation of the system, as well as providing demonstrations and help with installation of the system.

Finally, the results presented in this report, along with the results from the ongoing experiments with ABACUS.2, are to be presented at an upcoming workshop and conference. In order to meet submission deadlines, a great deal of time was spent preparing these papers, running experiments, and getting as many bugs out of the system as possible before sending it out to users, especially if the users are not within close proximity of the laboratory. In this case, ABACUS.2 was sent to researchers at M.I.T. and the University of Paris.

2. PORTING ABACUS.2

The copy of ABACUS.2 that Greg Greene forwarded to the laboratory runs on the Symbolics workstation, and contains Zeta LISP specific source code. A straight port from the Symbolics to the Vax or Sun workstations was not possible for two reasons. When the tape was received, the Symbolics was not connected to the local area network, and ABACUS.2 was only available on a Sun formatted cartridge tape. Secondly, the Zeta LISP specific code would not run under Vax or Sun Common LISP.

The Zeta LISP code was primarily concentrated in two files: *system-init.lisp* and *system.lisp*. These two files were used to accomplish the same task as a UNIX make file. The contents of these files are used to establish the order that the system files are to be loaded and compiled. Additional Zeta LISP specific code was included at the beginning of each file for specifying paths, text formatting, and performing checking of functions. The majority of ABACUS.2, however, was written in Common LISP, and that portion of the code was easily ported to the Vax and Sun workstations.

Many problems were encountered in generating a compiled version of ABACUS.2. The two major steps that were taken in order to get ABACUS.2 to compile were to load and compile the files such that the dependencies among software modules were correct, and the conversion to Common LISP. For example, both the Vax and Sun workstations require that a macro be defined before it is called by a function. Much of the reordering of macros and functions was performed on the files *form.lisp* and *io.lisp*. Any future changes to ABACUS.2 should be made taking into consideration the effects of changes on the dependencies already built into the system at the logic, function, macro, and file levels.

The other major problem was that the thesis is not a user's manual. The thesis does not provide enough details about the actual operation of the system, nor does it document known bugs and frequently encountered errors. Therefore it was difficult to distinguish between normal and abnormal program behavior. Much of the initial work on this project was done on a trial-and-error basis.

3. VALIDATION

Validation consisted of determining whether the system source and object code were consistent with the system specifications. Greene's thesis was used as the specifications document. Validation involved reviewing both the source code and the ability of ABACUS.2 to use dependencies to discover non-linear terms.

Inconsistencies and omissions were found in the ABACUS.2 specifications. and system. These inconsistencies and omissions, however, do not have severe affects on the ability of ABACUS.2 discover linear and non-linear terms. For example, references are made to features that are not part of the current version of the system. The print-flags parameter does

not support the 's' option. This was part of an earlier version of the system, and was intended to allow the user to specify various levels of debug output.

Overall, the system met the specifications provided in the thesis. One reason for the discrepancies is that the project was not under configuration management control. Specifications, features, data, and results span many versions of ABACUS.2. In addition, it is left up to the reader to determine, for instance, why the same values were used for some parameters over the set of experiments.

4. VERIFICATION

Verification consisted of a functional review of the system. This included checking the correctness of the program, whether the user interface worked as expected, and whether the program could be broken (i.e., error measurement).

The first stage in the verification process was to check the results reported by Greene. This was done by rerunning the experiments using the same datasets and parameters. Tables 1 and 2 summarize Greene's and the verification test results, respectively. Listings of the ABACUS.2 output for these experiments are located in Appendix A. Only two of the experiments had the same outcome. This was for the instantaneous current and angular conservation of momentum experiments. The same results were obtained for Stokes' Law (without equations supplied) only after the level of uncertainty was changed from 0.99 to 0.95. No results were obtained for Snell's Law or Stokes' Law (with equations supplied). Snell's Law experiment ran out of memory before an equation was output. Stokes' Law with equations supplied would not work with the equation form reported by Greene: $(=v (+ (* c1 t)))$. This program would not accept the c1 variable. c1 was not included in the declaration of

PARAMETER	Centrifugal Force	Steff's Law	Instantaneous Current	Stokes' Law	Stokes' Law with equation forms supplied	Angular Conservation of Momentum
MAXSTAR	10	10	10	10	10	10
COVERMODE	IC	IC	IC	IC	IC	IC
UNCERTAINTY	0.02	0.02	0.02	0.02	0.02	0.02
CONSTANCY	0.95	0.95	0.95	0.99	0.97	0.95
INVERSE-SW	30	30	30	30	30	30
DIRECT-SW	70	70	80	70	70	70
UNVIGNORE	NIL	NIL	NIL	(3 4)	(3 4)	NIL
DEBUG	NIL	NIL	T	NIL	NIL	NIL
PRINT-FLAGS	(P E)	(P E)	(P E)	(P S)	(P S)	(P E)
FILTER-LEVEL	2	2	2	2	2	2
SEARCH-LIMIT	3	3	3	3	3	3
MAX NODES	1000	1000	1000	1000	1000	1000
CLUSTER ON	T	T	T	T	T	T
CLUSTER ORDER	(GC NC RC)	(GC NC RC)	(NC GC RC)	(GC NC RC)	(GC NC RC)	(NC GC RC)
MIN-LINK-VALUE	50.0	50.0	50.0	50.0	50.0	50.0
starLEF	MAX-COV 0.5	MAX-COV 0.5	MAX-COV 0.5	MAX-COV 0.5	MAX-COV 0.5	MAX-COV 0.5
MIN-WEI 0.0	MIN-WEI 0.0	0.0	MIN-WEI	0.0	MIN-WEI 0.0	MIN-WEI 0.0
MIN-SEL 0.0	MIN-SEL 0.0	MIN-SEL 0.0	MIN-SEL 0.0	MIN-SEL 0.0		
BEAM-VALUE	no value included	no value included	0.5	0.5	0.5	0.5
DATA						
Number rows by columns (events x variables)	56 x 6	70 x 4	65 x 5	72 x 7	72 x 7	12 x 6
RESULTS						
Number of classes formed	1	1	1	3	3	1
Equations	$F^*R = \cos(\theta)M^*V^2$	$N^2 = ((\sin(\theta_1)N_1)/\sin(\theta_2))$	$I^*R = \cos((\omega * TIME)) * V_0$	A: $V = 9.8175^*T$ B: $R^*V = 0.9556^*M$ C: $R^*V = 0.7336^*M$	A: $V = 9.8175^*T$ B: $R^*V = 0.9556^*M$ C: $R^*V = 0.7336^*M$	$EDISS = 0.4999994^*(M^*V_0^*V_0) + (0.49999753^*(-(M^*V^*V))) + (0.49999753^*(-(I^*\omega * \omega)))$

TABLE 1. Summary of Greene's Experimental Data and Results.

RESULTS	Centrifugal Force	Stokes' Law	Instantaneous Current	Stokes' Law with equation forms supplied	Stokes' Law	Angular Conservation of Momentum
Same result as Greene reported?	No	No	Yes	No	No	Yes
Number of classes formed	7	5	1	0	0	1
Cover/Equation						
CLASSA	[LENGTH = 3546.7] F=R=41175.5	[N1=0.9334174;-1.5253406] [THETA1=0.4] N=0.7589*N2	$I^*R = \cos((\text{OMEGA} * \text{TIME})) * \text{VO}$			$\text{EDISS} = 0.49999222 * (M^* \text{VO}^* \text{VO}) + 0.4999954 * (-M^* \text{V}^* \text{V}) + 0.49998108 * (-I^* \text{OMEGA} * \text{OMEGA})$
CLASSB	[LENGTH = 4231.0] F=924766*M	[N1=0.5] $((\text{THETA2}^2)/\text{THETA}) + 2*((1^* \text{THETA2}^2)/\text{THETA1}) + (1^2 * \text{THETA1}) - 6.2035 + (2*(1^* \text{THETA2}) + ((\text{THETA2}^3)/(\text{THETA1}^2))) + (1^2 * \text{THETA2}))$				
CLASSC	[F = 430.0] R=5.5136*M	[N1=0.7] $2*((\text{THETA2}^2)/((\text{THETA2}^2) + (\text{THETA1}^2) + 2*(\text{THETA1} * \text{THETA2}) + 2*((\text{THETA2}^3)/\text{THETA1}))) + ((\text{THETA1}^2)/((\text{THETA2}^2) + (\text{THETA1}^2) + 2*(\text{THETA1} * \text{THETA2}) + 2*((\text{THETA2}^3)/\text{THETA1}))) + 2*((\text{THETA1} * \text{THETA2})/((\text{THETA2}^2) + (\text{THETA1}^2) + 2*(\text{THETA1} * \text{THETA2}) + 2*((\text{THETA2}^3)/\text{THETA1}))) - 1.0104$				
CLASSD	[LENGTH = 3211.8] F=1.4259*(V^2)	[N1=2.5] THETA2=0.8862*THETA1				
CLASSE	[F = 500.0] R=0.0878*(V^2)					
CLASSF	[F = 340.0] M*V^2=12348.6080					

TABLE 2. Summary of Verification of Greene's Results.

variables.

The second stage of the verification process consisted of performing error measurement. This in essence consisted of trying to "break" the program. While performing sensitivity analysis on the certainty and constancy parameters, undocumented bugs associated with boundary conditions were found. For example, when the parameter uncertainty was set at 0.0 or 1.0 (i.e., boundary conditions), the following error message was received for the speed of sound in air experiment:

```
Checking for COS(/(LAMBDA TIME) NIL))  
Fatal error in function SYSTEM::%EVAL (signal with  
error)
```

See Table 3 and Appendix B9. This error was the result of attempting division by zero since the term $(\text{number}/1 - \text{uncertainty})$ is in the equation. See the source file *abacus.lisp*. To fix this and other border value problems, the system needs to check for special cases. In this example, the system needs to check for the case where uncertainty is set to 1.0. The solution is to return a very large number for this case. Below is the patch to the code that was made is:

```
(defun calc-mean (number)  
  (if (= *uncertainty* 1.0)  
      1E8  
      (+ number (abs ....))))
```

5. EXPERIMENTATION

Experimentation consisted of running datasets through ABACUS.2. The sample datasets represented:

- computer distributed operating system statistics
- densities of liquids [CRC, 1979]

uncertainty	constancy	class(es)	formula(s)	error message
0.00	0.00			checking for cos ((- (%form -65)(%form -64))) ERROR: Fatal error in function SYSTEM::%EVAL
0.00	1.00			checking for cos ((- (%form -3)(%form -2))) ERROR: Fatal error in function SYSTEM::%EVAL
0.01	0.98	1	$V^2 = 401.3222 * T$	
0.01	0.95	1	$V^2 = 401.3222 * T$	
0.01	0.90	1	$V^2 = 401.3222 * T$	
0.01	0.70	3	class A: $T = 0.9033 * V$ class B: $T = 0.8248 * V$ class C: $T = 0.7893 * V$	
0.01	0.50	3	class A: $T = 0.9033 * V$ class B: $T = 0.8248 * V$ class C: $T = 0.7893 * V$	
0.01	0.30	3	class A: $T = 0.9033 * V$ class B: $T = 0.8248 * V$ class C: $T = 0.7893 * V$	
0.01	0.10	3	class A: $T = 0.9033 * V$ class B: $T = 0.8248 * V$ class C: $T = 0.7893 * V$	
0.01	0.02	3	class A: $T = 0.9033 * V$ class B: $T = 0.8248 * V$ class C: $T = 0.7893 * V$	
0.01	0.01	3	class A: $T = 0.9033 * V$ class B: $T = 0.8248 * V$ class C: $T = 0.7893 * V$	
0.01	0.00	3	class A: $T = 0.9033 * V$ class B: $T = 0.8248 * V$ class C: $T = 0.7893 * V$	
0.02	0.98	1	$V^2 = 401.3222 * T$	
0.02	0.95	1	$V^2 = 401.3222 * T$	
0.02	0.90	1	$V^2 = 401.3222 * T$	
0.02	0.70	3	class A: $T = 0.9033 * V$ class B: $T = 0.8248 * V$ class C: $T = 0.7893 * V$	
0.02	0.50	3	class A: $T = 0.9033 * V$ class B: $T = 0.8248 * V$ class C: $T = 0.7893 * V$	
0.02	0.30	3	class A: $T = 0.9033 * V$ class B: $T = 0.8248 * V$ class C: $T = 0.7893 * V$	
0.02	0.10	3	class A: $T = 0.9033 * V$ class B: $T = 0.8248 * V$ class C: $T = 0.7893 * V$	
0.02	0.02	3	class A: $T = 0.9033 * V$ class B: $T = 0.8248 * V$ class C: $T = 0.7893 * V$	
0.02	0.01	3	class A: $T = 0.9033 * V$ class B: $T = 0.8248 * V$ class C: $T = 0.7893 * V$	
0.02	0.00	3	class A: $T = 0.9033 * V$ class B: $T = 0.8248 * V$ class C: $T = 0.7893 * V$	
0.05	0.98	1	$V^2 = 401.3222 * T$	
0.05	0.95	1	$V^2 = 401.3222 * T$	
0.05	0.90	2	A: $T = 0.9033 * V$ B: $T = 0.8119 * V$	
0.05	0.70	2	A: $T = 0.9033 * V$ B: $T = 0.8119 * V$	

TABLE 3. Results from Varying the Uncertainty and Constancy Parameters.

uncertainty	constancy	class(es)	formula(s)	error message
0.05	0.50	2	A: $T = 0.9033 \cdot V$ B: $T = 0.8119 \cdot V$	
0.05	0.30	2	A: $T = 0.9033 \cdot V$ B: $T = 0.8119 \cdot V$	
0.05	0.10	2	A: $T = 0.9033 \cdot V$ B: $T = 0.8119 \cdot V$	
0.05	0.02	2	A: $T = 0.9033 \cdot V$ B: $T = 0.8119 \cdot V$	
0.05	0.01	2	A: $T = 0.9033 \cdot V$ B: $T = 0.8119 \cdot V$	
0.05	0.00	2	A: $T = 0.9033 \cdot V$ B: $T = 0.8119 \cdot V$	
0.10	0.98	1	$T = 0.8363 \cdot V$	
0.10	0.95	1	$T = 0.8363 \cdot V$	
0.10	0.90	1	$T = 0.8363 \cdot V$	
0.10	0.70	1	$T = 0.8363 \cdot V$	
0.10	0.50	1	$T = 0.8363 \cdot V$	
0.10	0.30	1	$T = 0.8363 \cdot V$	
0.10	0.10	1	$T = 0.8363 \cdot V$	
0.10	0.02	1	$T = 0.8363 \cdot V$	
0.10	0.01	1	$T = 0.8363 \cdot V$	
0.10	0.00	1	$T = 0.8363 \cdot V$	
0.30	0.98	1	$T = 0.8363 \cdot V$	
0.30	0.95	1	$T = 0.8363 \cdot V$	
0.30	0.90	1	$T = 0.8363 \cdot V$	
0.30	0.70	1	$T = 0.8363 \cdot V$	
0.30	0.50	1	$T = 0.8363 \cdot V$	
0.30	0.30	1	$T = 0.8363 \cdot V$	
0.30	0.10	1	$T = 0.8363 \cdot V$	
0.30	0.02	1	$T = 0.8363 \cdot V$	
0.30	0.01	1	$T = 0.8363 \cdot V$	
0.30	0.00	1	$T = 0.8363 \cdot V$	
0.50	0.98	1	$T = 0.8363 \cdot V$	
0.50	0.95	1	$T = 0.8363 \cdot V$	
0.50	0.90	1	$T = 0.8363 \cdot V$	
0.50	0.70	1	$T = 0.8363 \cdot V$	
0.50	0.50	1	$T = 0.8363 \cdot V$	
0.50	0.30	1	$T = 0.8363 \cdot V$	
0.50	0.10	1	$T = 0.8363 \cdot V$	
0.50	0.02	1	$T = 0.8363 \cdot V$	
0.50	0.01	1	$T = 0.8363 \cdot V$	
0.50	0.00	1	$T = 0.8363 \cdot V$	
0.70	0.98	1	$T = 0.8363 \cdot V$	
0.70	0.95	1	$T = 0.8363 \cdot V$	
0.70	0.90	1	$T = 0.8363 \cdot V$	
0.70	0.70	1	$T = 0.8363 \cdot V$	
0.70	0.50	1	$T = 0.8363 \cdot V$	
0.70	0.30	1	$T = 0.8363 \cdot V$	
0.70	0.10	1	$T = 0.8363 \cdot V$	
0.70	0.02	1	$T = 0.8363 \cdot V$	
0.70	0.01	1	$T = 0.8363 \cdot V$	

TABLE 3. Results from Varying the Uncertainty and Constancy Parameters.

uncertainty	constancy	class(es)	formula(s)	error message
0.70	0.00	1	$T = 0.8363 \cdot V$	
0.90	0.98	1	$T = 0.8363 \cdot V$	
0.90	0.95	1	$T = 0.8363 \cdot V$	
0.90	0.90	1	$T = 0.8363 \cdot V$	
0.90	0.70	1	$T = 0.8363 \cdot V$	
0.90	0.50	1	$T = 0.8363 \cdot V$	
0.90	0.30	1	$T = 0.8363 \cdot V$	
0.90	0.10	1	$T = 0.8363 \cdot V$	
0.90	0.02	1	$T = 0.8363 \cdot V$	
0.90	0.01	1	$T = 0.8363 \cdot V$	
0.90	0.00	1	$T = 0.8363 \cdot V$	
0.95	0.98	1	$T = 0.8363 \cdot V$	
0.95	0.95	1	$T = 0.8363 \cdot V$	
0.95	0.90	1	$T = 0.8363 \cdot V$	
0.95	0.70	1	$T = 0.8363 \cdot V$	
0.95	0.50	1	$T = 0.8363 \cdot V$	
0.95	0.30	1	$T = 0.8363 \cdot V$	
0.95	0.10	1	$T = 0.8363 \cdot V$	
0.95	0.02	1	$T = 0.8363 \cdot V$	
0.95	0.01	1	$T = 0.8363 \cdot V$	
0.95	0.00	1	$T = 0.8363 \cdot V$	
0.98	0.98	1	$T = 0.8363 \cdot V$	
0.98	0.95	1	$T = 0.8363 \cdot V$	
0.98	0.90	1	$T = 0.8363 \cdot V$	
0.98	0.70	1	$T = 0.8363 \cdot V$	
0.98	0.50	1	$T = 0.8363 \cdot V$	
0.98	0.30	1	$T = 0.8363 \cdot V$	
0.98	0.10	1	$T = 0.8363 \cdot V$	
0.98	0.02	1	$T = 0.8363 \cdot V$	
0.98	0.01	1	$T = 0.8363 \cdot V$	
0.98	0.00	1	$T = 0.8363 \cdot V$	
1.00	0.00			ERROR: floating divide by zero (/ 0.789266 0.0)
1.00	1.00			ERROR: floating divide by zero (/ 0.789266 0.0)

TABLE 3. Results from Varying the Uncertainty and Constancy Parameters.

- illumination of surfaces [CRC, 1979]
- thermal conductivity of copper compounds [CRC, 1979]
- speed of sound in air [CRC, 1979]

The output listings for these experiments are located in Appendix B. The first task was to generate datasets for use with ABACUS.2. The distributed operating system data was gathered from a local area network. The results of the experiments run on this dataset are summarized in Table 4. The other datasets were contrived in order to have sample data to experiment with.

It was confirmed that ABACUS.2 does not provide for independent variables. Instead, it searches for a combination of a subset of the variables. It searches for a combination of a subset of the variables or all of the variables that evaluate to a constant. If there is no combination of variables which evaluate to a constant, then no dependency can be found. For example, if you had three variables a , b , and c , and they formed the equation $a = bc/2$, if a is left out, then ABACUS.2 will not find that the combination $bc/2$ evaluates to a constant. This is evident from the results displayed in Appendix B. The parameters for these examples were varied to see the effect on the speed and precision of ABACUS.2. These results are described with each output listing. To get around the problem described above, it is sometimes advisable to set the `unignore` parameter for particular columns such that specific variables are not held constant while dependencies are being calculated.

Appendix C contains programs developed to gather data and/or build datasets.

PARAMETER	EXPERIMENT							
	1	2	3	4	5	6	7	8
MAXSTAR	10	10	10	10	10	10	10	10
COVERMODE	IC	IC	IC	IC	IC	IC	IC	IC
UNCERTAINTY	0.02	0.02	0.15	0.15	0.15	0.50	0.50	0.50
CONSTANCY	0.95	0.95	0.75	0.75	0.75	0.50	0.50	0.50
INVERSE-SW	30	30	20	30	30	30	30	30
DIRECT-SW	70	70	70	70	70	70	70	70
UNIVIONORE	NIL	NIL	(0..17)	(0..17)	(0..17)	NIL	(0..17)	(0..17)
DEBUG	T	T	NIL	NIL	NIL	NIL	NIL	NIL
PRINT-FLAGS	(P E)	(P)	(P)	(P)	(P)	(P)	(P)	(P)
FILTER-LEVEL	2	2	2	2	2	2	2	2
SEARCH-LIMIT	3	3	3	3	3	3	3	3
MAX NODES	1000	1000	1000	1000	1000	1000	1000	1000
CLUSTER ON	T	T	T	T	T	T	T	T
CLUSTER ORDER	(NC GC RC)	(GC NC RC)	(NC GC RC)	(NC GC RC)	(NC GC RC)	(NC GC RC)	(NC GC RC)	(GC NC RC)
MIN-LINK-VALUE	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
starLEF	MAX-COV 0.5 MIN-WEI 0.0 MIN-SEL 0.0	MAX-COV 0.5 MIN-WEI 0.0 MIN-SEL 0.0	MAX-COV 0.5 MIN-WEI 0.0 MIN-SEL 0.0	MAX-COV 0.5 MIN-WEI 0.0 MIN-SEL 0.0	MAX-COV 0.5 MIN-WEI 0.0 MIN-SEL 0.0	MAX-COV 0.5 MIN-WEI 0.0 MIN-SEL 0.0	MAX-COV 0.5 MIN-WEI 0.0 MIN-SEL 0.0	MAX-COV 0.5 MIN-WEI 0.0 MIN-SEL 0.0
BEAM-VALUE	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
DATA								
Number rows by columns (events x variables)	202 x 18	202 x 18	202 x 18	202 x 18	202 x 18	202 x 18	202 x 18	202 x 18
RESULTS								
Number of classes formed	0	0	0	0	0	0	1	1
Equations	none	none	none	none	none	none	((FREE + VIRT_MEM = 11938.6930))	((FREE + VIRTUAL_MEM = 11938.6930))

TABLE 4. Summary of Experiments with Network Dataset.

6. VERSION 1.0

The Sun and Vax workstation versions of ABACUS.2 were baselined so that when researchers report on their experiments with ABACUS.2, they can refer to a particular version. Baselining also makes it possible to rollback to a version of ABACUS.2 whose execution behavior and components are known or documented. This first baselined version is referred to as version 1.0.

Part of the reason this project took so long to complete was that there was very little documentation that provided information on design and implementation decisions, dependencies among the functions, and so on. This has been corrected to some degree for version 1.0, but a configuration management package, such as Unix's SCCS, should be used to formally record all future changes to ABACUS.2.

The cartridge tape accompanying this report contains Version 1.0 of ABACUS.2 for the Sun workstation, along with sample datasets.

7. USER INTERFACE

The design of a user interface for a scientific discovery system poses many problems. For ABACUS.2, these include

- slow response time
- handling quantitative and qualitative data
- user experimentation

The response time for medium to large datasets is poor. In addition, unless debug mode is invoked, the user does not receive any intermediate messages on the terminal screen until

ABACUS.2 has completed its search for linear and non-linear terms. What is required here is a way to maintain the user's interest in the system during the waiting period. One solution to the response problem would be to have a pop-up window display the progress of the system during the discovery process. In addition, this capability should be tailored such that the user can specify the amount of detail that is to be displayed in the pop-up window, and the help level.

Providing a means to handle a mixture of symbolic and quantitative data via a single interface is difficult. Unlike the EMERALD system which has a purely graphical interface, the quantitative flavor of ABACUS.2 dictates the use of both mouse and keyboard input, especially if the user is to be challenged to provide a dataset or parameter values. Otherwise, the user would be restricted to a predefined set of special characters (e.g., menu options and icons) as the input medium. However, the tradeoff here is that you may provide too much flexibility to the user, especially for novice users.

Familiarity with the host platform is another problem, especially if the users of the system are relatively unsophisticated. For example, to use ABACUS.2 version 1.0, the user must at a minimum have some knowledge of the following things:

- Unix or Ultrix
- LISP
- workstations

Users need at least a cursory knowledge of the Unix or Ultrix operating systems in order to log in, create, modify, and save files, and find the startup programs for ABACUS.2. Moreover, in order to understand the algorithms or determine that a software bug has been identified, the user needs to be fairly comfortable with using LISP. Even the datasets used by

ABACUS.2 are formatted for use with the LISP *car* and *cdr* operations. In addition, the user needs to have some familiarity with using workstations. In other words, experimenting with ABACUS.2 requires some hands on experience with computer equipment as well as some programming skills and knowledge.

In addition to all of the requirements that were just listed, the user will probably need some minimum amount of knowledge about the discovery system itself in order to effectively and efficiently use ABACUS.2, which is parameter-driven. One possible way to get around requiring the user to have a great deal of knowledge and experience in using computers and discovery systems would be to build a natural language interface that leads the user through the operation of the ABACUS.2 system. That is, the system would provide the user with help and feedback. For example, during data entry, the system would prompt the user for the data stored in each of the five data structures. Likewise, the system could provide the user with information about setting parameter values (e.g., for large datasets, a certain subset of the parameters should probably be set so that the system can find dependencies rather quickly). The development of a natural language interface, or for an intelligent editor for creation and modification of a dataset can be very difficult. In the case of an intelligent editor, the editor should have properties such as being language independent, automatic tabbing to next column, and so on.

Another issue that must be resolved is that of experimental design. There are very few guidelines on how to identify important variables and parameters, or for that matter, very few guidelines on how to structure an experiment. The user needs to have some *a priori* knowledge about the domain in which the data was obtained as well as about ABACUS.2, its parameters, and limitations and capabilities. In addition, ABACUS.2 is not the best

quantitative discovery system for all types of data analysis or domains. Therefore, it would be beneficial if results generated from ABACUS.2 for particular domains or types of datasets could be used to strategically structure experiments. For example, in some domains symbolic data is not available. If this is the case, ABACUS.2 will have problems discovering multiple equations since the system relies on nominal data to do so.

8. CONCLUSIONS

ABACUS.2 performs as stated in the documentation, but still needs a lot of refinement to improve the performance of the system and reduce the number of bugs and omissions. Reducing the search space appears to be the most effective way of increasing the speed of the system. However, there is a tradeoff between reducing the search space and increasing the precision of the resulting equation or equations. Additional experiments need to be conducted to further determine the capabilities of ABACUS.2. A plan needs to be formulated so that the experimentation procedure is successful (i.e., will be dealing with many combinations and permutations of parameters). Developing an intelligent user interface for a discovery system is difficult for the same reason as described above, and due to the need to represent, retrieve, and use this knowledge in constructing and executing experiments.

9. SUMMARY

The following objects were achieved:

ABACUS.2 was ported to the Dec and Sun platforms

- ABACUS.2 was converted to Common LISP
- ABACUS.2 was validated, verified, baselined, and documented

- experiments were conducted in order to better understand the system

10. FURTHER RESEARCH

In preparation for the upcoming IJCAI Workshop on Knowledge Discovery in Databases (Detroit, Michigan, August 22), and conference on quantitative discovery (France, September 11-14), experiments are being conducted to further test abilities of ABACUS.2 to discover linear and non-linear terms. See Appendix D. In addition, testing of a new heuristic that will permit the discovery of exponential and logarithmic terms is underway.

Development of a user interface is also planned. The user interface will be similar to that of Emerald, but will be separate from the Emerald system, and will run on Sun workstations.

11. REFERENCES

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APPENDIX A1 - Verification of Centrifugal Force Experiment

A different result was obtained from that reported by Greene for the centrifugal force experiment. In this case, six classes were formed, whereas Greene found only one class and one equation to describe the data for that class. The difference in results could be do to the way in which the Sun handles round-off error compared to the Symbolics. (i.e., the Sun math coprocessor may be more sensitive to round-off error than the Symobolics, which may in turn lead to the discovery of additional classes without changing the value of the uncertainty parameter.

```
;;; f = (* (/ (* m v v) r) (cos theta)), where f is the component of
;;; centrifugal force parallel to the road. This file is different from
;;; CENTRIF-FORCE1 in the respect that it has extra information which allows
;;; the proportionality graph search to find the equation and it has the
;;; irrelevant variable MASS.
```

```
(
(declarations
(f L ((meters 2) (seconds -2) (radians 1)))
(r L ((meters 1)))
(m L ((kilograms 1)))
(v L ((meters 1) (seconds -1)))
(theta L ((radians 1)))
(length L ((meters 1)))
)

(parameters
(*print-flags* (p e))
(*cluster-order* (gc nc rc))
)

(variables
(f r m v theta length)
)

(events
(3221.6787    30    50    75    1.22    3212.3)
(8121.7944    30    50    75    0.523    3212.3)
(-4909.809    30    50    75    2.122    3212.3)
(-3986.427    30    50    75    2.01    3212.3)
(-2885.0676   20    46    52    4.23    1755.3)
(1883.0659    20    46    52    5.02    1755.3)
(1584.4714    20    46    52    4.97    1755.3)
(-4994.303    20    46    52    3.78    1755.3)
(5521.2207    23    61    50    6.87    2112.4)
(6189.342     23    61    50    6.65    2112.4)
(4954.888     23    61    50    7.01    2112.4)
(6549.0776    23    61    50    6.44    2112.4)
(3927.5881    23    61    50    7.22    2112.4)
```

(1372.5167	30	50	60	1.34	3546.7)
(1029.3876	40	50	60	1.34	3546.7)
(823.5101	50	50	60	1.34	3546.7)
(2745.0334	15	50	60	1.34	3546.7)
(2774.2974	20	30	44	.3	4231)
(3699.0632	20	40	44	.3	4231)
(4623.829	20	50	44	.3	4231)
(5548.5947	20	60	44	.3	4231)
(8020.9575	27	48	75	.64	3211.8)
(5133.413	27	48	60	.64	3211.8)
(4005.488	27	48	53	.64	3211.8)
(9588.074	27	48	82	.64	3211.8)
(387.744	10	41	-10	.331	1998.7)
(2051.167	10	41	-23	.331	1998.7)
(5308.219	10	41	-37	.331	1998.7)
(9693.608	10	41	-50	.331	1998.7)
(340	30	2.1970859	75	.6	4538)
(340	30	4.9434433	50	.6	4538)
(340	30	13.731788	30	.6	4538)
(340	30	6.103017	45	.6	4538)
(612.4	13.1	27.725	-22.0	.93	1967.5)
(612.4	13.1	7.027	-43.7	.93	1967.5)
(612.4	13.1	5.001	-51.8	.93	1967.5)
(450		40 7.2548094	50	0.123	1455.6)
(450		50 9.068512	50	0.123	1455.6)
(450		60 10.882214	50	0.123	1455.6)
(450		70 12.695916	50	0.123	1455.6)
(500	493.6402	50	75	.5	2365.4)
(500	219.39563	50	50	.5	2365.4)
(500	140.41321	50	40	.5	2365.4)
(500	78.98243	50	30	.5	2365.4)
(210	7.37	34.5	-9	5.3	3425)
(210	87.52	34.5	-31	5.3	3425)
(210	141.38	34.5	-39.4	5.3	3425)
(300	20 -7.0686507	40	2.13	4368)	
(300	20 8.980002	40	1.14	4368)	
(300	20 3.7785466	40	0.123	4368)	
(300	20 3.9558227	40	0.324	4368)	
(540	25	50 16.51858	0.145	2436)	
(540	25	50 34.35569	1.34	2436)	
(540	25	50 17.540335	0.5	2436)	
(540	25	50 18.788666	0.7	2436)	
(540	25	50 27.296886	1.2	2436)	
(440	263.5919	60	75	1.22	3544)
(440	-463.71912	60	75	2.22	3544)
(440	-305.1912	60	75	1.98	3544)
(440	660.64606	60	75	0.533	3544)

)

)

== - - Parameters - - - ==

MAXSTAR = 10
 COVERMODE = IC
 UNCERTAINTY = 0.02
 CONSTANCY = 0.95
 INVERSE-SW = 30
 DIRECT-SW = 70

UNTVIGNORE = NIL
 DEBUG = NIL
 PRINT-FLAGS = (P E)
 FILTER-LEVEL = 2
 SEARCH-LIMIT = 3
 MAX-NODES = 1000
 CLUSTER-ON = #<Symbol 46001D>
 CLUSTER-ORDER = (GC NC RC)
 BEAM-VALUE = 0.5
 MINIMUM-LINK-VALUE = 50.0

 starLEF = ((MAX-COVERED 0.5) (MIN-WEIGHT 0.0) (MIN-SELECTORS 0.0))

--- Class Events ---

MISCELLANEOUS

(F R M V THETA LENGTH)
 (-4909.809 30.0 50.0 75.0 2.122 3212.3)
 (-3986.427 30.0 50.0 75.0 2.01 3212.3)
 (8121.7944 30.0 50.0 75.0 0.523 3212.3)
 (3221.6787 30.0 50.0 75.0 1.22 3212.3)
 (3927.5881 23.0 61.0 50.0 7.22 2112.4)
 (4954.888 23.0 61.0 50.0 7.01 2112.4)
 (6549.0776 23.0 61.0 50.0 6.44 2112.4)
 (5521.2207 23.0 61.0 50.0 6.87 2112.4)
 (6189.342 23.0 61.0 50.0 6.65 2112.4)
 (612.4 13.1 27.725 -22.0 0.93 1967.5)
 (612.4 13.1 7.027 -43.7 0.93 1967.5)
 (612.4 13.1 5.001 -51.8 0.93 1967.5)
 (210.0 141.38 34.5 -39.4 5.3 3425.0)
 (210.0 87.52 34.5 -31.0 5.3 3425.0)
 (210.0 7.37 34.5 -9.0 5.3 3425.0)
 (-4994.303 20.0 46.0 52.0 3.78 1755.3)
 (-2885.0676 20.0 46.0 52.0 4.23 1755.3)
 (1584.4714 20.0 46.0 52.0 4.97 1755.3)
 (1883.0659 20.0 46.0 52.0 5.02 1755.3)
 (9693.608 10.0 41.0 -50.0 0.331 1998.7)
 (5308.219 10.0 41.0 -37.0 0.331 1998.7)
 (2051.167 10.0 41.0 -23.0 0.331 1998.7)
 (387.744 10.0 41.0 -10.0 0.331 1998.7)
 (440.0 -463.71912 60.0 75.0 2.22 3544.0)
 (440.0 -305.1912 60.0 75.0 1.98 3544.0)
 (440.0 660.64606 60.0 75.0 0.533 3544.0)
 (440.0 263.5919 60.0 75.0 1.22 3544.0)
 (540.0 25.0 50.0 18.788666 0.7 2436.0)
 (540.0 25.0 50.0 17.540335 0.5 2436.0)
 (540.0 25.0 50.0 27.296886 1.2 2436.0)
 (540.0 25.0 50.0 34.35569 1.34 2436.0)
 (540.0 25.0 50.0 16.51858 0.145 2436.0)

CLASSF

(F R M V THETA LENGTH)
 (340.0 30.0 2.1970859 75.0 0.6 4538.0)
 (340.0 30.0 4.9434433 50.0 0.6 4538.0)
 (340.0 30.0 6.103017 45.0 0.6 4538.0)
 (340.0 30.0 13.731788 30.0 0.6 4538.0)

CLASSE

(F R M V THETA LENGTH)

(500.0 78.98243 50.0 30.0 0.5 2365.4)
 (500.0 140.41321 50.0 40.0 0.5 2365.4)
 (500.0 219.39563 50.0 50.0 0.5 2365.4)
 (500.0 493.6402 50.0 75.0 0.5 2365.4)

CLASSD
 (F R M V THETA LENGTH)
 (5133.413 27.0 48.0 60.0 0.64 3211.8)
 (8020.9575 27.0 48.0 75.0 0.64 3211.8)
 (9588.074 27.0 48.0 82.0 0.64 3211.8)
 (4005.488 27.0 48.0 53.0 0.64 3211.8)

CLASSC
 (F R M V THETA LENGTH)
 (450.0 40.0 7.2548094 50.0 0.123 1455.6)
 (450.0 50.0 9.068512 50.0 0.123 1455.6)
 (450.0 60.0 10.882214 50.0 0.123 1455.6)
 (450.0 70.0 12.695916 50.0 0.123 1455.6)

CLASSB
 (F R M V THETA LENGTH)
 (2774.2974 20.0 30.0 44.0 0.3 4231.0)
 (3699.0632 20.0 40.0 44.0 0.3 4231.0)
 (5548.5947 20.0 60.0 44.0 0.3 4231.0)
 (4623.829 20.0 50.0 44.0 0.3 4231.0)

CLASSA
 (F R M V THETA LENGTH)
 (1372.5167 30.0 50.0 60.0 1.34 3546.7)
 (2745.0334 15.0 50.0 60.0 1.34 3546.7)
 (823.5101 50.0 50.0 60.0 1.34 3546.7)
 (1029.3876 40.0 50.0 60.0 1.34 3546.7)

--- Hypotheses ---

CLASSA
 Cover:
 4 [LENGTH = 3546.7]
 Relation:
 $F * R = 41175.5000$

CLASSB
 Cover:
 4 [LENGTH = 4231.0]
 Relation:
 $F = 92.4766 * M$

CLASSC
 Cover:
 4 [F = 450.0]
 Relation:
 $R = 5.5136 * M$

CLASSD
 Cover:

4 [LENGTH = 3211.8]

Relation:

$$F = 1.4259 * (V^2)$$

CLASSE

Cover:

4 [F = 500.0]

Relation:

$$R = 0.0878 * (V^2)$$

CLASSF

Cover:

4 [F = 340.0]

Relation:

$$M * V^2 = 12358.6080$$

MISCELLANEOUS

Cover:

26 [LENGTH = 1755.3 .-. 3544.0][THETA = 0.523 .-. 7.22]

11 [V = -50.0 .-. 17.540335]

Relation:

No formula can be found

APPENDIX A2 - Verification of Snell Experiment

A different result was obtained from that reported by Greene for the Snell experiment. Once again, the the verification run generated four classes, whereas Greene reported one class and equation describing the member data of that class.

```
;;; Snell's law of refraction
;;;  $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$ 
;;; where  $n_1$  is the refractive index of medium #1,  $\theta_1$  is the angle of
;;; incidence,  $n_2$  is the refractive index of medium #2, and  $\theta_2$  is the
;;; angle of refraction. The refractive index  $n = c / v$ , where  $c$  is the
;;; velocity of light in free space and is approximately equal to  $3 \times 10^8$  m/s.
;;;  $v$  is the velocity of light in a medium. The desired equation is
;;;  $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$ .
```

```
(
(declarations
  (N1 L ((refractions 1)))
  (N2 L ((refractions 1)))
  (THETA1 L ((radians 1)))
  (THETA2 L ((radians 1)))
)
(parameters
  (*print-flags* (p e))
  (*cluster-order* (gc nc rc))
)
(variables
  (N1 N2 THETA2 THETA1)
)
```

```
(events
  (1.01 1.01 0.5 0.5)
  (0.06319116 1.01 0.03 0.5)
  (2.0299163 1.01 1.3 0.5)
  (1.7727169 1.01 1.0 0.5)
  (1.1895255 1.01 0.6 0.5)
  (1.4613272 1.24 1.7 1.0)
  (1.0988786 1.24 2.3 1.0)
  (-1.1152318 1.24 4.0 1.0)
  (0.20795585 1.24 3.0 1.0)
  (-0.51691765 1.24 3.5 1.0)
  (-7.2614417 1.8 5.0 0.24)
  (0.42999822 1.8 6.34 0.24)
  (5.587028 1.8 7.113 0.24)
  (-5.3426943 1.8 5.5 0.24)
  (3.7416677 1.8 6.8 0.24)
  (2.4451091 1.51 0.2 0.123)
  (0.5585338 1.51 0.2 0.567)
  (0.33661154 1.51 0.2 1.1)
  (0.39930588 1.51 0.2 0.85)
  (.033364605 1.51 .02 2.01)
  (.050458465 1.51 .02 2.5)
  (.21398798 1.51 .02 3.0)
```

```

(.09014642 1.51 .02 2.8)
(.12621978 1.51 .02 2.9)
(4.2597756 1.68 3 6.4)
(.85829824 1.68 3 6.9)
(.62554383 1.68 3 7.2)
(.52928984 1.68 3 7.5)
(1.5253406 2.01 0.3 0.4)
(1.5025743 1.98 0.3 0.4)
(0.9334174 1.23 0.3 0.4)
(1.1383139 1.5 0.3 0.4)
(2.35 6.77603 0.3 1.02)
(2.35 2.9377053 0.75 1.02)
(2.35 2.2468998 1.1 1.02)
(2.35 4.176778 0.5 1.02)
(.8 37252843 1.7 .48)
(.8 39038712 1.9 .48)
(.8 49540132 2.3 .48)
(.8 2.61779557 3.0 .48)
(1.44 7.6724596 6.4 .67)
(1.44 1.5459167 6.9 .67)
(1.44 1.126693 7.2 .67)
(1.44 .99500585 7.4 .67)
(2.6 0.62196493 0.98 0.2)
(2.6 1.8545122 0.98 0.634)
(2.6 2.917892 0.98 1.2)
(2.6 2.2457938 0.98 0.8)
(2.6 1.2191341 0.98 .4)
(2.6 1.636353 0.98 .55)
(1.08 1.3125678 0.19 2.91)
(1.08 0.69362235 0.19 3.02)
(1.08 -4.6793623 0.19 4.1)
(1.08 -2.0059733 0.19 3.5)
(1.788 -2.1388648 0.93 5)
(1.788 0.9030106 0.93 6.7)
(1.788 1.8968848 0.93 7.3)
(1.788 1.4653977 0.93 7.0)
(2.5 2.8 0.11956984 0.134)
(2.5 2.8 0.31908178 0.359)
(2.5 2.8 0.39885348 0.45)
(2.5 2.8 0.587704 0.67)
(.7 1.1 .6780504 1.74)
(.7 1.1 .6070415 2.03)
(.7 1.1 .2148228 2.8)
(.7 1.1 .070925675 3.03)
(.5 1.1 .05300177 6.4)
(.5 1.1 .14208728 6.6)
(.5 1.1 .33773357 7.1)
(.5 1.1 .44046894 7.5)
)
)

```

=== Parameters ===

```

*MAXSTAR* = 10
*COVERMODE* = IC
*UNCERTAINTY* = 0.02
*CONSTANCY* = 0.95
*INVERSE-SW* = 30
*DIRECT-SW* = 70

```

UNVIGNORE = NIL
 DEBUG = NIL
 PRINT-FLAGS = (P E)
 FILTER-LEVEL = 2
 SEARCH-LIMIT = 3
 MAX-NODES = 1000
 CLUSTER-ON = T
 CLUSTER-ORDER = (GC NC RC)
 BEAM-VALUE = 0.5
 MINIMUM-LINK-VALUE = 50.0

 starLEF = ((MAX-COVERED 0.5) (MIN-WEIGHT 0.0) (MIN-SELECTORS 0.0))

--- Class Events ---

MISCELLANEOUS
 (N1 N2 THETA2 THETA1)
 (2.4451091 1.51 0.2 0.123)
 (0.5585338 1.51 0.2 0.567)
 (0.39930588 1.51 0.2 0.85)
 (0.33661154 1.51 0.2 1.1)
 (0.52928984 1.68 0.3 7.5)
 (0.62554383 1.68 0.3 7.2)
 (0.85829824 1.68 0.3 6.9)
 (4.2597756 1.68 0.3 6.4)
 (1.44 0.99500585 7.4 0.67)
 (1.44 1.126693 7.2 0.67)
 (1.44 1.5459167 6.9 0.67)
 (1.44 7.6724596 6.4 0.67)
 (2.35 6.77603 0.3 1.02)
 (2.35 4.176778 0.5 1.02)
 (2.35 2.9377053 0.75 1.02)
 (2.35 2.2468998 1.1 1.02)
 (1.788 -2.1388648 0.93 5.0)
 (1.788 0.9030106 0.93 6.7)
 (1.788 1.4653977 0.93 7.0)
 (1.788 1.8968848 0.93 7.3)
 (2.6 2.917892 0.98 1.2)
 (2.6 2.2457938 0.98 0.8)
 (2.6 1.8545122 0.98 0.634)
 (2.6 1.636353 0.98 0.55)
 (2.6 1.2191341 0.98 0.4)
 (2.6 0.62196493 0.98 0.2)
 (1.08 -4.6793623 0.19 4.1)
 (1.08 -2.0059733 0.19 3.5)
 (1.08 0.69362235 0.19 3.02)
 (1.08 1.3125678 0.19 2.91)
 (0.8 0.39038712 1.9 0.48)
 (0.8 0.49540132 2.3 0.48)
 (0.8 0.37252843 1.7 0.48)
 (0.8 2.6177955 3.0 0.48)
 (0.050458465 1.51 0.02 2.5)
 (0.033364605 1.51 0.02 2.01)
 (0.09014642 1.51 0.02 2.8)
 (0.12621978 1.51 0.02 2.9)
 (0.21398798 1.51 0.02 3.0)
 (0.06319116 1.01 0.03 0.5)
 (1.01 1.01 0.5 0.5)
 (1.1895255 1.01 0.6 0.5)

(2.0299163 1.01 1.3 0.5)
 (1.7727169 1.01 1.0 0.5)
 (-1.1152318 1.24 4.0 1.0)
 (-0.51691765 1.24 3.5 1.0)
 (0.20795585 1.24 3.0 1.0)
 (1.4613272 1.24 1.7 1.0)
 (1.0988786 1.24 2.3 1.0)
 (-7.2614417 1.8 5.0 0.24)
 (-5.3426943 1.8 5.5 0.24)
 (0.42999822 1.8 6.34 0.24)
 (3.7416677 1.8 6.8 0.24)
 (5.587028 1.8 7.113 0.24)

CLASSD

(N1 N2 THETA2 THETA1)
 (2.5 2.8 0.587704 0.67)
 (2.5 2.8 0.39885348 0.45)
 (2.5 2.8 0.31908178 0.359)
 (2.5 2.8 0.11956984 0.134)

CLASSC

(N1 N2 THETA2 THETA1)
 (0.7 1.1 0.070925675 3.03)
 (0.7 1.1 0.2148228 2.8)
 (0.7 1.1 0.6780504 1.74)
 (0.7 1.1 0.6070415 2.03)

CLASSB

(N1 N2 THETA2 THETA1)
 (0.5 1.1 0.33773357 7.1)
 (0.5 1.1 0.14208728 6.6)
 (0.5 1.1 0.05300177 6.4)
 (0.5 1.1 0.44046894 7.5)

CLASSA

(N1 N2 THETA2 THETA1)
 (1.5253406 2.01 0.3 0.4)
 (1.5025743 1.98 0.3 0.4)
 (1.1383139 1.5 0.3 0.4)
 (0.9334174 1.23 0.3 0.4)

--- Hypotheses ---

CLASSA

Cover:

$$4 [N1 = 0.9334174 \wedge 1.5253406][THETA1 = 0.4]$$

Relation:

$$N1 = 0.7589 * N2$$

CLASSB

Cover:

$$4 [N1 = 0.5]$$

Relation:

$$((THETA2^2) / THETA1) + 2*((1 * THETA2^2) / THETA1) + (1^2 * THETA1) = 6.2035 + (2*(1 * THETA2) + ((THETA2^3) / (THETA1^2)) + (1^2 * THETA2))$$

CLASSC

Cover:

4 [N1 = 0.7]

Relation:

$$\frac{2*((\text{THETA2}^2) / ((\text{THETA2}^2) + (\text{THETA1}^2) + 2*(\text{THETA1} * \text{THETA2}) + 2*((\text{THETA2}^3) / \text{THETA1}))) + ((\text{THETA1}^2) / ((\text{THETA2}^2) + (\text{THETA1}^2) + 2*(\text{THETA1} * \text{THETA2}) + 2*((\text{THETA2}^3) / \text{THETA1}))) + 2*((\text{THETA1} * \text{THETA2}) / ((\text{THETA2}^2) + (\text{THETA1}^2) + 2*(\text{THETA1} * \text{THETA2}) + 2*((\text{THETA2}^3) / \text{THETA1})))}{2*((\text{THETA2}^3) / \text{THETA1})))} = 1.0104$$

CLASSD

Cover:

4 [N1 = 2.5]

Relation:

$$\text{THETA2} = 0.8862 * \text{THETA1}$$

MISCELLANEOUS

Cover: Relation:

No formula can be found

APPENDIX A3 - Verification of Instantaneous Current Experiment

Both Greene's and the test results match for the instantaneous current experiment.

```
;;; Data set for the equation  $i = V_o \cos(\omega \text{ time}) / R$ , where  $i$  is the  
;;; instantaneous current,  $V_o$  is the peak voltage,  $\omega$  is the angular  
;;; velocity for an AC supply.
```

```
;;; (defun i (R Vo omega tme) (/ (* Vo (cos (* omega tme))) R))  
;;; (defun R (i Vo omega tme) (/ (* Vo (cos (* omega tme))) i))  
;;; (defun Vo (i R omega tme) (/ (* R i) (cos (* omega tme))))  
;;; (defun omega (i R Vo tme) (/ (acos (/ (* R i) Vo)) tme))  
;;; (defun tme (i R Vo omega) (/ (acos (/ (* R i) Vo)) omega))
```

```
(  
  (declarations  
    (i L ((amperes 1)))  
    (R L ((volts 1) (amperes -1)))  
    (Vo L ((volts 1)))  
    (omega L ((radians 1) (seconds -1)))  
    (time L ((seconds 1)))  
  )
```

```
(parameters  
  (*inverse-sw* 20)  
  (*direct-sw* 80)  
  (*print-flags* (p e))  
  (*cluster-order* (nc gc rc))  
)
```

```
(variables  
  (i R Vo omega time)  
)
```

```
(events  
  (3.293959 65.8 220 337 0.13)  
  (1.7229134 125.8 220 337 0.13)  
  (2.8526256 75.98 220 337 0.13) ;i <-> R  
  (0.4027866 110 198 412 0.5)  
  (0.36616963 121 198 412 0.5)  
  (0.24891306 178 198 412 0.5)  
  
  (1.0244168 35.4 110 451 0.6018)  
  (2.0488336 35.4 220 451 0.6018) ;i <-> Vo  
  (0.70684767 35.4 75.9 451 0.6018)  
  
  (0.0107615795 167 312 857.3 0.273)  
  (-1.4499393 167 312 857.3 0.8473)  
  (-1.5206991 167 312 857.3 0.472) ;i <-> time  
  (-0.21546386 408 110 483.2 0.9284)  
  (0.0381025 408 110 483.2 0.172)  
  (0.22920708 408 110 483.2 0.584)  
  
  (0.9 -130.74095 121 575 0.1034)  
  (0.9 -123.15287 121 575 0.5744)  
  (0.9 129.87405 121 575 0.382)  
  (2.87 -55.08361 289 4428 0.832) ;R <-> time  
  (2.87 -77.89037 289 4428 0.7072)  
  (2.87 -91.08931 289 4428 0.344)
```

```

(2.87 -93.95117 289 4428 0.54)

(0.8212155 143 273.3 3948.3 0.8453)
(0.3551025 143 273.3 4837.3 0.8453)
(0.674562 143 273.3 938 0.8453)
(-0.2674998 523.4 214.4 3824.3 0.712)
(0.40340808 523.4 214.4 873.4 0.712) ;i <-> omega
(-0.11722943 523.4 214.4 8363.2 0.712)
(0.0056263898 1200 56.56 3827 0.745)
(0.002560051 1200 56.56 2937 0.745)
(0.046101775 1200 56.56 928 0.745)

(14.3 445.4 13580.729 382 0.743)
(14.3 445.4 -11384.064 2483.3 0.743)
(14.3 445.4 6490.2524 338 0.743)
(7.5 323 20376.268 3289 413) ;Vo <-> omega
(7.5 323 6500.317 3289 0.413)
(7.5 323 5997.8525 2918.2 0.413)
(7.5 323 -3920.034 238 0.413)

(5.16 318 -1654.2872 2872.3 0.1323)
(5.16 318 -2484.7036 2872.3 0.4383)
(5.16 318 -8932.971 2872.3 0.2828)
(0.987 102.3 2877.3518 382.3 0.1439) ;Vo <-> time
(0.987 102.3 183.18755 382.3 0.1289)
(0.987 102.3 -24831.43 382.3 0.415)

(0.41 220 103 1.6253369 0.31)
(0.41 220 103 3.5432804 0.1422)
(0.41 220 103 7.453469 0.0676)
(0.132 98 217 2.727704 0.554) ;omega <-> time
(0.132 98 217 3.5699222 0.4233)
(0.132 98 217 16.979193 0.089)

(2.09 31.04176 75 848.8 0.3621)
(2.09 49.666813 120 848.8 0.3621) ;R <-> Vo
(2.09 72.43076 175 848.8 0.3621)

(4.53 39.961258 313 773.3 0.83)
(4.53 7.392655 313 773.3 0.9)
(4.53 54.87472 313 773.3 0.2771)
(4.53 45.117954 313 773.3 0.708) ;R <-> time
(10.2 19.9217 266 998.1 1.82)
(10.2 -19.182266 266 998.1 0.783)
(10.2 -17.272163 266 998.1 0.388)

(7.1 543 -3858.9182 416.3 0.098)
(7.1 543 4160.0293 416.3 0.799)
(7.1 543 18707.385 416.3 0.691) ;Vo <-> time
(0.96 612 591.68634 588 0.823)
(0.96 612 3503.0276 588 0.3823)
(0.96 612 638.5944 588 0.384)
)
)

```

== - Parameters - ==

MAXSTAR = 10
COVERMODE = IC

UNCERTAINTY = 0.02
 CONSTANCY = 0.95
 INVERSE-SW = 20
 DIRECT-SW = 80
 UNIVIGNORE = NIL
 DEBUG = NIL
 PRINT-FLAGS = (P E)
 FILTER-LEVEL = 2
 SEARCH-LIMIT = 3
 MAX-NODES = 1000
 CLUSTER-ON = T
 CLUSTER-ORDER = (NC GC RC)
 BEAM-VALUE = 0.5
 MINIMUM-LINK-VALUE = 50.0

starLEF = ((MAX-COVERED 0.5) (MIN-WEIGHT 0.0) (MIN-SELECTORS 0.0))

== - - - Class Events - - - ==

CLASSA

(I R VO OMEGA TIME)

(10.2 -17.272163 266.0 998.1 0.388)
 (7.1 543.0 -3858.9182 416.3 0.098)
 (2.87 -93.95117 289.0 4428.0 0.54)
 (7.1 543.0 18707.385 416.3 0.691)
 (5.16 318.0 -8932.971 2872.3 0.2828)
 (0.41 220.0 103.0 7.453469 0.0676)
 (0.41 220.0 103.0 3.5432804 0.1422)
 (0.41 220.0 103.0 1.6253369 0.31)
 (0.36616963 121.0 198.0 412.0 0.5)
 (0.987 102.3 -24831.43 382.3 0.415)
 (2.87 -77.89037 289.0 4428.0 0.7072)
 (0.9 -123.15287 121.0 575.0 0.5744)
 (0.96 612.0 3503.0276 588.0 0.3823)
 (7.1 543.0 4160.0293 416.3 0.799)
 (0.40340808 523.4 214.4 873.4 0.712)
 (-0.11722943 523.4 214.4 8363.2 0.712)
 (-0.2674998 523.4 214.4 3824.3 0.712)
 (14.3 445.4 13580.729 382.0 0.743)
 (14.3 445.4 6490.2524 338.0 0.743)
 (14.3 445.4 -11384.064 2483.3 0.743)
 (-0.21546386 408.0 110.0 483.2 0.9284)
 (7.5 323.0 20376.268 3289.0 413.0)
 (7.5 323.0 6500.317 3289.0 0.413)
 (7.5 323.0 5997.8525 2918.2 0.413)
 (7.5 323.0 -3920.034 238.0 0.413)
 (5.16 318.0 -1654.2872 2872.3 0.1323)
 (5.16 318.0 -2484.7036 2872.3 0.4383)
 (0.24891306 178.0 198.0 412.0 0.5)
 (-1.5206991 167.0 312.0 857.3 0.472)
 (0.0107615795 167.0 312.0 857.3 0.273)
 (-1.4499393 167.0 312.0 857.3 0.8473)
 (0.674562 143.0 273.3 938.0 0.8453)
 (0.3551025 143.0 273.3 4837.3 0.8453)
 (0.8212155 143.0 273.3 3948.3 0.8453)
 (0.9 129.87405 121.0 575.0 0.382)
 (0.987 102.3 183.18755 382.3 0.1289)
 (2.09 72.43076 175.0 848.8 0.3621)
 (3.293959 65.8 220.0 337.0 0.13)

(4.53 54.87472 313.0 773.3 0.2771)
 (4.53 39.961258 313.0 773.3 0.83)
 (2.0488336 35.4 220.0 451.0 0.6018)
 (1.0244168 35.4 110.0 451.0 0.6018)
 (10.2 19.9217 266.0 998.1 1.82)
 (4.53 7.392655 313.0 773.3 0.9)
 (10.2 -19.182266 266.0 998.1 0.783)
 (2.87 -55.08361 289.0 4428.0 0.832)
 (0.9 -130.74095 121.0 575.0 0.1034)
 (0.0056263898 1200.0 56.56 3827.0 0.745)
 (0.002560051 1200.0 56.56 2937.0 0.745)
 (0.046101775 1200.0 56.56 928.0 0.745)
 (0.96 612.0 638.5944 588.0 0.384)
 (0.96 612.0 591.68634 588.0 0.823)
 (0.22920708 408.0 110.0 483.2 0.584)
 (0.0381025 408.0 110.0 483.2 0.172)
 (1.7229134 125.8 220.0 337.0 0.13)
 (0.4027866 110.0 198.0 412.0 0.5)
 (0.987 102.3 2877.3518 382.3 0.1439)
 (2.8526256 75.98 220.0 337.0 0.13)
 (2.09 49.666813 120.0 848.8 0.3621)
 (4.53 45.117954 313.0 773.3 0.708)
 (0.70684767 35.4 75.9 451.0 0.6018)
 (2.87 -91.08931 289.0 4428.0 0.344)
 (2.09 31.04176 75.0 848.8 0.3621)
 (0.132 98.0 217.0 3.5699222 0.4233)
 (0.132 98.0 217.0 2.727704 0.554)
 (0.132 98.0 217.0 16.979193 0.089)

--- Hypotheses ---

CLASSA

Cover: Relation:

$$I * R = \text{COS}((\text{OMEGA} * \text{TIME})) * \text{VO}$$

APPENDIX A4.1 - Verification of Stoke Experiment #1

The result of this experiment did not match the result obtained by Greene. In this case, the system ran out of memory before an answer could be obtained. The reason why the Symbolics version was able to obtain an answer was due to the special memory and caching capabilities of the Symbolics workstation. At a lower level of uncertainty, the result reported by Greene was obtained.

```
(
(declarations
  (v L ((meter 1) (sec -1)))
  (r L ((meter 1)))
  (m L ((kg 1)))
  (t L ((sec 1)))
  (h L ((meter 1)))
  (substance N nil)
  (location N nil)
)

(parameters
  (*univignore* (3 4))
  (*uncertainty* 0.02)
  (*cluster-order* (gc nc rc))
  (*constancy* .99)
  (*print-flags* (p e))
)

(variables
  (v r m t h substance location)
)

(events
  (11.9425 0.05 0.6231 0.0837 1.0 Glycerol DeathValley)
  (26.8705 0.075 2.1029 0.0372 1.0 Glycerol DeathValley)
  (47.7698 0.1 4.9847 0.0209 1.0 Glycerol DeathValley)
  (18.0642 0.05 0.9425 0.0554 1.0 Glycerol DeathValley)
  (40.6445 0.075 3.1809 0.0246 1.0 Glycerol DeathValley)
  (72.2569 0.1 7.5398 0.0138 1.0 Glycerol DeathValley)
  (9.1671 0.05 0.6231 0.1091 1.0 CastorOil DeathValley)
  (20.6260 0.075 2.1029 0.0485 1.0 CastorOil DeathValley)
  (36.6684 0.1 4.9847 0.0273 1.0 CastorOil DeathValley)
  (13.8662 0.05 0.9425 0.0721 1.0 CastorOil DeathValley)
  (31.1989 0.075 3.1809 0.0321 1.0 CastorOil DeathValley)
  (55.4648 0.1 7.5398 0.0180 1.0 CastorOil DeathValley)
  (11.8757 0.05 0.6231 0.0842 1.0 Glycerol Denver)
  (26.7204 0.075 2.1029 0.0374 1.0 Glycerol Denver)
  (47.5030 0.1 4.9847 0.0211 1.0 Glycerol Denver)
  (17.9633 0.05 0.9425 0.0557 1.0 Glycerol Denver)
  (40.4174 0.075 3.1809 0.0247 1.0 Glycerol Denver)
  (71.8532 0.1 7.5398 0.0139 1.0 Glycerol Denver)
  (9.1159 0.05 0.6231 0.1097 1.0 CastorOil Denver)
  (20.5107 0.075 2.1029 0.0488 1.0 CastorOil Denver)
```

(36.4635 0.1 4.9847 0.0274 1.0 CastorOil Denver)
 (13.7887 0.05 0.9425 0.0725 1.0 CastorOil Denver)
 (31.0246 0.075 3.1809 0.0322 1.0 CastorOil Denver)
 (55.1549 0.1 7.5398 0.0181 1.0 CastorOil Denver)
 (11.9425 0.05 0.6231 0.1675 2.0 Glycerol DeathValley)
 (26.8705 0.075 2.1029 0.0744 2.0 Glycerol DeathValley)
 (47.7698 0.1 4.9847 0.0419 2.0 Glycerol DeathValley)
 (18.0642 0.05 0.9425 0.1107 2.0 Glycerol DeathValley)
 (40.6445 0.075 3.1809 0.0492 2.0 Glycerol DeathValley)
 (72.2569 0.1 7.5398 0.0277 2.0 Glycerol DeathValley)
 (9.1671 0.05 0.6231 0.2182 2.0 CastorOil DeathValley)
 (20.6260 0.075 2.1029 0.0970 2.0 CastorOil DeathValley)
 (36.6684 0.1 4.9847 0.0545 2.0 CastorOil DeathValley)
 (13.8662 0.05 0.9425 0.1442 2.0 CastorOil DeathValley)
 (31.1989 0.075 3.1809 0.0641 2.0 CastorOil DeathValley)
 (55.4648 0.1 7.5398 0.0361 2.0 CastorOil DeathValley)
 (11.8757 0.05 0.6231 0.1684 2.0 Glycerol Denver)
 (26.7204 0.075 2.1029 0.0748 2.0 Glycerol Denver)
 (47.5030 0.1 4.9847 0.0421 2.0 Glycerol Denver)
 (17.9633 0.05 0.9425 0.1113 2.0 Glycerol Denver)
 (40.4174 0.075 3.1809 0.0495 2.0 Glycerol Denver)
 (71.8532 0.1 7.5398 0.0278 2.0 Glycerol Denver)
 (9.1159 0.05 0.6231 0.2194 2.0 CastorOil Denver)
 (20.5107 0.075 2.1029 0.0975 2.0 CastorOil Denver)
 (36.4635 0.1 4.9847 0.0548 2.0 CastorOil Denver)
 (13.7887 0.05 0.9425 0.1450 2.0 CastorOil Denver)
 (31.0246 0.075 3.1809 0.0645 2.0 CastorOil Denver)
 (55.1549 0.1 7.5398 0.0363 2.0 CastorOil Denver)
 (4.4373 0.05 0.6231 0.4507 1.0 Vacuum DeathValley)
 (4.4373 0.075 2.1029 0.4507 1.0 Vacuum DeathValley)
 (4.4373 0.1 4.9847 0.4507 1.0 Vacuum DeathValley)
 (4.4373 0.05 0.9425 0.4507 1.0 Vacuum DeathValley)
 (4.4373 0.075 3.1809 0.4507 1.0 Vacuum DeathValley)
 (4.4373 0.1 7.5398 0.4507 1.0 Vacuum DeathValley)
 (4.4249 0.05 0.6231 0.4520 1.0 Vacuum Denver)
 (4.4249 0.075 2.1029 0.4520 1.0 Vacuum Denver)
 (4.4249 0.1 4.9847 0.4520 1.0 Vacuum Denver)
 (4.4249 0.05 0.9425 0.4520 1.0 Vacuum Denver)
 (4.4249 0.075 3.1809 0.4520 1.0 Vacuum Denver)
 (4.4249 0.1 7.5398 0.4520 1.0 Vacuum Denver)
 (6.2753 0.05 0.6231 0.6374 2.0 Vacuum DeathValley)
 (6.2753 0.075 2.1029 0.6374 2.0 Vacuum DeathValley)
 (6.2753 0.1 4.9847 0.6374 2.0 Vacuum DeathValley)
 (6.2753 0.05 0.9425 0.6374 2.0 Vacuum DeathValley)
 (6.2753 0.075 3.1809 0.6374 2.0 Vacuum DeathValley)
 (6.2753 0.1 7.5398 0.6374 2.0 Vacuum DeathValley)
 (6.2578 0.05 0.6231 0.6392 2.0 Vacuum Denver)
 (6.2578 0.075 2.1029 0.6392 2.0 Vacuum Denver)
 (6.2578 0.1 4.9847 0.6392 2.0 Vacuum Denver)
 (6.2578 0.05 0.9425 0.6392 2.0 Vacuum Denver)
 (6.2578 0.075 3.1809 0.6392 2.0 Vacuum Denver)
 (6.2578 0.1 7.5398 0.6392 2.0 Vacuum Denver)
)
)

;;; Dribble file #P"/usr.MC68020/mlis3a/jmichael/SUN_AB2/stoke.out2" started

***** ABACUS MENU *****

select one:

q: quit
g: get and initialize a dataset
r: run program
next: run abacus on the next set of clustered groups
p: change program parameters
s: show results
f: output full results to a file
ab: run equation-learning program only
aq: run Aq (ab or r must have been run at least once)
setq: setq given atom to dataset
eval: evaluate an s-expression

= > g

Enter file name => "abacus2/data/stoke.data"

Dataset loaded

***** ABACUS MENU *****

select one:

q: quit
g: get and initialize a dataset
r: run program
next: run abacus on the next set of clustered groups
p: change program parameters
s: show results
f: output full results to a file
ab: run equation-learning program only
aq: run Aq (ab or r must have been run at least once)
setq: setq given atom to dataset
eval: evaluate an s-expression

= > r

>> Error: There are 730 words left and GC is disabled.
This is the last warning before memory is exhausted.

LUCID:NEW-SIMPLE-VECTOR:

:A Abort to Lisp Top Level
:C GC Will remain Disabled
-> :C
GC Will remain Disabled
;;; GC cannot obtain space

APPENDIX A4.2 - Verification of Stoke Experiment #2

The stoke experiment provided the same results as reported by Greene with the uncertainty value increased from 0.99 to 0.95. The greater the uncertainty allowed, the fewer the number of nodes that must be created, and therefore the less memory that needs to be used to discover the regularities.

```
(
(declarations
  (v L ((meter 1) (sec -1)))
  (r L ((meter 1)))
  (m L ((kg 1)))
  (t L ((sec 1)))
  (h L ((meter 1)))
  (substance N nil)
  (location N nil)
)

(parameters
  (*unvignore* (3 4))
  (*uncertainty* 0.02)
  (*cluster-order* (gc nc rc))
  (*print-flags* (p e))
)

(variables
  (v r m t h substance location)
)

(events
  (11.9425 0.05 0.6231 0.0837 1.0 Glycerol DeathValley)
  (26.8705 0.075 2.1029 0.0372 1.0 Glycerol DeathValley)
  (47.7698 0.1 4.9847 0.0209 1.0 Glycerol DeathValley)
  (18.0642 0.05 0.9425 0.0554 1.0 Glycerol DeathValley)
  (40.6445 0.075 3.1809 0.0246 1.0 Glycerol DeathValley)
  (72.2569 0.1 7.5398 0.0138 1.0 Glycerol DeathValley)
  (9.1671 0.05 0.6231 0.1091 1.0 CastorOil DeathValley)
  (20.6260 0.075 2.1029 0.0485 1.0 CastorOil DeathValley)
  (36.6684 0.1 4.9847 0.0273 1.0 CastorOil DeathValley)
  (13.8662 0.05 0.9425 0.0721 1.0 CastorOil DeathValley)
  (31.1989 0.075 3.1809 0.0321 1.0 CastorOil DeathValley)
  (55.4648 0.1 7.5398 0.0180 1.0 CastorOil DeathValley)
  (11.8757 0.05 0.6231 0.0842 1.0 Glycerol Denver)
  (26.7204 0.075 2.1029 0.0374 1.0 Glycerol Denver)
  (47.5030 0.1 4.9847 0.0211 1.0 Glycerol Denver)
  (17.9633 0.05 0.9425 0.0557 1.0 Glycerol Denver)
  (40.4174 0.075 3.1809 0.0247 1.0 Glycerol Denver)
  (71.8532 0.1 7.5398 0.0139 1.0 Glycerol Denver)
  (9.1159 0.05 0.6231 0.1097 1.0 CastorOil Denver)
  (20.5107 0.075 2.1029 0.0488 1.0 CastorOil Denver)
  (36.4635 0.1 4.9847 0.0274 1.0 CastorOil Denver)
  (13.7887 0.05 0.9425 0.0725 1.0 CastorOil Denver)
  (31.0246 0.075 3.1809 0.0322 1.0 CastorOil Denver)
  (55.1549 0.1 7.5398 0.0181 1.0 CastorOil Denver)
```

(11.9425 0.05 0.6231 0.1675 2.0 Glycerol DeathValley)
 (26.8705 0.075 2.1029 0.0744 2.0 Glycerol DeathValley)
 (47.7698 0.1 4.9847 0.0419 2.0 Glycerol DeathValley)
 (18.0642 0.05 0.9425 0.1107 2.0 Glycerol DeathValley)
 (40.6445 0.075 3.1809 0.0492 2.0 Glycerol DeathValley)
 (72.2569 0.1 7.5398 0.0277 2.0 Glycerol DeathValley)
 (9.1671 0.05 0.6231 0.2182 2.0 CastorOil DeathValley)
 (20.6260 0.075 2.1029 0.0970 2.0 CastorOil DeathValley)
 (36.6684 0.1 4.9847 0.0545 2.0 CastorOil DeathValley)
 (13.8662 0.05 0.9425 0.1442 2.0 CastorOil DeathValley)
 (31.1989 0.075 3.1809 0.0641 2.0 CastorOil DeathValley)
 (55.4648 0.1 7.5398 0.0361 2.0 CastorOil DeathValley)
 (11.8757 0.05 0.6231 0.1684 2.0 Glycerol Denver)
 (26.7204 0.075 2.1029 0.0748 2.0 Glycerol Denver)
 (47.5030 0.1 4.9847 0.0421 2.0 Glycerol Denver)
 (17.9633 0.05 0.9425 0.1113 2.0 Glycerol Denver)
 (40.4174 0.075 3.1809 0.0495 2.0 Glycerol Denver)
 (71.8532 0.1 7.5398 0.0278 2.0 Glycerol Denver)
 (9.1159 0.05 0.6231 0.2194 2.0 CastorOil Denver)
 (20.5107 0.075 2.1029 0.0975 2.0 CastorOil Denver)
 (36.4635 0.1 4.9847 0.0548 2.0 CastorOil Denver)
 (13.7887 0.05 0.9425 0.1450 2.0 CastorOil Denver)
 (31.0246 0.075 3.1809 0.0645 2.0 CastorOil Denver)
 (55.1549 0.1 7.5398 0.0363 2.0 CastorOil Denver)
 (4.4373 0.05 0.6231 0.4507 1.0 Vacuum DeathValley)
 (4.4373 0.075 2.1029 0.4507 1.0 Vacuum DeathValley)
 (4.4373 0.1 4.9847 0.4507 1.0 Vacuum DeathValley)
 (4.4373 0.05 0.9425 0.4507 1.0 Vacuum DeathValley)
 (4.4373 0.075 3.1809 0.4507 1.0 Vacuum DeathValley)
 (4.4373 0.1 7.5398 0.4507 1.0 Vacuum DeathValley)
 (4.4249 0.05 0.6231 0.4520 1.0 Vacuum Denver)
 (4.4249 0.075 2.1029 0.4520 1.0 Vacuum Denver)
 (4.4249 0.1 4.9847 0.4520 1.0 Vacuum Denver)
 (4.4249 0.05 0.9425 0.4520 1.0 Vacuum Denver)
 (4.4249 0.075 3.1809 0.4520 1.0 Vacuum Denver)
 (4.4249 0.1 7.5398 0.4520 1.0 Vacuum Denver)
 (6.2753 0.05 0.6231 0.6374 2.0 Vacuum DeathValley)
 (6.2753 0.075 2.1029 0.6374 2.0 Vacuum DeathValley)
 (6.2753 0.1 4.9847 0.6374 2.0 Vacuum DeathValley)
 (6.2753 0.05 0.9425 0.6374 2.0 Vacuum DeathValley)
 (6.2753 0.075 3.1809 0.6374 2.0 Vacuum DeathValley)
 (6.2753 0.1 7.5398 0.6374 2.0 Vacuum DeathValley)
 (6.2578 0.05 0.6231 0.6392 2.0 Vacuum Denver)
 (6.2578 0.075 2.1029 0.6392 2.0 Vacuum Denver)
 (6.2578 0.1 4.9847 0.6392 2.0 Vacuum Denver)
 (6.2578 0.05 0.9425 0.6392 2.0 Vacuum Denver)
 (6.2578 0.075 3.1809 0.6392 2.0 Vacuum Denver)
 (6.2578 0.1 7.5398 0.6392 2.0 Vacuum Denver)
))

=== Parameters ===

MAXSTAR = 10
 COVERMODE = IC
 UNCERTAINTY = 0.02
 CONSTANCY = 0.95
 INVERSE-SW = 30
 DIRECT-SW = 70
 UNIVIGNORE = (3 4)
 DEBUG = NIL

PRINT-FLAGS = (P E)
 FILTER-LEVEL = 2
 SEARCH-LIMIT = 3
 MAX-NODES = 1000
 CLUSTER-ON = # <Symbol 46001D>
 CLUSTER-ORDER = (GC NC RC)
 BEAM-VALUE = 0.5
 MINIMUM-LINK-VALUE = 50.0

 starLEF = ((MAX-COVERED 0.5) (MIN-WEIGHT 0.0) (MIN-SELECTORS 0.0))

--- Class Events ---

CLASSC

(V R M # <Symbol 46001D> H SUBSTANCE LOCATION)
 (9.1159 0.05 0.6231 0.1097 1.0 CASTOROIL DENVER)
 (9.1159 0.05 0.6231 0.2194 2.0 CASTOROIL DENVER)
 (13.7887 0.05 0.9425 0.0725 1.0 CASTOROIL DENVER)
 (13.7887 0.05 0.9425 0.145 2.0 CASTOROIL DENVER)
 (31.0246 0.075 3.1809 0.0322 1.0 CASTOROIL DENVER)
 (31.0246 0.075 3.1809 0.0645 2.0 CASTOROIL DENVER)
 (36.4635 0.1 4.9847 0.0274 1.0 CASTOROIL DENVER)
 (36.4635 0.1 4.9847 0.0548 2.0 CASTOROIL DENVER)
 (20.5107 0.075 2.1029 0.0975 2.0 CASTOROIL DENVER)
 (20.5107 0.075 2.1029 0.0488 1.0 CASTOROIL DENVER)
 (55.1549 0.1 7.5398 0.0181 1.0 CASTOROIL DENVER)
 (55.1549 0.1 7.5398 0.0363 2.0 CASTOROIL DENVER)
 (9.1671 0.05 0.6231 0.1091 1.0 CASTOROIL DEATHVALLEY)
 (9.1671 0.05 0.6231 0.2182 2.0 CASTOROIL DEATHVALLEY)
 (13.8662 0.05 0.9425 0.0721 1.0 CASTOROIL DEATHVALLEY)
 (13.8662 0.05 0.9425 0.1442 2.0 CASTOROIL DEATHVALLEY)
 (31.1989 0.075 3.1809 0.0641 2.0 CASTOROIL DEATHVALLEY)
 (31.1989 0.075 3.1809 0.0321 1.0 CASTOROIL DEATHVALLEY)
 (36.6684 0.1 4.9847 0.0545 2.0 CASTOROIL DEATHVALLEY)
 (36.6684 0.1 4.9847 0.0273 1.0 CASTOROIL DEATHVALLEY)
 (20.626 0.075 2.1029 0.0485 1.0 CASTOROIL DEATHVALLEY)
 (20.626 0.075 2.1029 0.097 2.0 CASTOROIL DEATHVALLEY)
 (55.4648 0.1 7.5398 0.018 1.0 CASTOROIL DEATHVALLEY)
 (55.4648 0.1 7.5398 0.0361 2.0 CASTOROIL DEATHVALLEY)

CLASSB

(V R M # <Symbol 46001D> H SUBSTANCE LOCATION)
 (11.8757 0.05 0.6231 0.0842 1.0 GLYCEROL DENVER)
 (11.8757 0.05 0.6231 0.1684 2.0 GLYCEROL DENVER)
 (17.9633 0.05 0.9425 0.1113 2.0 GLYCEROL DENVER)
 (17.9633 0.05 0.9425 0.0557 1.0 GLYCEROL DENVER)
 (40.4174 0.075 3.1809 0.0247 1.0 GLYCEROL DENVER)
 (40.4174 0.075 3.1809 0.0495 2.0 GLYCEROL DENVER)
 (47.503 0.1 4.9847 0.0421 2.0 GLYCEROL DENVER)
 (47.503 0.1 4.9847 0.0211 1.0 GLYCEROL DENVER)
 (26.7204 0.075 2.1029 0.0374 1.0 GLYCEROL DENVER)
 (26.7204 0.075 2.1029 0.0748 2.0 GLYCEROL DENVER)
 (71.8532 0.1 7.5398 0.0139 1.0 GLYCEROL DENVER)
 (71.8532 0.1 7.5398 0.0278 2.0 GLYCEROL DENVER)
 (18.0642 0.05 0.9425 0.1107 2.0 GLYCEROL DEATHVALLEY)
 (18.0642 0.05 0.9425 0.0554 1.0 GLYCEROL DEATHVALLEY)
 (11.9425 0.05 0.6231 0.0837 1.0 GLYCEROL DEATHVALLEY)
 (11.9425 0.05 0.6231 0.1675 2.0 GLYCEROL DEATHVALLEY)
 (40.6445 0.075 3.1809 0.0246 1.0 GLYCEROL DEATHVALLEY)

(40.6445 0.075 3.1809 0.0492 2.0 GLYCEROL DEATHVALLEY)
 (47.7698 0.1 4.9847 0.0209 1.0 GLYCEROL DEATHVALLEY)
 (47.7698 0.1 4.9847 0.0419 2.0 GLYCEROL DEATHVALLEY)
 (26.8705 0.075 2.1029 0.0372 1.0 GLYCEROL DEATHVALLEY)
 (26.8705 0.075 2.1029 0.0744 2.0 GLYCEROL DEATHVALLEY)
 (72.2569 0.1 7.5398 0.0138 1.0 GLYCEROL DEATHVALLEY)
 (72.2569 0.1 7.5398 0.0277 2.0 GLYCEROL DEATHVALLEY)

CLASSA

(V R M # <Symbol 46001D> H SUBSTANCE LOCATION)
 (4.4249 0.05 0.6231 0.452 1.0 VACUUM DENVER)
 (4.4249 0.05 0.9425 0.452 1.0 VACUUM DENVER)
 (4.4249 0.075 2.1029 0.452 1.0 VACUUM DENVER)
 (4.4249 0.075 3.1809 0.452 1.0 VACUUM DENVER)
 (4.4249 0.1 4.9847 0.452 1.0 VACUUM DENVER)
 (4.4249 0.1 7.5398 0.452 1.0 VACUUM DENVER)
 (6.2578 0.05 0.6231 0.6392 2.0 VACUUM DENVER)
 (6.2578 0.05 0.9425 0.6392 2.0 VACUUM DENVER)
 (6.2578 0.075 2.1029 0.6392 2.0 VACUUM DENVER)
 (6.2578 0.075 3.1809 0.6392 2.0 VACUUM DENVER)
 (6.2578 0.1 4.9847 0.6392 2.0 VACUUM DENVER)
 (6.2578 0.1 7.5398 0.6392 2.0 VACUUM DENVER)
 (6.2753 0.05 0.6231 0.6374 2.0 VACUUM DEATHVALLEY)
 (6.2753 0.05 0.9425 0.6374 2.0 VACUUM DEATHVALLEY)
 (6.2753 0.075 2.1029 0.6374 2.0 VACUUM DEATHVALLEY)
 (6.2753 0.075 3.1809 0.6374 2.0 VACUUM DEATHVALLEY)
 (6.2753 0.1 4.9847 0.6374 2.0 VACUUM DEATHVALLEY)
 (6.2753 0.1 7.5398 0.6374 2.0 VACUUM DEATHVALLEY)
 (4.4373 0.05 0.6231 0.45069999 1.0 VACUUM DEATHVALLEY)
 (4.4373 0.05 0.9425 0.45069999 1.0 VACUUM DEATHVALLEY)
 (4.4373 0.075 2.1029 0.45069999 1.0 VACUUM DEATHVALLEY)
 (4.4373 0.075 3.1809 0.45069999 1.0 VACUUM DEATHVALLEY)
 (4.4373 0.1 4.9847 0.45069999 1.0 VACUUM DEATHVALLEY)
 (4.4373 0.1 7.5398 0.45069999 1.0 VACUUM DEATHVALLEY)

=== Hypotheses ===

CLASSA

Cover:

24 [SUBSTANCE = VACUUM]

Relation:

$V = 9.8175 * \# < \text{Symbol 46001D} >$

CLASSB

Cover:

24 [SUBSTANCE = GLYCEROL]

Relation:

$R * V = 0.9556 * M$

CLASSC

Cover:

24 [SUBSTANCE = CASTOROIL]

Relation:

$R * V = 0.7336 * M$

APPENDIX A5 - Verification of Conservation Experiment

The results of the conservation of momentum experiment were the same as those reported by Greene.

```
(
(declarations
(m L ((meters 1)))
(Vo L ((meters 1) (seconds -1)))
(V L ((meters 1) (seconds -1)))
(I L ((kilogram 1) (meters 2)))
(omega L ((radians 1) (seconds -2)))
(Ediss L ((joules 1)))
)

(parameters
(*print-flags* (p e))
(*cluster-order* (nc gc rc))
)

(variables
(m Vo V I omega Ediss)
)

(equations
(= Ediss (+ (* c1 (* m Vo Vo)) (* c2 (- (* I omega omega)))))
(= Ediss (+ (* c1 (* m V V)) (* c2 (- (* m V V)))))
(= Ediss (+ (* c1 (* m Vo Vo)) (* c2 (- (* m V V)) (* c3 (- (* I omega omega)))))
)

(events
(40.0          6.77    3.11    720.0    0.667    563.05)
(55.6          5.96    4.89    801.7    0.501    222.13)
(34.65  4.86    2.3    685.6    0.473    240.86)
(69.48  5.21    4.5    699.0    0.801    15.26)
(34.5        4.09    2.51    598.8    0.542    91.93)
(43.0        6.34    4.4    740.9    0.603    313.26)
(50.2        7.21    4.89    798.2    0.66    530.75)
(28.9        3.87    1.98    402.0    0.329    138.01)
(47.5        4.32    3.95    574.5    0.354    36.67)
(38.5        4.68    3.75    583.4    0.473    85.65)
(47.5        5.68    4.53    754.0    0.385    222.97)
(51.0        5.84    4.367    803    0.574    251.11)
)
)
```

=== Parameters ===

```
*MAXSTAR* = 10
*COVERMODE* = IC
*UNCERTAINTY* = 0.02
*CONSTANCY* = 0.95
*INVERSE-SW* = 30
*DIRECTIONS-SW* = 70
```

UNVIGNORE = NIL
 DEBUG = NIL
 PRINT-FLAGS = (P E)
 FILTER-LEVEL = 2
 SEARCH-LIMIT = 3
 MAX-NODES = 1000
 CLUSTER-ON = T
 CLUSTER-ORDER = (NC GC RC)
 BEAM-VALUE = 0.5
 MINIMUM-LINK-VALUE = 50.0

starLEF = ((MAX-COVERED 0.5) (MIN-WEIGHT 0.0) (MIN-SELECTORS 0.0))

--- Class Events ---

CLASSA
 (M VO V I OMEGA EDISS)
 (69.48 5.21 4.5 699.0 0.801 15.26)
 (47.5 4.32 3.95 574.5 0.354 36.67)
 (38.5 4.68 3.75 583.4 0.473 85.65)
 (34.5 4.09 2.51 598.8 0.542 91.93)
 (28.9 3.87 1.98 402.0 0.329 138.01)
 (55.6 5.96 4.89 801.7 0.501 222.13)
 (47.5 5.68 4.53 754.0 0.385 222.97)
 (34.65 4.86 2.3 685.6 0.473 240.86)
 (51.0 5.84 4.367 803.0 0.574 251.11)
 (43.0 6.34 4.4 740.9 0.603 313.26)
 (50.2 7.21 4.89 798.2 0.66 530.75)
 (40.0 6.77 3.11 720.0 0.667 563.05)

--- Hypotheses ---

CLASSA

Cover: Relation:

EDISS = 0.49999222 * (M * VO * VO) + 0.49999544 * (- (M * V * V)) + 0.49998108 * (- (I * OMEGA * OMEGA))

APPENDIX B1 - Network Experiment #1

This experiment was run using network operating system statistics from a network of 65 Sun 3/50 and 3/60 workstations. The statistics were recorded during the middle of the workday every five seconds.

The statistics consisted of three load averages (*loadav1*, *loadav2*, and *loadav3*), the current time (*cur_time*), the number of existing processes (*exist_proc*), the number of processes in each state:

- sleeping (*sleeping*)
- abandoned (*abandoned*)
- running (*running*)
- starting (*starting*)

percentage of time spent in each of the processor states (*user*, *nice*, *system*, and *idle*), and memory statistics:

- amount of virtual memory in use (*virt_mem*)
- amount of active virtual memory in use (*virt_act*)
- amount of real memory in use (*real_mem*)
- amount of active real memory in use (*real_act*)
- amount of free memory (*free*)

The raw data had to be extracted from the Top program output. Top is a public domain program that is included as a utility program with the Sun Unix operating system. The data capture was achieved by setting the terminal type to 'dumb' in order to write to a file ASCII output (the output would otherwise have been binary since Top uses the Sun Curses features), and then running the output through Unix sed scripts that format the data for use with ABACUS.2.

After manipulating the various parameters, it became apparent that ABACUS.2

could not find dependencies between variables for this dataset unless the variables were allowed to vary during the calculation of the dependencies. To achieve this, the universal ignore (*univignore*) parameter used. In this case, the first seven variables were allowed to vary. The same result was obtained when all 18 variables were allowed to vary.

```
(
(declarations
(loadav1 L ((load 1)))
(loadav2 L ((load 1)))
(loadav3 L ((load 1)))
(cur_time N nil)
(exist_proc L ((process 1)))
(sleeping L ((process 1)))
(abandoned L ((process 1)))
(running L ((process 1)))
(starting L ((process 1)))
(user L ((time 1)))
(nice L ((time 1)))
(system L ((time 1)))
(idle L ((time 1)))
(virt_mem L ((kilobyte 1)))
(virt_act L ((kilobyte 1)))
(real_mem L ((kilobyte 1)))
(real_act L ((kilobyte 1)))
(free L ((kilobyte 1)))
)

(parameters
(*print-flags* (p e))
)

(variables
(loadav1 loadav2 loadav3 cur_time exist_proc sleeping abandoned running
starting user nice system idle virt_mem virt_act real_mem rel_act
free)
)

(events
(1.36 2.36 1.64 15_14_34 73 68 1 1 3 6.0 4.4 20.3 69.3 6624 2560 21976 13392 4536)
(1.25 2.32 1.63 15_14_39 73 67 2 1 3 2.0 0.0 9.1 88.9 6624 2576 21976 13416 4472)
(1.14 2.27 1.61 15_14_45 73 68 1 1 3 1.2 0.0 7.1 91.7 6624 2576 21976 13720 4384)
(1.05 2.22 1.60 15_14_50 73 67 2 1 3 2.8 0.0 21.3 75.9 6624 2752 21976 13904 4344)
(1.13 2.21 1.60 15_14_55 73 68 1 1 3 1.5 0.0 78.7 19.9 6624 2568 21976 13720 4344)
(1.20 2.21 1.60 15_15_00 73 68 1 1 3 1.6 0.0 19.1 79.3 6624 2552 21976 13696 4344)
(1.26 2.20 1.60 15_15_05 73 68 1 1 3 3.9 0.0 18.4 77.6 6672 2528 22040 13704 4328)
(1.23 2.18 1.59 15_15_10 73 68 1 1 3 2.0 0.0 24.9 73.1 6624 2496 21976 13664 4328)
(1.13 2.13 1.57 15_15_15 73 68 1 1 3 0.8 0.0 18.9 80.3 6624 2664 21976 14080 4328)
(1.03 2.07 1.55 15_15_20 73 68 1 1 3 1.2 0.0 9.1 89.7 6624 2736 21976 14136 4328)
(1.02 2.04 1.54 15_15_25 73 68 1 1 3 12.6 0.0 27.7 59.7 6536 3248 21896 14728 4368)
(1.02 2.02 1.54 15_15_30 74 68 2 1 3 5.9 0.0 47.0 47.0 6584 3248 21960 14536 4344)
(1.02 2.00 1.53 15_15_36 73 67 2 1 3 2.8 0.0 43.5 53.8 6528 3208 21888 14488 4296)
(1.09 2.00 1.53 15_15_41 73 68 1 1 3 2.0 0.0 27.7 70.4 6520 3200 21872 14472 4384)
)
```

```

(1.00 1.96 1.51 15_15_46 73 68 1 1 3 0.8 0.0 12.2 87.0 6528 2856 21888 14096 4408)
(0.92 1.92 1.50 15_15_51 73 68 1 1 3 0.4 0.0 7.1 92.5 6528 2800 21888 14024 4408)
(0.84 1.88 1.48 15_15_56 73 68 1 1 3 1.2 0.0 5.2 93.7 6528 2304 21888 13424 4408 )
(1.34 1.98 1.53 15_16_01 73 68 1 1 3 3.2 0.0 16.2 80.6 6528 2304 21888 13424 4424)
(1.31 1.96 1.52 15_16_06 73 67 2 1 3 0.8 0.0 4.7 94.5 6528 2320 21888 13448 4424)
(1.20 1.92 1.50 15_16_11 73 68 1 1 3 0.7 0.0 5.1 94.2 6528 2320 21888 13448 4424)
(1.10 1.88 1.49 15_16_16 73 68 1 1 3 2.8 0.0 12.6 84.6 6528 2320 21888 13448 4424)
(1.01 1.84 1.48 15_16_22 73 68 1 1 3 0.8 0.0 1.2 98.0 6528 2320 21888 13448 4424)
(0.92 1.80 1.46 15_16_27 73 68 1 1 3 0.8 0.0 1.6 97.6 6528 2304 21888 13424 4424)
(0.84 1.76 1.45 15_16_32 73 68 1 1 3 2.0 0.0 7.9 90.1 6528 2304 21888 13424 4424)
(0.77 1.73 1.44 15_16_37 72 67 1 1 3 0.8 0.0 4.7 94.5 6448 2392 21792 13528 4464)
(0.71 1.69 1.43 15_16_42 72 67 1 1 3 0.8 0.0 2.0 97.2 6448 2408 21792 13552 4504)
(0.65 1.66 1.42 15_16_47 73 66 3 1 3 4.4 0.0 23.4 72.2 6448 2600 21792 13752 4496)
(0.68 1.64 1.41 15_16_52 74 69 1 1 3 5.5 0.0 4.2 80.3 6592 3024 21968 14272 4480)
(0.62 1.61 1.40 15_16_57 75 70 1 1 3 1.2 0.0 13.8 85.0 6592 3024 21968 14416 4392)
(1.22 1.73 1.45 15_17_02 74 70 1 3 0 2.0 0.0 42.0 56.1 6672 3216 22064 14648 4304)
(1.12 1.69 1.44 15_17_07 74 69 2 3 0 1.6 0.0 22.5 75.9 6672 3448 22064 14904 4296)
)
)

```

--- Parameters ---

```

*MAXSTAR* = 10
*COVERMODE* = IC
*UNCERTAINTY* = 0.02
*CONSTANCY* = 0.95
*INVERSE-SW* = 30
*DIREKT-SW* = 70
*UNVIGNORE* = (0 1 2 3 4 5 6)
*DEBUG* = TRUE
*PRINT-FLAGS* = (P E)
*FILTER-LEVEL* = 2
*SEARCH-LIMIT* = 3
*MAX-NODES* = 1000
*CLUSTER-ON* = T
*CLUSTER-ORDER* = (NC GC RC)
*BEAM-VALUE* = 0.5
*MINIMUM-LINK-VALUE* = 50.0

*starLEF* = ((MAX-COVERED 0.5) (MIN-WEIGHT 0.0) (MIN-SELECTORS 0.0))

```

--- Class Events ---

CLASSA

```

(LOADAV1 LOADAV2 LOADAV3 CUR_TIME EXIST_PROC SLEEPING ABANDONED RUNNING
STARTING USER NICE SYSTEM IDLE VIRT_MEM VIRT_ACT REAL_MEM REAL_ACT FREE)

```

(0.92 1.92 1.5 15 15 51 73.0 68.0 1.0 1.0 3.0 0.4 0.0 7.1 92.5 6528.0 2800.0 21888.0 14024.0 4408.0)
 (1.2 1.92 1.5 15 16 11 73.0 68.0 1.0 1.0 3.0 0.7 0.0 5.1 94.2 6528.0 2320.0 21888.0 13448.0 4424.0)
 (0.77 1.73 1.44 15 16 37 72.0 67.0 1.0 1.0 3.0 0.8 0.0 4.7 94.5 6448.0 2392.0 21792.0 13528.0 4464.0)
 (0.71 1.69 1.43 15 16 42 72.0 67.0 1.0 1.0 3.0 0.8 0.0 2.0 97.2 6448.0 2408.0 21792.0 13552.0 4504.0)
 (0.92 1.8 1.46 15 16 27 73.0 68.0 1.0 1.0 3.0 0.8 0.0 1.6 97.6 6528.0 2304.0 21888.0 13424.0 4424.0)
 (1.31 1.96 1.52 15 16 06 73.0 67.0 2.0 1.0 3.0 0.8 0.0 4.7 94.5 6528.0 2320.0 21888.0 13448.0 4424.0)
 (1.01 1.84 1.48 15 16 22 73.0 68.0 1.0 1.0 3.0 0.8 0.0 1.2 98.0 6528.0 2320.0 21888.0 13448.0 4424.0)
 (1.0 1.96 1.51 15 15 46 73.0 68.0 1.0 1.0 3.0 0.8 0.0 12.2 87.0 6528.0 2856.0 21888.0 14096.0 4408.0)
 (1.13 2.13 1.57 15 15 15 73.0 68.0 1.0 1.0 3.0 0.8 0.0 18.9 80.3 6624.0 2664.0 21976.0 14080.0 4328.0)
 (0.84 1.88 1.48 15 15 56 73.0 68.0 1.0 1.0 3.0 1.2 0.0 5.2 93.7 6528.0 2304.0 21888.0 13424.0 4408.0)
 (0.62 1.61 1.4 15 16 57 75.0 70.0 1.0 1.0 3.0 1.2 0.0 13.8 85.0 6592.0 3024.0 21968.0 14416.0 4392.0)
 (1.14 2.27 1.61 15 14 45 73.0 68.0 1.0 1.0 3.0 1.2 0.0 7.1 91.7 6624.0 2576.0 21976.0 13720.0 4384.0)
 (1.03 2.07 1.55 15 15 20 73.0 68.0 1.0 1.0 3.0 1.2 0.0 9.1 89.7 6624.0 2736.0 21976.0 14136.0 4328.0)
 (1.13 2.21 1.6 15 14 55 73.0 68.0 1.0 1.0 3.0 1.5 0.0 78.7 19.9 6624.0 2568.0 21976.0 13720.0 4344.0)
 (1.2 2.21 1.6 15 15 00 73.0 68.0 1.0 1.0 3.0 1.6 0.0 19.1 79.3 6624.0 2552.0 21976.0 13696.0 4344.0)
 (1.12 1.69 1.44 15 17 07 74.0 69.0 2.0 3.0 0.0 1.6 0.0 22.5 75.9 6672.0 3448.0 22064.0 14904.0 4296.0)
 (1.22 1.73 1.45 15 17 02 74.0 70.0 1.0 3.0 0.0 2.0 0.0 42.0 56.1 6672.0 3216.0 22064.0 14648.0 4304.0)
 (1.09 2.0 1.53 15 15 41 73.0 68.0 1.0 1.0 3.0 2.0 0.0 27.7 70.4 6520.0 3200.0 21872.0 14472.0 4384.0)
 (0.84 1.76 1.45 15 16 32 73.0 68.0 1.0 1.0 3.0 2.0 0.0 7.9 90.1 6528.0 2304.0 21888.0 13424.0 4424.0)
 (1.23 2.18 1.59 15 15 10 73.0 68.0 1.0 1.0 3.0 2.0 0.0 24.9 73.1 6624.0 2496.0 21976.0 13664.0 4328.0)
 (1.25 2.32 1.63 15 14 39 73.0 67.0 2.0 1.0 3.0 2.0 0.0 9.1 88.9 6624.0 2576.0 21976.0 13416.0 4472.0)
 (1.02 2.0 1.53 15 15 36 73.0 67.0 2.0 1.0 3.0 2.8 0.0 43.5 53.8 6528.0 3208.0 21888.0 14488.0 4296.0)
 (1.1 1.88 1.49 15 16 16 73.0 68.0 1.0 1.0 3.0 2.8 0.0 12.6 84.6 6528.0 2320.0 21888.0 13448.0 4424.0)
 (1.05 2.22 1.6 15 14 50 73.0 67.0 2.0 1.0 3.0 2.8 0.0 21.3 75.9 6624.0 2752.0 21976.0 13904.0 4344.0)
 (1.34 1.98 1.53 15 16 01 73.0 68.0 1.0 1.0 3.0 3.2 0.0 16.2 80.6 6528.0 2304.0 21888.0 13424.0 4424.0)
 (1.26 2.2 1.6 15 15 05 73.0 68.0 1.0 1.0 3.0 3.9 0.0 18.4 77.6 6672.0 2528.0 22040.0 13704.0 4328.0)
 (0.65 1.66 1.42 15 16 47 73.0 66.0 3.0 1.0 3.0 4.4 0.0 23.4 72.2 6448.0 2600.0 21792.0 13752.0 4496.0)
 (1.02 2.02 1.54 15 15 30 74.0 68.0 2.0 1.0 3.0 5.9 0.0 47.0 47.0 6584.0 3248.0 21960.0 14536.0 4344.0)
 (0.68 1.64 1.41 15 16 52 74.0 69.0 1.0 1.0 3.0 5.5 0.0 4.2 80.3 6592.0 3024.0 21968.0 14272.0 4480.0)
 (1.36 2.36 1.64 15 14 34 73.0 68.0 1.0 1.0 3.0 6.0 4.4 20.3 69.3 6624.0 2560.0 21976.0 13392.0 4536.0)
 (1.02 2.04 1.54 15 15 25 73.0 68.0 1.0 1.0 3.0 12.6 0.0 27.7 59.7 6536.0 3248.0 21896.0 14728.0 4368.0)

=== Hypotheses ===

CLASSA

Cover: Relation:

$$\begin{aligned}
 & ((IDLE^2) / ((IDLE^2) + (SYSTEM^2) + 2*(IDLE * SYSTEM))) + ((SYSTEM^2) / ((IDLE^2) + \\
 & (SYSTEM^2) + 2*(IDLE * SYSTEM))) + 2*((IDLE * SYSTEM) / ((IDLE^2) + (SYSTEM^2) + 2*(IDLE * \\
 & SYSTEM))) + ((USER^2) / ((IDLE^2) + (SYSTEM^2) + 2*(IDLE * SYSTEM))) = 1.0014
 \end{aligned}$$

APPENDIX B2 - Network Experiment #2

This experiment was run using network operating system statistics that gathered using the Unix Top program as discussed in Appendix B1. This dataset consisted of 202 events and 20 variables. The following is a listing of the debug output as well as the equation that ABACUS.2 discovered. The debug output was included only for illustrative purposes. When working with large datasets, it is not advisable to use the debug option since this option slows down the system (i.e., there is overhead associated with writing the debug data to the screen). It was evident from the high frequency of invocation of garbage collection that that improvements need to be made to the algorithms to affect a reduction in the amount of garbage collection required (e.g., pruning algorithms that could be used to discard extraneous and/or redundant information).

Two additional variables were used in this experiment. They are the process states *zombie* and *stopped*.

Unlike the first experiment with the network data, 19 of the variables had to be permitted to vary for ABACUS.2 to discover dependencies between variables.


```
Clustering projection (17,8)...
Clustering projection (17,9)...
Clustering projection (17,10)...
Clustering projection (17,11)...
Clustering projection (17,12)...
Clustering projection (17,13)...
Clustering projection (17,14)...
Clustering projection (17,15)...
Clustering projection (17,16)...
Merging projection (17,16)...
Before merge: 1 groups.
After merge: 1 groups.
```

Merging projection (17 15)...

Merging projection (17 14)...

Merging projection (17 13)...

Merging projection (17 12)...

Merging projection (17 11)...

Merging projection (17 9)...

Merging projection (17 7)...

Merging projection (17 1)...

Merging projection (17 0)...

Merging projection (16 7)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (15 17)...

Merging projection (15 12)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (15 11)..
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (15 10)...

Merging projection (13 1)..
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (9 11)...

Before merge:	1 groups.
After merge:	1 groups.

Merging projection (9 10)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (9 7)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (9 2)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (9 1)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (9 0)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (8 17)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (8 16)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (8 15)...

Merging projection (8 14)...

Merging projection (8 13)...

Merging projection (8 12)...

Merging projection (8 11)...

Merging projection (8 10)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (8 9)...

Merging projection (8 6)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (8 2)...

Merging projection (7 0)...

Merging projection (6 17)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (6 16)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (6 15)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (6 14)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (6 13)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (6 12)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (6 11)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (6 10)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (6 9)...

Merging projection (6 8)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (6 7)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (6 5)...

Merging projection (6 4)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (6 2)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (6 1)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (6 0)...

Merging projection (4 14)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (4 13)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (4 12)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (4 11)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (4 10)...

Before merge: 1 groups.

After merge: 1 groups.

Merging projection (4 9)...

Merging projection (4 6)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (4 5)...

Merging projection (4 2)...

Merging projection (4 1)...

Merging projection (4 0)...

Merging projection (2 17)...

Merging projection (2 16)...

Merging projection (2 15)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (2 14)...

Merging projection (2 13)...

Merging projection (2 12)...

Merging projection (2 11)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (2 10)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (2 9)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (2 8)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (2 7)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (2 6)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (2 5)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (2 4)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (2 0)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (1 17)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (1 15)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (1 14)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (1 12)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (1 11)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (1 10)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (1 9)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (1 8)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (1 7)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (0 8)...
Before merge: 1 groups.
After merge: 1 groups.

Merging projection (0 7)...
Before merge: 1 groups.
After merge: 1 groups.

Cover for group <GROUP 155> is: NIL
Trying first cluster(8) is a zero-variable
(12) is a zero-variable

SEARCH TECHNIQUE NUMBER 5

REL (0, 1) 1 47.0
REL (0, 2) 1 44.0
REL (0, 4) 1 44.0

REL (0, 5) 1 43.0
REL (0, 6) 1 44.0
REL (0, 7) 1 80.0

REL (0, 8) 1 83.0
REL (0, 9) 1 38.0
REL (0, 10) 1 38.0
REL (0, 11) 1 37.0
REL (0, 12) 1 41.0
REL (0, 13) 1 33.0

REL (0, 14) 1 38.0
REL (0, 15) 1 38.0
REL (0, 16) 1 36.0
REL (0, 17) 1 38.0
REL (1, 2) 1 57.0
REL (1, 4) 1 36.0

REL (1, 5) 1 33.0
REL (1, 6) 1 33.0
REL (1, 7) 1 84.0
REL (1, 8) 1 92.0
REL (1, 9) 1 26.0
REL (1, 10) 1 27.0

REL (1, 11) 1 29.0
REL (1, 12) 1 29.0
REL (1, 13) 1 30.0
REL (1, 14) 1 27.0
REL (1, 15) 1 28.0
REL (1, 16) 1 30.0
REL (1, 17) 1 24.0

REL (2, 4) 1 23.0
REL (2, 5) 1 28.0
REL (2, 6) 1 26.0
REL (2, 7) 1 82.0
REL (2, 8) 1 86.0
REL (2, 9) 1 22.0

REL (2, 10) 1 19.0
REL (2, 11) 1 22.0
REL (2, 12) 1 20.0
REL (2, 13) 1 14.0
REL (2, 14) 1 25.0
REL (2, 15) 1 17.0

REL (2, 16) 1 25.0
 REL (2, 17) 1 17.0
 REL (4, 5) 1 0.0
 REL (4, 6) 1 8.0
 REL (4, 7) 1 41.0
 REL (4, 8) 1 33.0

 REL (4, 9) 1 0.0
 REL (4, 10) 1 0.0
 REL (4, 11) 1 0.0
 REL (4, 12) 1 8.0
 REL (4, 13) 1 0.0
 REL (4, 14) 1 0.0

 REL (4, 15) 1 0.0
 REL (4, 16) 1 0.0
 REL (4, 17) 1 0.0
 REL (5, 6) 1 0.0
 REL (5, 7) 1 47.0
 REL (5, 8) 1 41.0

 REL (5, 9) 1 11.0
 REL (5, 10) 1 11.0
 REL (5, 11) 1 11.0
 REL (5, 12) 1 5.0
 REL (5, 13) 1 5.0
 REL (5, 14) 1 5.0

 REL (5, 15) 1 11.0
 REL (5, 16) 1 17.0
 REL (5, 17) 1 11.0
 REL (6, 7) 1 22.0
 REL (6, 8) 1 22.0
 REL (6, 9) 1 11.0
 REL (6, 10) 1 11.0

 REL (6, 11) 1 0.0
 REL (6, 12) 1 22.0
 REL (6, 13) 1 0.0
 REL (6, 14) 1 11.0
 REL (6, 15) 1 0.0
 REL (6, 16) 1 11.0

 REL (6, 17) 1 0.0
 REL (7, 8) 1 0.0
 REL (7, 9) 1 0.0
 REL (7, 10) 1 0.0
 REL (7, 11) 1 0.0
 REL (7, 12) 1 0.0

 REL (7, 13) 1 0.0
 REL (7, 14) 1 0.0
 REL (7, 15) 1 0.0
 REL (7, 16) 1 0.0
 REL (7, 17) 1 0.0
 REL (8, 9) 1 0.0

 REL (8, 10) 1 0.0
 REL (8, 11) 1 0.0
 REL (8, 12) 1 0.0
 REL (8, 13) 1 0.0
 REL (8, 14) 1 0.0
 REL (8, 15) 1 0.0

 REL (8, 16) 1 0.0
 REL (8, 17) 1 0.0
 REL (9, 10) 1 23.0
 REL (9, 11) 1 27.0

REL (9, 12) 1 23.0
 REL (9, 13) 1 25.0

 REL (9, 14) 1 26.0
 REL (9, 15) 1 23.0
 REL (9, 16) 1 23.0
 REL (9, 17) 1 26.0
 REL (10, 11) 1 33.0
 REL (10, 12) 1 28.0

 REL (10, 13) 1 29.0
 REL (10, 14) 1 35.0
 REL (10, 15) 1 30.0
 REL (10, 16) 1 35.0
 REL (10, 17) 1 24.0
 REL (11, 12) 1 40.0

 REL (11, 13) 1 36.0
 REL (11, 14) 1 35.0
 REL (11, 15) 1 41.0
 REL (11, 16) 1 37.0
 REL (11, 17) 1 42.0
 REL (12, 13) 1 42.0

 REL (12, 14) 1 39.0
 REL (12, 15) 1 41.0
 REL (12, 16) 1 42.0
 REL (12, 17) 1 42.0
 REL (13, 14) 1 25.0
 REL (13, 15) 1 31.0
 REL (13, 16) 1 25.0

 REL (13, 17) 1 21.0
 REL (14, 15) 1 35.0
 REL (14, 16) 1 34.0
 REL (14, 17) 1 31.0
 REL (15, 16) 1 30.0
 REL (15, 17) 1 27.0

 REL (16, 17) 1 39.0
 doing edge = >(17 13)

Column 19 has constancy 1.0 (202 / 202)

Column 18 has constancy 1.0 (202 / 202)

Column 20 has constancy 1.0 (202 / 202)

Relation found: (#S(COLUMN ID 19 PROP+ NIL PROP- NIL PROP? NIL CONSTANCY 1.0 ZAPPED NIL SUBCOMPONENTS (17 13) FORM (- (FREE VIRT_MEM) NIL) RELMATRIX NIL UNTIS ((KILOBYTE 1)) ADJ NIL MARK NIL DFNUMBER NIL LOW NIL FATHER NIL) 1.0)

(8) is a zero-variable
(12) is a zero-variable

== - Parameters - ==

MAXSTAR = 10
 COVERMODE = IC
 UNCERTAINTY = 0.15
 CONSTANCY = 0.75
 INVERSE-SW = 30
 DIRECT-SW = 70
 UNVIGNORE = (0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17)
 DEBUG = TRUE
 PRINT-FLAGS = (P E)
 FILTER-LEVEL = 2
 SEARCH-LIMIT = 5
 MAX-NODES = 1000
 CLUSTER-ON = T
 CLUSTER-ORDER = (NC GC RC)
 BEAM-VALUE = 0.5
 MINIMUM-LINK-VALUE = 50.0

 starLEF = ((MAX-COVERED 0.5) (MIN-WEIGHT 0.0) (MIN-SELECTORS 0.0))

== - Class Events - ==

CLASSA

(LOADAV1 LOADAV2 LOADAV3 CUR_TIME EXIST_PROC SLEEPING ABANDONED RUNNING STARTING USER NICE SYSTEM IDLE VIRT_MEM VIRT_ACT REAL_MEM REAL_ACT FREE)

(2.63 2.52 2.46 11_44_32 87.0 83.0 1.0 1.0 2.0 5.9 3.9 53.9 36.3 7960.0 2960.0 10456.0 3688.0 3504.0)
 (3.05 2.97 2.64 11_55_06 86.0 82.0 3.0 1.0 0.0 6.9 9.2 52.5 31.4 7848.0 2800.0 10336.0 3600.0 3792.0)
 (3.47 2.86 2.58 11_51_41 89.0 84.0 3.0 2.0 0.0 6.7 11.4 71.0 11.0 8240.0 3744.0 11160.0 5000.0 3464.0)
 (3.89 3.07 2.67 11_52_37 87.0 83.0 2.0 2.0 0.0 6.3 15.7 59.4 18.5 8160.0 3328.0 11064.0 4528.0 3560.0)
 (2.57 2.51 2.46 11_44_37 86.0 82.0 1.0 1.0 2.0 3.5 5.8 40.5 50.2 7928.0 3408.0 10400.0 4344.0 3800.0)
 (3.84 3.1 2.68 11_54_35 86.0 83.0 2.0 1.0 0.0 1.9 11.7 44.4 42.0 8216.0 3336.0 10968.0 4448.0 3520.0)
 (2.83 2.59 2.5 11_49_38 91.0 86.0 2.0 1.0 2.0 4.3 11.3 57.0 27.3 8144.0 3080.0 11016.0 4360.0 3592.0)
 (2.73 2.82 2.57 11_54_04 88.0 85.0 1.0 2.0 0.0 1.2 10.1 37.4 51.4 8288.0 2928.0 11128.0 4008.0 3456.0)
 (2.56 2.41 2.43 11_47_41 86.0 81.0 2.0 1.0 2.0 4.7 6.7 42.0 46.7 7632.0 2528.0 10208.0 3336.0 4112.0)
 (3.33 2.82 2.57 11_51_36 90.0 85.0 3.0 2.0 0.0 9.4 9.0 58.2 23.4 8456.0 3856.0 11280.0 5152.0 3312.0)
 (2.77 2.52 2.48 11_48_42 88.0 84.0 1.0 1.0 2.0 2.7 7.8 36.1 53.3 8072.0 2968.0 11056.0 4224.0 3704.0)
 (2.96 3.39 2.98 12_01_25 81.0 77.0 3.0 1.0 0.0 3.1 12.8 46.5 37.6 7856.0 3000.0 10608.0 4072.0 3928.0)
 (2.95 2.61 2.51 11_49_23 89.0 85.0 1.0 1.0 2.0 9.3 13.6 48.8 28.3 8200.0 2688.0 11152.0 3832.0 3600.0)
 (4.44 3.7 3.08 12_00_13 84.0 82.0 1.0 1.0 0.0 6.6 7.8 53.7 31.9 8040.0 3248.0 10760.0 4488.0 3760.0)
 (2.7 2.45 2.45 11_47_36 86.0 82.0 1.0 1.0 2.0 3.1 5.4 37.0 54.5 7840.0 2592.0 10288.0 3264.0 3968.0)
 (2.44 2.38 2.42 11_47_25 86.0 81.0 2.0 1.0 2.0 4.3 16.1 46.1 33.5 7880.0 2688.0 10368.0 3408.0 3936.0)
 (2.8 2.65 2.5 11_51_11 88.0 83.0 2.0 1.0 2.0 3.1 10.5 63.7 22.7 8472.0 3824.0 10944.0 4768.0 3352.0)
 (2.71 2.82 2.57 11_53_59 91.0 88.0 1.0 2.0 0.0 1.2 4.3 37.0 57.5 8384.0 3216.0 11264.0 4344.0 3440.0)
 (3.7 2.99 2.64 11_52_22 90.0 86.0 2.0 2.0 0.0 3.5 37.9 26.6 32.0 8272.0 3184.0 11104.0 4256.0 3552.0)
 (2.57 2.58 2.48 11_50_45 91.0 86.0 2.0 1.0 2.0 12.1 5.1 48.8 34.0 8168.0 3144.0 10992.0 4224.0 3656.0)
 (2.21 2.42 2.44 11_46_09 89.0 84.0 2.0 1.0 2.0 2.0 13.7 48.6 35.7 8136.0 2952.0 10944.0 4016.0 3688.0)
 (2.5 2.5 2.47 11_45_49 89.0 84.0 2.0 1.0 2.0 2.4 7.9 37.2 52.6 8136.0 3032.0 10944.0 4072.0 3688.0)
 (2.64 2.54 2.48 11_45_43 90.0 84.0 3.0 1.0 2.0 4.3 16.7 46.3 32.7 8176.0 3144.0 10984.0 4200.0 3656.0)
 (3.3 2.65 2.54 11_48_52 91.0 87.0 1.0 1.0 2.0 6.7 14.9 62.0 16.5 8112.0 2680.0 11056.0 3832.0 3720.0)
 (2.93 2.66 2.52 11_50_30 89.0 85.0 1.0 1.0 2.0 8.7 12.6 49.6 29.1 8424.0 3792.0 11272.0 4960.0 3416.0)
 (3.42 2.98 2.64 11_54_25 90.0 83.0 4.0 1.0 2.0 13.3 8.6 78.1 0.0 8416.0 3408.0 11264.0 4544.0 3424.0)
 (2.72 2.82 2.57 11_53_18 86.0 82.0 2.0 2.0 0.0 2.7 18.0 58.8 20.4 8160.0 2960.0 11064.0 4080.0 3680.0)
 (2.64 2.53 2.47 11_44_47 88.0 83.0 2.0 1.0 2.0 3.5 5.1 44.3 47.1 7832.0 2968.0 10280.0 3848.0 4008.0)
 (2.51 2.5 2.46 11_44_52 87.0 81.0 3.0 1.0 2.0 3.5 28.7 22.4 45.3 7760.0 2840.0 10208.0 3720.0 4080.0)
 (2.96 2.68 2.52 11_51_05 90.0 85.0 2.0 1.0 2.0 14.1 7.4 61.3 17.2 8624.0 4168.0 11416.0 5440.0 3224.0)

(3.27 2.82 2.57 11 51 46 89.0 86.0 1.0 2.0 0.0 4.7 12.5 64.8 18.0 8320.0 3736.0 11240.0 4968.0 3528.0)
 (3.11 2.97 2.64 11 55 37 84.0 79.0 4.0 1.0 0.0 6.7 3.5 54.1 35.7 8072.0 3568.0 10864.0 4792.0 3776.0)
 (3.48 3.07 2.68 11 55 57 80.0 78.0 1.0 1.0 0.0 9.2 11.1 59.8 19.9 8032.0 3168.0 10824.0 4312.0 3816.0)
 (3.58 2.86 2.58 11 51 26 91.0 84.0 4.0 1.0 2.0 15.2 5.5 74.2 5.1 8560.0 3600.0 11416.0 4864.0 3296.0)
 (3.94 3.04 2.65 11 52 17 92.0 89.0 1.0 2.0 0.0 9.8 8.6 34.9 46.7 8360.0 3288.0 11280.0 4456.0 3496.0)
 (3.08 2.78 2.55 11 51 51 89.0 84.0 3.0 2.0 0.0 5.1 11.0 46.7 37.3 8288.0 3392.0 11200.0 4584.0 3568.0)
 (2.92 2.63 2.51 11 49 49 89.0 84.0 2.0 1.0 2.0 3.1 11.4 47.1 38.4 8280.0 3344.0 11152.0 4632.0 3576.0)
 (3.23 3.02 2.66 11 55 01 86.0 80.0 5.0 1.0 0.0 3.1 15.1 52.9 29.0 8224.0 2864.0 10680.0 3608.0 3632.0)
 (2.99 2.88 2.6 11 53 34 87.0 80.0 5.0 2.0 0.0 4.7 23.5 69.4 2.4 8120.0 2944.0 11064.0 4152.0 3736.0)
 (2.16 2.38 2.43 11 46 45 89.0 85.0 1.0 1.0 2.0 3.1 9.0 47.5 40.4 8064.0 2888.0 10856.0 3920.0 3792.0)
 (3.46 3.5 3.03 12 01 30 84.0 81.0 2.0 1.0 0.0 1.6 16.5 43.7 38.2 7864.0 2792.0 10608.0 3824.0 3992.0)
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 (3.36 3.5 3.02 12 01 09 83.0 78.0 4.0 1.0 0.0 10.0 12.0 53.7 24.3 8096.0 3168.0 10808.0 4352.0 3768.0)
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 (3.8 3.31 2.83 11 57 55 82.0 79.0 2.0 1.0 0.0 2.7 5.5 51.8 40.0 7936.0 3208.0 10352.0 3968.0 3936.0)
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 (3.2 2.94 2.62 11 52 58 89.0 86.0 1.0 2.0 0.0 2.7 9.4 40.8 47.1 8136.0 2920.0 11064.0 4096.0 3744.0)
 (2.46 2.48 2.54 11 45 08 89.0 85.0 1.0 1.0 2.0 3.5 14.8 53.9 27.7 8040.0 3344.0 10848.0 4440.0 3840.0)
 (2.91 2.86 2.59 11 53 29 90.0 83.0 5.0 2.0 0.0 7.5 9.4 75.7 7.5 8032.0 3048.0 10928.0 4216.0 3848.0)
 (3.27 3.03 2.67 11 56 02 82.0 78.0 3.0 1.0 0.0 4.3 8.2 50.4 37.1 7840.0 3168.0 10544.0 4256.0 4040.0)
 (2.87 2.86 2.59 11 53 54 89.0 84.0 3.0 2.0 0.0 5.1 5.5 36.2 53.1 8440.0 3216.0 11304.0 4320.0 3448.0)
 (2.91 2.74 2.54 11 51 57 89.0 86.0 1.0 2.0 0.0 10.5 4.7 52.3 32.4 8320.0 3304.0 11216.0 4456.0 3568.0)
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 (3.23 2.66 2.54 11 49 08 91.0 86.0 2.0 1.0 2.0 2.4 6.3 29.1 62.2 8312.0 3552.0 11208.0 4776.0 3576.0)
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 (2.32 2.43 2.45 11 46 29 87.0 81.0 3.0 1.0 2.0 5.1 12.1 42.2 40.6 8144.0 3264.0 10904.0 4296.0 3760.0)
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 (3.16 2.99 2.65 11 55 52 83.0 78.0 4.0 1.0 0.0 7.0 15.6 52.3 25.0 8112.0 3248.0 10880.0 4368.0 3800.0)
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 (2.99 2.53 2.48 11 48 11 89.0 84.0 2.0 1.0 2.0 8.6 9.4 40.4 41.6 8008.0 3048.0 11032.0 4336.0 3904.0)
 (4.09 3.29 2.79 11 56 33 82.0 69.0 12.0 1.0 0.0 6.2 12.0 51.2 30.6 7952.0 3264.0 10704.0 4400.0 3960.0)
 (3.59 3.34 2.88 11 58 41 82.0 78.0 3.0 1.0 0.0 7.1 9.5 60.9 22.5 7768.0 2952.0 10536.0 4088.0 4144.0)
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 (1.99 2.29 2.39 11 47 10 88.0 81.0 4.0 1.0 2.0 4.3 10.6 48.2 36.9 8168.0 3040.0 10936.0 4048.0 3752.0)
 (4.73 3.79 3.12 12 00 18 83.0 80.0 2.0 1.0 0.0 2.4 17.6 60.4 19.6 8160.0 3368.0 10896.0 4624.0 3760.0)
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 (2.7 2.82 2.57 11 53 13 88.0 85.0 1.0 2.0 0.0 1.6 21.5 59.4 17.6 8280.0 3360.0 11192.0 4552.0 3656.0)
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 (4.48 3.7 3.08 12 00 08 86.0 83.0 2.0 1.0 0.0 2.3 7.8 35.7 54.3 8208.0 3400.0 10936.0 4640.0 3728.0)
 (1.98 2.31 2.4 11 47 00 87.0 82.0 2.0 1.0 2.0 6.3 8.6 39.1 46.1 8136.0 2944.0 10912.0 3952.0 3800.0)

(2.48 2.39 2.43 11_47_20 87.0 83.0 1.0 1.0 2.0 7.1 17.3 50.4 25.2 8128.0 2880.0 10896.0 3872.0 3808.0)
 (2.34 2.45 2.44 11_45_13 88.0 84.0 1.0 1.0 2.0 5.0 8.9 43.6 42.5 8112.0 3272.0 10904.0 4344.0 3824.0)
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 (3.65 2.92 2.61 11_52_02 89.0 83.0 4.0 2.0 0.0 5.9 4.7 52.9 36.5 8360.0 3216.0 11328.0 4440.0 3584.0)
 (2.66 2.88 2.61 11_55_26 86.0 83.0 2.0 1.0 0.0 4.7 24.7 54.9 15.7 8208.0 4048.0 11008.0 5288.0 3736.0)
 (2.27 2.41 2.44 11_46_40 87.0 83.0 1.0 1.0 2.0 2.4 15.4 49.2 33.1 8144.0 3280.0 10912.0 4328.0 3800.0)
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 (1.99 2.28 2.38 11_47_15 89.0 83.0 3.0 1.0 2.0 3.1 15.7 55.1 26.0 8160.0 2912.0 10936.0 3912.0 3792.0)
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 (1.89 2.29 2.39 11_47_05 87.0 82.0 2.0 1.0 2.0 7.9 6.3 36.6 49.2 8128.0 2936.0 10912.0 3952.0 3824.0)
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 (3.61 3.13 2.71 11_56_18 83.0 80.0 2.0 1.0 0.0 12.5 4.3 46.7 36.5 8040.0 3832.0 10800.0 5032.0 3912.0)
 (3.53 3.31 2.86 11_58_25 83.0 80.0 2.0 1.0 0.0 7.1 7.5 47.1 38.4 7920.0 3320.0 10696.0 4456.0 4032.0)
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 (2.87 2.85 2.58 11_54_15 89.0 84.0 3.0 2.0 0.0 45.2 11.6 43.2 0.0 8440.0 3560.0 11288.0 4704.0 3520.0)
 (1.98 2.32 2.4 11_46_55 87.0 83.0 1.0 1.0 2.0 1.6 6.7 33.3 58.4 8168.0 2696.0 10968.0 3696.0 3792.0)
 (2.27 2.43 2.44 11_46_14 92.0 81.0 8.0 1.0 2.0 49.6 8.3 31.9 10.2 8296.0 3304.0 11640.0 4912.0 3672.0)
 (4.23 3.71 3.1 12_00_34 83.0 81.0 1.0 1.0 0.0 6.2 11.7 39.3 42.8 8152.0 3320.0 10864.0 4528.0 3816.0)
 (2.89 2.49 2.46 11_48_01 89.0 84.0 2.0 1.0 2.0 4.3 5.0 42.6 48.1 8056.0 2592.0 10912.0 3656.0 3912.0)
 (4.09 3.39 2.87 11_58_05 82.0 79.0 2.0 1.0 0.0 7.1 11.8 54.1 27.1 7816.0 3088.0 10216.0 3856.0 4152.0)
 (3.02 2.68 2.53 11_50_24 89.0 85.0 1.0 1.0 2.0 3.9 13.4 53.1 29.5 8744.0 4008.0 11552.0 5232.0 3232.0)
 (2.77 2.63 2.5 11_50_35 91.0 86.0 2.0 1.0 2.0 5.1 8.6 38.7 47.7 8496.0 4048.0 11352.0 5240.0 3480.0)
 (2.79 2.57 2.49 11_45_38 89.0 83.0 3.0 1.0 2.0 6.3 10.2 59.4 24.0 8480.0 3576.0 11256.0 4744.0 3496.0)
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 (3.48 2.95 2.62 11_52_27 89.0 84.0 3.0 2.0 0.0 7.1 21.6 34.5 36.9 8520.0 3552.0 11376.0 4664.0 3464.0)
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 (2.98 3.07 2.71 11_57_09 83.0 78.0 4.0 1.0 0.0 10.2 8.3 41.3 40.2 8136.0 3248.0 10856.0 4344.0 3848.0)
 (3.27 3.15 2.75 11_57_29 83.0 81.0 1.0 1.0 0.0 5.9 8.7 34.3 51.2 8120.0 3224.0 10856.0 4296.0 3864.0)
 (2.52 2.5 2.46 11_44_42 86.0 82.0 1.0 1.0 2.0 2.8 5.9 36.6 54.7 8080.0 3376.0 10560.0 4320.0 3904.0)
 (3.71 3.11 2.69 11_54_45 88.0 83.0 4.0 1.0 0.0 5.9 5.1 50.8 38.2 8416.0 4112.0 11192.0 5432.0 3576.0)
 (3.71 3.61 3.06 12_00_44 83.0 80.0 2.0 1.0 0.0 2.4 9.1 31.9 56.7 8176.0 3144.0 10864.0 4304.0 3816.0)
 (3.67 3.44 2.96 11_59_42 84.0 79.0 4.0 1.0 0.0 5.4 5.8 56.4 32.4 7976.0 3760.0 10736.0 5120.0 4016.0)
 (3.25 3.45 3.01 12_01_35 82.0 80.0 1.0 1.0 0.0 1.6 12.6 52.8 33.1 7944.0 2904.0 10664.0 3928.0 4048.0)
 (2.96 2.68 2.52 11_51_00 91.0 86.0 3.0 2.0 0.0 3.1 18.2 60.1 18.6 8680.0 4264.0 11504.0 5568.0 3320.0)
 (3.31 2.77 2.55 11_51_16 91.0 87.0 1.0 1.0 2.0 2.4 9.0 34.5 54.1 8600.0 3736.0 11480.0 4960.0 3400.0)
 (3.44 3.06 2.68 11_54_56 88.0 79.0 8.0 1.0 0.0 3.9 9.8 79.2 7.1 8432.0 3432.0 11200.0 4512.0 3568.0)
 (3.0 2.95 2.64 11_55_47 86.0 84.0 1.0 1.0 0.0 5.9 2.8 46.1 45.3 8224.0 3576.0 11024.0 4768.0 3776.0)
 (3.24 2.94 2.62 11_53_39 89.0 82.0 5.0 2.0 0.0 26.8 14.4 58.8 0.0 8208.0 3248.0 11064.0 4384.0 3792.0)
 (3.31 2.96 2.63 11_52_53 90.0 87.0 1.0 2.0 0.0 1.6 7.5 37.5 53.4 8192.0 3240.0 11104.0 4432.0 3808.0)
 (2.44 2.38 2.42 11_47_56 87.0 83.0 1.0 1.0 2.0 3.9 8.6 47.1 40.4 8104.0 2832.0 11024.0 3968.0 3896.0)
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 (3.76 3.27 2.81 11_57_39 84.0 81.0 2.0 1.0 0.0 8.7 7.9 40.2 43.3 8192.0 3160.0 10976.0 4280.0 3816.0)
 (4.05 3.25 2.76 11_56_23 82.0 80.0 1.0 1.0 0.0 6.7 6.3 44.9 42.1 8144.0 4048.0 10888.0 5248.0 3864.0)
 (3.39 3.17 2.76 11_57_24 83.0 78.0 4.0 1.0 0.0 5.5 6.3 36.9 51.4 8136.0 3224.0 10856.0 4272.0 3872.0)
 (3.92 3.26 2.78 11_56_38 82.0 79.0 2.0 1.0 0.0 13.4 13.4 54.3 18.9 8040.0 3136.0 10760.0 4216.0 3968.0)
 (4.55 3.61 3.01 11_59_22 82.0 79.0 2.0 1.0 0.0 2.4 17.3 58.4 22.0 7848.0 3216.0 10624.0 4360.0 4160.0)
 (3.43 2.7 2.55 11_49_03 88.0 82.0 3.0 1.0 2.0 14.1 5.1 51.4 29.4 8504.0 3904.0 11456.0 5184.0 3512.0)
 (2.38 2.47 2.46 11_45_54 88.0 82.0 3.0 1.0 2.0 5.1 6.7 39.6 48.6 8336.0 3264.0 11144.0 4296.0 3680.0)
 (3.4 3.52 3.03 12_01_04 85.0 83.0 1.0 1.0 0.0 4.3 5.5 53.5 36.7 8304.0 3440.0 11008.0 4624.0 3712.0)
 (4.5 3.65 3.05 11_59_53 84.0 82.0 1.0 1.0 0.0 6.2 5.8 77.1 10.9 8200.0 3984.0 10936.0 5448.0 3816.0)
 (3.35 3.16 2.75 11_56_58 85.0 73.0 11.0 1.0 0.0 1.6 7.9 60.6 29.9 8136.0 3720.0 10856.0 4872.0 3880.0)
 (3.82 3.47 2.97 11_59_37 84.0 73.0 10.0 1.0 0.0 4.3 7.5 50.6 37.6 7904.0 3360.0 10720.0 4704.0 4112.0)
 (4.25 3.55 2.99 11_59_27 84.0 82.0 1.0 1.0 0.0 2.7 16.1 46.7 34.5 7864.0 3232.0 10640.0 4376.0 4152.0)
 (3.64 3.37 2.91 11_59_12 83.0 79.0 3.0 1.0 0.0 2.3 20.3 58.2 19.1 7776.0 2928.0 10536.0 4024.0 4240.0)
 (2.23 2.43 2.44 11_46_04 90.0 86.0 1.0 1.0 2.0 4.7 10.2 44.9 40.2 8336.0 3200.0 11144.0 4264.0 3688.0)
 (2.79 2.58 2.5 11_49_28 91.0 86.0 2.0 1.0 2.0 3.5 11.3 52.3 32.8 8472.0 3080.0 11352.0 4168.0 3560.0)
 (2.77 2.63 2.5 11_50_50 90.0 86.0 1.0 1.0 2.0 8.3 3.5 44.9 43.3 8440.0 3392.0 11288.0 4480.0 3592.0)

(4.83 3.83 3.14 12_00_23 83.0 81.0 1.0 1.0 0.0 5.9 9.4 54.7 29.9 8256.0 3616.0 10976.0 4864.0 3776.0)
 (4.05 3.36 2.85 11_57_50 82.0 80.0 1.0 1.0 0.0 1.9 7.4 37.0 53.7 8136.0 3240.0 10856.0 4312.0 3896.0)
 (3.81 3.38 2.89 11_58_31 82.0 79.0 2.0 1.0 0.0 17.3 2.4 52.9 27.5 7928.0 3280.0 10728.0 4440.0 4104.0)
 (3.74 3.26 2.8 11_57_34 85.0 82.0 2.0 1.0 0.0 9.3 10.5 59.3 20.9 8184.0 3152.0 10968.0 4272.0 3856.0)
 (2.39 2.37 2.42 11_47_51 87.0 83.0 1.0 1.0 2.0 4.3 6.6 34.0 55.1 8176.0 2848.0 11056.0 3936.0 3864.0)
 (3.13 2.92 2.61 11_53_44 89.0 84.0 3.0 2.0 0.0 39.4 15.4 45.3 0.0 8448.0 3680.0 11320.0 4840.0 3600.0)
 (3.95 3.15 2.71 11_54_40 89.0 85.0 3.0 1.0 0.0 12.5 8.2 50.2 29.2 8440.0 4136.0 11232.0 5472.0 3608.0)
 (4.8 3.75 3.1 12_00_03 83.0 78.0 4.0 1.0 0.0 4.7 2.7 53.5 39.1 8360.0 3688.0 11096.0 4920.0 3688.0)
 (2.07 2.35 2.41 11_46_50 88.0 82.0 3.0 1.0 2.0 5.5 11.7 36.7 46.1 8264.0 3064.0 11056.0 4080.0 3784.0)
 (3.42 3.18 2.76 11_57_19 83.0 80.0 2.0 1.0 0.0 8.7 8.7 41.9 40.7 8224.0 3360.0 10968.0 4432.0 3824.0)
 (3.7 3.23 2.77 11_56_48 85.0 83.0 1.0 1.0 0.0 11.6 5.4 50.8 32.2 8104.0 3384.0 10840.0 4488.0 3944.0)
 (3.68 3.21 2.76 11_56_43 82.0 80.0 1.0 1.0 0.0 6.3 10.2 46.7 36.9 8096.0 3312.0 10824.0 4384.0 3952.0)
 (4.18 3.56 3.02 11_59_47 86.0 82.0 3.0 1.0 0.0 11.7 8.9 63.4 16.0 8048.0 3832.0 10904.0 5416.0 4000.0)
 (3.79 3.4 2.92 11_59_06 83.0 81.0 1.0 1.0 0.0 1.2 14.1 30.2 54.5 7800.0 2952.0 10552.0 4040.0 4248.0)
 (2.86 2.58 2.5 11_45_33 88.0 81.0 4.0 1.0 2.0 6.6 13.9 54.8 24.7 8568.0 4064.0 11344.0 5312.0 3488.0)
 (3.49 3.55 3.04 12_00_49 85.0 81.0 3.0 1.0 0.0 5.9 9.0 63.3 21.9 8240.0 3272.0 10976.0 4488.0 3816.0)
 (3.83 3.36 2.87 11_58_15 84.0 82.0 1.0 1.0 0.0 3.1 13.7 52.2 31.0 8040.0 3232.0 10824.0 4376.0 4016.0)
 (3.11 2.61 2.52 11_48_57 90.0 85.0 2.0 1.0 2.0 5.9 16.9 52.9 24.3 8456.0 3760.0 11352.0 4992.0 3608.0)
 (4.37 3.34 2.8 11_56_28 82.0 80.0 1.0 1.0 0.0 6.3 9.0 54.7 30.1 8160.0 3888.0 10904.0 5056.0 3904.0)
 (2.78 2.55 2.48 11_44_27 90.0 86.0 1.0 1.0 2.0 7.7 33.8 41.1 17.4 8912.0 3480.0 11472.0 4200.0 3160.0)
 (3.96 3.66 3.08 12_00_39 84.0 81.0 2.0 1.0 0.0 3.6 9.9 27.3 59.3 8296.0 3496.0 11008.0 4696.0 3776.0)
 (3.75 3.16 2.72 11_56_13 86.0 83.0 2.0 1.0 0.0 10.5 6.6 57.4 25.6 8104.0 3896.0 10840.0 5072.0 3968.0)
 (2.46 2.48 2.45 11_44_57 89.0 84.0 2.0 1.0 2.0 4.3 24.1 32.4 39.1 8088.0 3056.0 10904.0 4120.0 3984.0)
 (2.79 2.63 2.5 11_50_55 90.0 86.0 1.0 1.0 2.0 6.7 9.4 48.2 35.7 8504.0 3504.0 11328.0 4712.0 3576.0)
 (3.55 3.2 2.77 11_57_14 84.0 80.0 3.0 1.0 0.0 3.4 9.5 52.9 34.2 8256.0 3600.0 11000.0 4720.0 3824.0)
 (2.59 2.51 2.46 11_45_03 87.0 83.0 1.0 1.0 2.0 7.9 17.4 53.0 21.7 8240.0 3336.0 11112.0 4464.0 3840.0)
 (4.0 3.38 2.87 11_58_10 84.0 79.0 4.0 1.0 0.0 10.2 8.2 52.2 29.4 7904.0 3320.0 10704.0 4496.0 4176.0)
 (3.76 3.36 2.87 11_58_20 84.0 82.0 1.0 1.0 0.0 10.2 9.4 42.7 37.6 8096.0 3456.0 10888.0 4600.0 3992.0)
 (3.38 3.3 2.86 11_58_46 81.0 79.0 1.0 1.0 0.0 3.9 13.7 57.4 25.0 7920.0 3008.0 10696.0 4120.0 4168.0)
 (3.82 3.39 2.89 11_58_36 82.0 79.0 2.0 1.0 0.0 12.0 6.6 43.4 38.0 8080.0 3496.0 10888.0 4672.0 4016.0)
 (2.96 2.65 2.52 11_50_14 91.0 85.0 3.0 1.0 2.0 12.6 6.1 66.0 15.3 8672.0 3704.0 11472.0 4888.0 3432.0)
 (3.41 3.3 2.87 11_58_56 83.0 72.0 10.0 1.0 0.0 2.7 2.7 71.2 23.5 7920.0 3320.0 10696.0 4456.0 4184.0)
 (4.78 3.73 3.09 11_59_58 85.0 74.0 9.0 1.0 1.0 5.0 10.4 62.9 21.6 8360.0 3704.0 11096.0 4944.0 3752.0)
 (2.67 2.53 2.48 11_45_28 89.0 85.0 1.0 1.0 2.0 7.9 5.5 46.1 40.6 8568.0 4064.0 11344.0 5312.0 3560.0)
 (4.24 3.52 2.97 11_59_17 82.0 73.0 8.0 1.0 0.0 4.6 12.4 67.6 15.4 8000.0 3416.0 10784.0 4568.0 4176.0)
 (3.82 3.16 2.72 11_56_07 84.0 82.0 1.0 1.0 0.0 7.8 7.4 55.6 29.2 8200.0 3864.0 10976.0 5080.0 4016.0)

==-- Hypotheses ==--

CLASSA

Cover:

202

Relation:

FREE + VIRT_MEM = 11938.6930

APPENDIX B3 - Density of Liquids Experiment #1

This experiment resulted in the discovery of two equations: one for gasoline, and the other for milk and sea water. The result was somewhat unexpected since the density of a liquid can be stated as $\text{density} = \text{mass} / \text{volume}$. The result demonstrates the ability of ABACUS.2 to use symbolic data in the classification process, and the discovery of equations by class. Also note that the ABACUS.2 was able to distinguish between relevant and non-relevant symbolic data.

```
(
(declarations
  (rho L ((grams 1)(cm -3)))
  (mass L ((grams 1)))
  (volume L ((cm 3)))
  (substance N nil)
  (sample N nil)
)

(parameters
  (*print-flags* (p e))
)

(variables
  (rho mass volume substance sample)
)

(events
  (0.68 51.0 75 GASOLINE SAMPLE1)
  (0.68 54.4 80 GASOLINE SAMPLE1)
  (0.68 61.2 90 GASOLINE SAMPLE1)
  (0.68 68.0 100 GASOLINE SAMPLE1)
  (0.66 49.5 75 GASOLINE SAMPLE2)
  (0.65999997 52.8 80 GASOLINE SAMPLE2)
  (0.66 59.4 90 GASOLINE SAMPLE2)
  (0.66 66.0 100 GASOLINE SAMPLE2)
  (0.6666667 50.0 75 GASOLINE SAMPLE3)
  (0.66999996 53.6 80 GASOLINE SAMPLE3)
  (0.67 60.3 90 GASOLINE SAMPLE3)
  (0.67 67.0 100 GASOLINE SAMPLE3)
  (1.025 76.875 75 SEA_WATER SAMPLE1)
  (1.025 82.0 80 SEA_WATER SAMPLE1)
  (1.025 92.25 90 SEA_WATER SAMPLE1)
  (1.025 102.5 100 SEA_WATER SAMPLE1)
  (1.0320001 77.4 75 MILK SAMPLE1)
  (1.032 82.56 80 MILK SAMPLE1)
  (1.032 92.88 90 MILK SAMPLE1)
  (1.032 103.2 100 MILK SAMPLE1)
```

(1.028 77.1 75 MILK SAMPLE2)
 (1.028 82.24 80 MILK SAMPLE2)
 (1.028 92.52 90 MILK SAMPLE2)
 (1.028 102.8 100 MILK SAMPLE2)
)
)

=== Parameters ===

MAXSTAR = 10
 COVERMODE = IC
 UNCERTAINTY = 0.02
 CONSTANCY = 0.95
 INVERSE-SW = 30
 DIRECT-SW = 70
 UNVIGNORE = NIL
 DEBUG = NIL
 PRINT-FLAGS = (P E)
 FILTER-LEVEL = 2
 SEARCH-LIMIT = 3
 MAX-NODES = 1000
 CLUSTER-ON = T
 CLUSTER-ORDER = (NC GC RC)
 BEAM-VALUE = 0.5
 MINIMUM-LINK-VALUE = 50.0

starLEF = ((MAX-COVERED 0.5) (MIN-WEIGHT 0.0) (MIN-SELECTORS 0.0))

=== Class Events ===

CLASSB
 (RHO MASS VOLUME SUBSTANCE SAMPLE)
 (1.0320001 77.4 75.0 MILK SAMPLE1)
 (1.032 82.56 80.0 MILK SAMPLE1)
 (1.032 92.88 90.0 MILK SAMPLE1)
 (1.032 103.2 100.0 MILK SAMPLE1)
 (1.028 77.1 75.0 MILK SAMPLE2)
 (1.028 82.24 80.0 MILK SAMPLE2)
 (1.028 102.8 100.0 MILK SAMPLE2)
 (1.028 92.52 90.0 MILK SAMPLE2)
 (1.025 76.875 75.0 SEA_WATER SAMPLE1)
 (1.025 82.0 80.0 SEA_WATER SAMPLE1)
 (1.025 92.25 90.0 SEA_WATER SAMPLE1)
 (1.025 102.5 100.0 SEA_WATER SAMPLE1)

CLASSA
 (RHO MASS VOLUME SUBSTANCE SAMPLE)
 (0.68 51.0 75.0 GASOLINE SAMPLE1)
 (0.68 54.4 80.0 GASOLINE SAMPLE1)
 (0.68 61.2 90.0 GASOLINE SAMPLE1)
 (0.68 68.0 100.0 GASOLINE SAMPLE1)
 (0.67 67.0 100.0 GASOLINE SAMPLE3)
 (0.66999996 53.6 80.0 GASOLINE SAMPLE3)
 (0.67 60.3 90.0 GASOLINE SAMPLE3)

(0.6666667 50.0 75.0 GASOLINE SAMPLE3)
(0.65999997 52.8 80.0 GASOLINE SAMPLE2)
(0.66 49.5 75.0 GASOLINE SAMPLE2)
(0.66 59.4 90.0 GASOLINE SAMPLE2)
(0.66 66.0 100.0 GASOLINE SAMPLE2)

= - - Hypotheses - - =

CLASSA

Cover:

12 [SUBSTANCE = GASOLINE]

Relation:

VOLUME = 1.4934 * MASS

CLASSB

Cover:

12 [SUBSTANCE != GASOLINE]

Relation:

VOLUME = 0.9725 * MASS

APPENDIX B4 - Density of Liquids Experiment #2

In this second experiment with the density data, the level of acceptable uncertainty was decreased from 0.02 to 0.005. This resulted in the discovery of an equation for density that includes all three variables. Notice in the first experiment (AppendixB3) that only mass and volume appeared in the equation. It appears that the higher lower the uncertainty the higher the degree of accuracy that is achieved.

--- Parameters ---

MAXSTAR = 10
COVERMODE = IC
UNCERTAINTY = 0.005
CONSTANCY = 0.95
INVERSE-SW = 30
DIRECT-SW = 70
UNVIGNORE = NIL
DEBUG = NIL
PRINT-FLAGS = (P E)
FILTER-LEVEL = 2
SEARCH-LIMIT = 3
MAX-NODES = 1000
CLUSTER-ON = #<Symbol 46001D>
CLUSTER-ORDER = (NC GC RC)
BEAM-VALUE = 0.5
MINIMUM-LINK-VALUE = 50.0

starLEF = ((MAX-COVERED 0.5) (MIN-WEIGHT 0.0) (MIN-SELECTORS 0.0))

--- Class Events ---

CLASSA
(RHO MASS VOLUME SUBSTANCE SAMPLE)
(0.65999997 52.8 80.0 GASOLINE SAMPLE2)
(1.0320001 77.4 75.0 MILK SAMPLE1)
(1.032 103.2 100.0 MILK SAMPLE1)
(1.032 92.88 90.0 MILK SAMPLE1)
(1.032 82.56 80.0 MILK SAMPLE1)
(1.028 102.8 100.0 MILK SAMPLE2)
(1.028 82.24 80.0 MILK SAMPLE2)
(1.028 77.1 75.0 MILK SAMPLE2)
(1.028 92.52 90.0 MILK SAMPLE2)
(1.025 102.5 100.0 SEA WATER SAMPLE1)
(1.025 92.25 90.0 SEA WATER SAMPLE1)
(1.025 82.0 80.0 SEA WATER SAMPLE1)
(1.025 76.875 75.0 SEA WATER SAMPLE1)
(0.68 68.0 100.0 GASOLINE SAMPLE1)
(0.68 61.2 90.0 GASOLINE SAMPLE1)

(0.68 54.4 80.0 GASOLINE SAMPLE1)
(0.68 51.0 75.0 GASOLINE SAMPLE1)
(0.67 67.0 100.0 GASOLINE SAMPLE3)
(0.66999996 53.6 80.0 GASOLINE SAMPLE3)
(0.6666667 50.0 75.0 GASOLINE SAMPLE3)
(0.66 66.0 100.0 GASOLINE SAMPLE2)
(0.66 59.4 90.0 GASOLINE SAMPLE2)
(0.66 49.5 75.0 GASOLINE SAMPLE2)
(0.67 60.3 90.0 GASOLINE SAMPLE3)

= - - - Hypotheses - - - =

CLASSA

Cover: Relation:

$\text{RHO} * \text{VOLUME} = \text{MASS}$

APPENDIX B5 - Illumination of Surfaces Experiment #1

This experiment used contrived data that simulated observations of the illumination of a surface. The illumination of a surface E is the luminous flux incident per unit area of the surface. If a point source of luminous intensity I is distant r from a surface of area A and if the luminous flux makes an angle i with the normal to the surface, then the illuminance of the surface is

$$E = I / r^2 \cos i$$

In this example, ABACUS.2 identified seven classes of data. The equations describing the data for each class displayed varying levels of precision. A miscellaneous class was also formed. No relation was found for this class. Note that none of the equations incorporated the cosine relationship.

```
(
(declarations
  (E L ((lumens 1)(meters -2)))
  (I L ((candles 1)))
  (r L ((meters 2)))
  (angle L ((radians 1)))
)

(parameters
  (*debug* true)
  (*print-flags* (p e))
)

(variables
  (E I r angle)
)

(events
  (32.393932 72 1.2 0.866)
  (32.41678 72 1.2 0.8654)
  (39.361927 73 1.1 0.86)
  (28.151255 74 1.3 0.8725)
  (32.02282 71 1.2 0.8639)
  (13.046136 68 1.8 0.9)
  (52.847862 100.1 1.2 0.7071)
  (8.215724 100.2 3.4 0.3244)
  (10.605741 101 3.0 0.333)
  (119.83027 250 1.2 0.809)
```

```

(227.74919 600 1.2 0.9925)
(146.73123 604 1.5 0.9925)
(227.3696 599 1.2 0.9925)
(23.708344 588 4.9 0.2533)
(23.781572 602 5.0 0.1576)
(37.135017 20 0.6 0.8387)
(30.733662 20 0.3 0.8 0.19)
(33.96329 22 0.79 0.2711)
(24.124243 21.26 0.88 0.4976)
(104.77243 60 0.6 0.891)
(2.1318607 65 5.5 0.1255)
(2.3262053 63.4 5.2 0.1256)
(128.90459 78 0.6 0.9336)
(0.7425692 77 9.0 0.6743)
(0.71998465 76 9.3 0.6105)
(0.55620784 79 10.7 0.6333)
(225.00339 110 0.6 0.7431)
(266.06866 110 0.55 0.75)
(1265.4518 111 0.25 0.7777)
(77.331215 109 0.99 0.8019)
(1252.2725 114 0.3 0.1509)
))

```

== - Parameters - - - =

```

*MAXSTAR* = 10
*COVERMODE* = IC
*UNCERTAINTY* = 0.02
*CONSTANCY* = 0.95
*INVERSE-SW* = 30
*DIRECT-SW* = 70
*UNIVIGNORE* = NIL
*DEBUG* = TRUE
*PRINT-FLAGS* = (P E)
*FILTER-LEVEL* = 2
*SEARCH-LIMIT* = 3
*MAX-NODES* = 1000
*CLUSTER-ON* = T
*CLUSTER-ORDER* = (NC GC RC)
*BEAM-VALUE* = 0.5
*MINIMUM-LINK-VALUE* = 50.0

```

```

*starLEF* = ((MAX-COVERED 0.5) (MIN-WEIGHT 0.0) (MIN-SELECTORS 0.0))

```

== - Class Events - - - =

MISCELLANEOUS

```

(E I R ANGLE)
(128.90459 78.0 0.6 0.9336)
(0.7425692 77.0 9.0 0.6743)
(0.55620784 79.0 10.7 0.6333)
(0.71998465 76.0 9.3 0.6105)
(24.124243 21.26 0.88 0.4976)

```

CLASSG

```

(E I R ANGLE)
(227.3696 599.0 1.2 0.9925)

```

(227.74919 600.0 1.2 0.9925)
(146.73123 604.0 1.5 0.9925)

CLASSF
(E I R ANGLE)
(13.046136 68.0 1.8 0.9)
(104.77243 60.0 0.6 0.891)
(28.151255 74.0 1.3 0.8725)

CLASSE
(E I R ANGLE)
(32.393932 72.0 1.2 0.866)
(32.41678 72.0 1.2 0.8654)
(32.02282 71.0 1.2 0.8639)
(39.361927 73.0 1.1 0.86)
(37.135017 20.0 0.6 0.8387)

CLASSD
(E I R ANGLE)
(119.83027 250.0 1.2 0.809)
(77.331215 109.0 0.99 0.8019)
(1265.4518 111.0 0.25 0.7777)

CLASSC
(E I R ANGLE)
(266.06866 110.0 0.55 0.75)
(225.00339 110.0 0.6 0.7431)
(52.847862 100.1 1.2 0.7071)

CLASSB
(E I R ANGLE)
(10.605741 101.0 3.0 0.333)
(8.215724 100.2 3.4 0.3244)
(33.96329 22.0 0.79 0.2711)
(23.708344 588.0 4.9 0.2533)
(30.733662 20.03 0.8 0.19)

CLASSA
(E I R ANGLE)
(23.781572 602.0 5.0 0.1576)
(1252.2725 114.0 0.3 0.1509)
(2.3262053 63.4 5.2 0.1256)
(2.1318607 65.0 5.5 0.1255)

--- Hypotheses ---

CLASSA
Cover:
4 [ANGLE = 0.1255 .. 0.1576]
Relation:
 $E * R^2 = 0.9901 * I$

CLASSB
Cover:

5 [ANGLE = 0.19 .-. 0.333]
Relation:
 $E * R^2 = 0.9613 * I$

CLASSC
Cover:
3 [ANGLE = 0.7071 .-. 0.75]
Relation:
 $E * R^2 = 0.7428 * I$

CLASSD
Cover:
3 [ANGLE = 0.7777 .-. 0.809]
Relation:
 $E * R^2 = 0.6994 * I$

CLASSE
Cover:
5 [ANGLE = 0.8387 .-. 0.866]
Relation:
 $E * R^2 = 0.6533 * I$

CLASSF
Cover:
3 [ANGLE = 0.8725 .-. 0.9]
Relation:
 $E * R^2 = 0.6311 * I$

CLASSG
Cover:
3 [ANGLE = 0.9925]
Relation:
 $E * R^2 = 0.5466 * I$

MISCELLANEOUS
Cover:
4 [ANGLE = 0.4976 .-. 0.6743]

1 [ANGLE = 0.9336]
Relation:
No formula can be found

APPENDIX B6 - Illumination of Surfaces Experiment #2

For this experiment, the *inverse-sw* and *direct-sw* parameters were modified to see what effect, if any, these two parameters had on speed of discovery. The hypothesis was that the lower the ratio of direct-sw to inverse-sw, the faster the system would be able to find dependencies between variables. Upon running this experiment, it was observed that there was a speed up in the equation discovery process without a loss in precision in the resulting equations. Additional experiments were run, and the same observations were made with respect to speedup and precision. However, it is possible that using a low ratio could affect the precision of the data when other parameters such as the beam value are reduced.

```
(
(declarations
  (E L ((lumens 1)(meters -2)))
  (I L ((candles 1)))
  (r L ((meters 2)))
  (angle L ((radians 1)))
)
```

```
(parameters
  (*debug* true)
  (*print-flags* (p e))
  (*direct-sw* 20)
  (*inverse-sw* 80)
)
```

```
(variables
  (E I r angle)
)
```

```
(events
(32.393932 72 1.2 0.866)
(32.41678 72 1.2 0.8654)
(39.361927 73 1.1 0.86)
(28.151255 74 1.3 0.8725)
(32.02282 71 1.2 0.8639)
(13.046136 68 1.8 0.9)
(52.847862 100.1 1.2 0.7071)
(8.215724 100.2 3.4 0.3244)
(10.605741 101 3.0 0.333)
(119.83027 250 1.2 0.809)
(227.74919 600 1.2 0.9925)
```

(146.73123 604 1.5 0.9925)
 (227.3696 599 1.2 0.9925)
 (23.708344 588 4.9 0.2533)
 (23.781572 602 5.0 0.1576)
 (37.135017 20 0.6 0.8387)
 (30.733662 20.03 0.8 0.19)
 (33.96329 22 0.79 0.2711)
 (24.124243 21.26 0.88 0.4976)
 (104.77243 60 0.6 0.891)
 (2.1318607 65 5.5 0.1255)
 (2.3262053 63.4 5.2 0.1256)
 (128.90459 78 0.6 0.9336)
 (0.7425692 77 9.0 0.6743)
 (0.71998465 76 9.3 0.6105)
 (0.55620784 79 10.7 0.6333)
 (225.00339 110 0.6 0.7431)
 (266.06866 110 0.55 0.75)
 (1265.4518 111 0.25 0.7777)
 (77.331215 109 0.99 0.8019)
 (1252.2725 114 0.3 0.1509)
))

MAXSTAR = 10
 COVERMODE = IC
 UNCERTAINTY = 0.02
 CONSTANCY = 0.95
 INVERSE-SW = 80
 DIRECT-SW = 20
 UNIVIGNORE = NIL
 DEBUG = TRUE
 PRINT-FLAGS = (P E)
 FILTER-LEVEL = 2
 SEARCH-LIMIT = 3
 MAX-NODES = 1000
 CLUSTER-ON = T
 CLUSTER-ORDER = (NC GC RC)
 BEAM-VALUE = 0.5
 MINIMUM-LINK-VALUE = 50.0

starLEF = ((MAX-COVERED 0.5) (MIN-WEIGHT 0.0) (MIN-SELECTORS 0.0))

== - - - Class Events - - - ==

MISCELLANEOUS

(E I R ANGLE)
 (128.90459 78.0 0.6 0.9336)
 (0.7425692 77.0 9.0 0.6743)
 (0.55620784 79.0 10.7 0.6333)
 (0.71998465 76.0 9.3 0.6105)
 (24.124243 21.26 0.88 0.4976)

CLASSG

(E I R ANGLE)
 (227.3696 599.0 1.2 0.9925)
 (227.74919 600.0 1.2 0.9925)

(146.73123 604.0 1.5 0.9925)

CLASSF

(E I R ANGLE)

(13.046136 68.0 1.8 0.9)

(104.77243 60.0 0.6 0.891)

(28.151255 74.0 1.3 0.8725)

CLASSE

(E I R ANGLE)

(32.393932 72.0 1.2 0.866)

(32.41678 72.0 1.2 0.8654)

(32.02282 71.0 1.2 0.8639)

(39.361927 73.0 1.1 0.86)

(37.135017 20.0 0.6 0.8387)

CLASSD

(E I R ANGLE)

(119.83027 250.0 1.2 0.809)

(77.331215 109.0 0.99 0.8019)

(1265.4518 111.0 0.25 0.7777)

CLASSC

(E I R ANGLE)

(266.06866 110.0 0.55 0.75)

(225.00339 110.0 0.6 0.7431)

(52.847862 100.1 1.2 0.7071)

CLASSB

(E I R ANGLE)

(10.605741 101.0 3.0 0.333)

(8.215724 100.2 3.4 0.3244)

(33.96329 22.0 0.79 0.2711)

(23.708344 588.0 4.9 0.2533)

(30.733662 20.03 0.8 0.19)

CLASSA

(E I R ANGLE)

(23.781572 602.0 5.0 0.1576)

(1252.2725 114.0 0.3 0.1509)

(2.3262053 63.4 5.2 0.1256)

(2.1318607 65.0 5.5 0.1255)

--- Hypotheses ---

CLASSA

Cover:

4 [ANGLE = 0.1255 .. 0.1576]

Relation:

$E * R^2 = 0.9901 * I$

CLASSB

Cover:

5 [ANGLE = 0.19 .. 0.333]

Relation:

$$E * R^2 = 0.9613 * I$$

CLASSC

Cover:

$$3 \text{ [ANGLE = 0.7071 } \dots \text{ 0.75]}$$

Relation:

$$E * R^2 = 0.7428 * I$$

CLASSD

Cover:

$$3 \text{ [ANGLE = 0.7777 } \dots \text{ 0.809]}$$

Relation:

$$E * R^2 = 0.6994 * I$$

CLASSE

Cover:

$$5 \text{ [ANGLE = 0.8387 } \dots \text{ 0.866]}$$

Relation:

$$E * R^2 = 0.6533 * I$$

CLASSF

Cover:

$$3 \text{ [ANGLE = 0.8725 } \dots \text{ 0.9]}$$

Relation:

$$E * R^2 = 0.6311 * I$$

CLASSG

Cover:

$$3 \text{ [ANGLE = 0.9925]}$$

Relation:

$$E * R^2 = 0.5466 * I$$

MISCELLANEOUS

Cover:

$$4 \text{ [ANGLE = 0.4976 } \dots \text{ 0.6743]}$$

$$1 \text{ [ANGLE = 0.9336]}$$

Relation:

No formula can be found

APPENDIX B7 - Thermal Conductivity Experiment #1

The thermal conductivity of copper compounds is described by the equation

$$\lambda = (M * L) / (T * \Theta^3)$$

where:

λ is the thermal conductivity of the substance

M is the mass

L is the length of the rod of the substance sampled

T is the temperature of the substance

Θ is the time it takes an electron to travel the distance of the rod

This experiment demonstrates the ability of ABACUS.2, via the Star algorithm, to create multi-attribute covers. In CLASSD, the cover consists of three attributes:

[COMPOUND = CUTE02] AND [LAMBDA = 4.8148E-4..0.00385185]
OR [LAMBDA = 3.081E-5]

[LAMBDA = 4.8148E-4..0.00385185] is read "all events whose value of λ is in the range [4.8148E-4..0.00385185]."

In CLASSF, the cover consists of two attributes:

[LAMBDA = 2.632E-5..0.00168474] AND [MASS = 71.075]

This capability is important because ABACUS.2 can find regularities for subsets of the data when regularities may not be able to be identified for the entire set of events and/or variables. Moreover, each cover differed in the types of variables that are included (e.g., CLASSD is the only cover that includes the symbolic variable COMPOUND).

In order for all of the variables and events to be included in a cover, additional events need to be supplied to ABACUS.2. This is one of the weaknesses of this

discovery system. ABACUS.2 requires a lot of examples to be provided.

```
(
(declarations
(MASS L ((kilograms 1)))
(LENGTH L ((meters 1)))
(TEMP L ((kelvin 1)))
(TIME L ((seconds 1)))
(LAMBDA L ((kilograms 1)(meters 1)(kelvin -1)(seconds -3)))
(COMPOUND N nil)
)

(parameters
(*uncertainty* 0.05)
(*constancy* 0.95)
(*print-flags* (p e))
(*debug* t)
)

(variables
(MASS LENGTH TEMP TIME LAMBDA COMPOUND)
)

(events
(181.75 24      323.0  15      0.00400138      Cu2Te)
(181.75 24      323.0  15      0.00400138      Cu2Te)
(181.75 24      323.0  15      0.00400138      Cu2Te)
(181.75 24      323.0  15      0.00400138      Cu2Te)
(185.02 24      323.0  15      0.00400138      Cu2Te)
(181.75 24      323.0  15      0.00400138      Cu2Te)
(181.75 24      323.0  15      0.00400138      Cu2Te)
(181.75 24      323.0  15      0.00400138      Cu2Te)
(185.02 24      323.0  15      0.00400138      Cu2Te)
(181.75 24      323.0  15      0.00400138      Cu2Te)
(181.75 24      323.0  15      0.00400138      Cu2Te)
(181.75 24      323.0  15      0.00400138      Cu2Te)
(181.75 24      323.0  15      0.00400138      Cu2Te)
(181.75 24      323.0  30      0.00050017      Cu2Te)
(181.75 24      323.0  30      0.00050017      Cu2Te)
(181.75 24      323.0  30      0.00050017      Cu2Te)
(181.75 24      323.0  30      0.00050017      Cu2Te)
(181.75 24      323.0  30      0.00050017      Cu2Te)
(181.75 24      323.0  30      0.00050017      Cu2Te)
(181.75 24      323.0  60      0.00006252      Cu2Te)
(181.75 24      323.0  60      0.00006252      Cu2Te)
(181.75 24      323.0  60      0.00006252      Cu2Te)
(181.75 24      323.0  60      0.00006252      Cu2Te)
(181.75 24      323.0  60      0.00006252      Cu2Te)
(181.75 24      323.0  60      0.00006252      Cu2Te)
(181.75 24      323.0  60      0.00006252      Cu2Te)
(181.75 24      323.0  60      0.00006252      Cu2Te)
(181.75 24      323.0  60      0.00006252      Cu2Te)
(181.75 24      323.0  60      0.00006252      Cu2Te)
(181.75 24      323.0  75      0.00003201      Cu2Te)
(181.75 24      323.0  75      0.00003201      Cu2Te)
(188.50 24      348.0  15      0.00385185      CuTeO2)
(188.50 24      348.0  15      0.00385185      CuTeO2)
(188.50 24      348.0  15      0.00385185      CuTeO2)
```

(188.50 24	348.0	15	0.00385185	CuTeO2)
(188.50 24	348.0	30	0.00048148	CuTeO2)
(188.50 24	348.0	60	0.00006019	CuTeO2)
(188.50 24	348.0	75	0.00003081	CuTeO2)
(188.50 24	348.0	75	0.00003081	CuTeO2)
(188.50 24	348.0	75	0.00003081	CuTeO2)
(188.50 24	348.0	75	0.00003081	CuTeO2)
(188.50 24	348.0	75	0.00003081	CuTeO2)
(188.50 24	348.0	75	0.00003081	CuTeO2)
(188.50 24	348.0	75	0.00003081	CuTeO2)
(71.075 24	300.0	15	0.00168474	CuSCN)
(71.075 24	300.0	15	0.00168474	CuSCN)
(71.075 24	300.0	15	0.00168474	CuSCN)
(71.075 24	300.0	15	0.00168474	CuSCN)
(71.075 24	300.0	15	0.00168474	CuSCN)
(71.075 24	300.0	15	0.00168474	CuSCN)
(71.075 24	300.0	15	0.00168474	CuSCN)
(71.075 24	300.0	30	0.00021059	CuSCN)
(71.075 24	300.0	60	0.00002632	CuSCN)
(71.075 24	300.0	75	0.00001348	CuSCN)
(71.075 24	300.0	75	0.00001348	CuSCN)
(71.075 24	300.0	75	0.00001348	CuSCN)
(71.075 24	300.0	75	0.00001348	CuSCN)
(71.075 24	300.0	75	0.00001348	CuSCN)
(981.45 30	323.0	15	0.02700929	Cu2Te)
(981.45 30	323.0	15	0.02700929	Cu2Te)
(981.45 30	323.0	30	0.00337616	Cu2Te)
(981.45 30	323.0	60	0.00042202	Cu2Te)
(383.81 30	300.0	15	0.01137215	CuSCN)
(383.81 30	300.0	15	0.01137215	CuSCN)
(383.81 30	300.0	15	0.01137215	CuSCN)
(383.81 30	300.0	15	0.01137215	CuSCN)
(383.81 30	300.0	15	0.01137215	CuSCN)
(383.81 30	300.0	15	0.01137215	CuSCN)
(383.81 30	300.0	75	0.00009098	CuSCN)
(383.81 30	300.0	75	0.00009098	CuSCN)

)
)

-- Parameters --

MAXSTAR = 10
 COVERMODE = IC
 UNCERTAINTY = 1.0E-4
 CONSTANCY = 0.99
 INVERSE-SW = 30
 DIRECT-SW = 70
 UNVIGNORE = NIL
 DEBUG = T
 PRINT-FLAGS = (P E)
 FILTER-LEVEL = 2
 SEARCH-LIMIT = 3
 MAX-NODES = 1000
 CLUSTER-ON = T
 CLUSTER-ORDER = (NC GC RC)
 BEAM-VALUE = 0.5
 MINIMUM-LINK-VALUE = 50.0

starLEF = ((MAX-COVERED 0.5) (MIN-WEIGHT 0.0) (MIN-SELECTORS 0.0))

= - - - Class Events - - -

MISCELLANEOUS
(MASS LENGTH TEMP TIME LAMBDA COMPOUND)
(188.5 24.0 348.0 60.0 6.019E-5 CUTE02)

CLASSF
(MASS LENGTH TEMP TIME LAMBDA COMPOUND)
(71.075 24.0 300.0 60.0 2.632E-5 CUSCN)
(71.075 24.0 300.0 30.0 2.1059E-4 CUSCN)
(71.075 24.0 300.0 15.0 0.00168474 CUSCN)
(71.075 24.0 300.0 15.0 0.00168474 CUSCN)
(71.075 24.0 300.0 15.0 0.00168474 CUSCN)
(71.075 24.0 300.0 15.0 0.00168474 CUSCN)
(71.075 24.0 300.0 15.0 0.00168474 CUSCN)
(71.075 24.0 300.0 15.0 0.00168474 CUSCN)
(71.075 24.0 300.0 15.0 0.00168474 CUSCN)

CLASSE
(MASS LENGTH TEMP TIME LAMBDA COMPOUND)
(71.075 24.0 300.0 75.0 1.348E-5 CUSCN)
(71.075 24.0 300.0 75.0 1.348E-5 CUSCN)
(71.075 24.0 300.0 75.0 1.348E-5 CUSCN)
(71.075 24.0 300.0 75.0 1.348E-5 CUSCN)
(71.075 24.0 300.0 75.0 1.348E-5 CUSCN)

CLASSD
(MASS LENGTH TEMP TIME LAMBDA COMPOUND)
(188.5 24.0 348.0 75.0 3.081E-5 CUTE02)
(188.5 24.0 348.0 75.0 3.081E-5 CUTE02)
(188.5 24.0 348.0 75.0 3.081E-5 CUTE02)
(188.5 24.0 348.0 75.0 3.081E-5 CUTE02)
(188.5 24.0 348.0 75.0 3.081E-5 CUTE02)
(188.5 24.0 348.0 75.0 3.081E-5 CUTE02)
(188.5 24.0 348.0 75.0 3.081E-5 CUTE02)
(188.5 24.0 348.0 30.0 4.8148E-4 CUTE02)
(188.5 24.0 348.0 15.0 0.00385185 CUTE02)
(188.5 24.0 348.0 15.0 0.00385185 CUTE02)
(188.5 24.0 348.0 15.0 0.00385185 CUTE02)
(188.5 24.0 348.0 15.0 0.00385185 CUTE02)

CLASSC
(MASS LENGTH TEMP TIME LAMBDA COMPOUND)
(181.75 24.0 323.0 75.0 3.201E-5 CU2TE)
(181.75 24.0 323.0 75.0 3.201E-5 CU2TE)
(181.75 24.0 323.0 60.0 6.252E-5 CU2TE)
(181.75 24.0 323.0 60.0 6.252E-5 CU2TE)
(181.75 24.0 323.0 60.0 6.252E-5 CU2TE)
(181.75 24.0 323.0 60.0 6.252E-5 CU2TE)
(181.75 24.0 323.0 60.0 6.252E-5 CU2TE)
(181.75 24.0 323.0 60.0 6.252E-5 CU2TE)
(181.75 24.0 323.0 60.0 6.252E-5 CU2TE)
(181.75 24.0 323.0 60.0 6.252E-5 CU2TE)

(181.75 24.0 323.0 60.0 6.252E-5 CU2TE)
 (181.75 24.0 323.0 30.0 5.0017E-4 CU2TE)
 (181.75 24.0 323.0 30.0 5.0017E-4 CU2TE)
 (181.75 24.0 323.0 30.0 5.0017E-4 CU2TE)
 (181.75 24.0 323.0 30.0 5.0017E-4 CU2TE)
 (181.75 24.0 323.0 30.0 5.0017E-4 CU2TE)
 (181.75 24.0 323.0 15.0 0.00400138 CU2TE)
 (181.75 24.0 323.0 15.0 0.00400138 CU2TE)
 (181.75 24.0 323.0 15.0 0.00400138 CU2TE)
 (181.75 24.0 323.0 15.0 0.00400138 CU2TE)
 (181.75 24.0 323.0 15.0 0.00400138 CU2TE)
 (181.75 24.0 323.0 15.0 0.00400138 CU2TE)
 (181.75 24.0 323.0 15.0 0.00400138 CU2TE)
 (181.75 24.0 323.0 15.0 0.00400138 CU2TE)
 (181.75 24.0 323.0 15.0 0.00400138 CU2TE)
 (181.75 24.0 323.0 15.0 0.00400138 CU2TE)
 (185.02 24.0 323.0 15.0 0.00400138 CU2TE)
 (185.02 24.0 323.0 15.0 0.00400138 CU2TE)

CLASSB
 (MASS LENGTH TEMP TIME LAMBDA COMPOUND)
 (383.81 30.0 300.0 15.0 0.01137215 CUSCN)
 (383.81 30.0 300.0 15.0 0.01137215 CUSCN)
 (383.81 30.0 300.0 15.0 0.01137215 CUSCN)
 (383.81 30.0 300.0 15.0 0.01137215 CUSCN)
 (383.81 30.0 300.0 15.0 0.01137215 CUSCN)
 (383.81 30.0 300.0 75.0 9.098E-5 CUSCN)
 (383.81 30.0 300.0 75.0 9.098E-5 CUSCN)

CLASSA
 (MASS LENGTH TEMP TIME LAMBDA COMPOUND)
 (981.45 30.0 323.0 30.0 0.00337616 CU2TE)
 (981.45 30.0 323.0 60.0 4.2202E-4 CU2TE)
 (981.45 30.0 323.0 15.0 0.02700929 CU2TE)
 (981.45 30.0 323.0 15.0 0.02700929 CU2TE)

--- Hypotheses ---

CLASSA
 Cover:
 4 [MASS = 981.45]
 Relation:
 $LAMBDA * TIME^3 = 91.1563$

CLASSB
 Cover:
 7 [MASS = 383.81]
 Relation:
 $LAMBDA * TIME^3 = 38.3813$

CLASSC
 Cover:
 29 [MASS = 181.75 .. 185.02]

Relation:

$$\text{LAMBDA} * \text{TIME}^3 = 13.5045$$

CLASSD

Cover:

$$5 \text{ [COMPOUND = CUTE02][LAMBDA = } 4.8148\text{E-4} \text{ } \therefore \text{ } 0.00385185]$$

$$7 \text{ [LAMBDA = } 3.081\text{E-5]}$$

Relation:

$$\text{LAMBDA} * \text{TIME}^3 = 12.9988$$

CLASSE

Cover:

$$5 \text{ [LAMBDA = } 1.348\text{E-5]}$$

Relation:

$$\text{LAMBDA} * \text{TIME}^3 = 5.6869$$

CLASSF

Cover:

$$9 \text{ [LAMBDA = } 2.632\text{E-5} \text{ } \therefore \text{ } 0.00168474][\text{MASS = } 71.075]$$

Relation:

$$\text{LAMBDA} * \text{TIME}^3 = 5.6859$$

MISCELLANEOUS

Cover:

$$1 \text{ [LAMBDA = } 6.019\text{E-5]}$$

Relation:

No formula can be found

APPENDIX B8 - Thermal Conductivity Experiment

This experiment was run with the clustering component switched off. When the *cluster-on* parameter is switched off, the events will not be clustered and ABACUS.2 will attempt to fit one equation to the events. In this case, ABACUS.2 was not successful in finding a single equation that fits the events.

--- Parameters ---

```
*MAXSTAR* = 10
*COVERMODE* = IC
*UNCERTAINTY* = 0.05
*CONSTANCY* = 0.95
*INVERSE-SW* = 30
*DIRECT-SW* = 70
*UNVIGNORE* = NIL
*DEBUG* = T
*PRINT-FLAGS* = (P E)
*FILTER-LEVEL* = 2
*SEARCH-LIMIT* = 3
*MAX-NODES* = 1000
*CLUSTER-ON* = NIL
*CLUSTER-ORDER* = (NC GC RC)
*BEAM-VALUE* = 0.5
*MINIMUM-LINK-VALUE* = 50.0

*starLEF* = ((MAX-COVERED 0.5) (MIN-WEIGHT 0.0) (MIN-SELECTORS 0.0))
```

--- Class Events ---

```
CLASSD
(MASS LENGTH TEMP TIME LAMBDA COMPOUND)
(71.075 24.0 300.0 60.0 2.632E-5 CUSCN)
(71.075 24.0 300.0 30.0 2.1059E-4 CUSCN)
(71.075 24.0 300.0 15.0 0.00168474 CUSCN)
(71.075 24.0 300.0 15.0 0.00168474 CUSCN)
(71.075 24.0 300.0 15.0 0.00168474 CUSCN)
(71.075 24.0 300.0 15.0 0.00168474 CUSCN)
(71.075 24.0 300.0 15.0 0.00168474 CUSCN)
(71.075 24.0 300.0 15.0 0.00168474 CUSCN)
(71.075 24.0 300.0 15.0 0.00168474 CUSCN)
(71.075 24.0 300.0 75.0 1.348E-5 CUSCN)
(71.075 24.0 300.0 75.0 1.348E-5 CUSCN)
(71.075 24.0 300.0 75.0 1.348E-5 CUSCN)
(71.075 24.0 300.0 75.0 1.348E-5 CUSCN)
(71.075 24.0 300.0 75.0 1.348E-5 CUSCN)
```

```
CLASSC
(MASS LENGTH TEMP TIME LAMBDA COMPOUND)
(188.5 24.0 348.0 75.0 3.081E-5 CUTE02)
(188.5 24.0 348.0 75.0 3.081E-5 CUTE02)
(188.5 24.0 348.0 75.0 3.081E-5 CUTE02)
(188.5 24.0 348.0 75.0 3.081E-5 CUTE02)
(188.5 24.0 348.0 75.0 3.081E-5 CUTE02)
(188.5 24.0 348.0 75.0 3.081E-5 CUTE02)
```

(188.5 24.0 348.0 75.0 3.081E-5 CUTE02)
 (188.5 24.0 348.0 30.0 4.8148E-4 CUTE02)
 (188.5 24.0 348.0 15.0 0.00385185 CUTE02)
 (188.5 24.0 348.0 15.0 0.00385185 CUTE02)
 (188.5 24.0 348.0 15.0 0.00385185 CUTE02)
 (188.5 24.0 348.0 15.0 0.00385185 CUTE02)
 (188.5 24.0 348.0 60.0 6.019E-5 CUTE02)
 (181.75 24.0 323.0 75.0 3.201E-5 CUZTE)
 (181.75 24.0 323.0 75.0 3.201E-5 CUZTE)
 (181.75 24.0 323.0 60.0 6.252E-5 CUZTE)
 (181.75 24.0 323.0 60.0 6.252E-5 CUZTE)
 (181.75 24.0 323.0 60.0 6.252E-5 CUZTE)
 (181.75 24.0 323.0 60.0 6.252E-5 CUZTE)
 (181.75 24.0 323.0 60.0 6.252E-5 CUZTE)
 (181.75 24.0 323.0 60.0 6.252E-5 CUZTE)
 (181.75 24.0 323.0 60.0 6.252E-5 CUZTE)
 (181.75 24.0 323.0 60.0 6.252E-5 CUZTE)
 (181.75 24.0 323.0 60.0 6.252E-5 CUZTE)
 (181.75 24.0 323.0 30.0 5.0017E-4 CUZTE)
 (181.75 24.0 323.0 30.0 5.0017E-4 CUZTE)
 (181.75 24.0 323.0 30.0 5.0017E-4 CUZTE)
 (181.75 24.0 323.0 30.0 5.0017E-4 CUZTE)
 (181.75 24.0 323.0 15.0 0.00400138 CUZTE)
 (181.75 24.0 323.0 15.0 0.00400138 CUZTE)
 (181.75 24.0 323.0 15.0 0.00400138 CUZTE)
 (181.75 24.0 323.0 15.0 0.00400138 CUZTE)
 (181.75 24.0 323.0 15.0 0.00400138 CUZTE)
 (181.75 24.0 323.0 15.0 0.00400138 CUZTE)
 (181.75 24.0 323.0 15.0 0.00400138 CUZTE)
 (181.75 24.0 323.0 15.0 0.00400138 CUZTE)
 (181.75 24.0 323.0 15.0 0.00400138 CUZTE)
 (181.75 24.0 323.0 15.0 0.00400138 CUZTE)
 (185.02 24.0 323.0 15.0 0.00400138 CUZTE)
 (185.02 24.0 323.0 15.0 0.00400138 CUZTE)

CLASSB

(MASS LENGTH TEMP TIME LAMBDA COMPOUND)

(383.81 30.0 300.0 15.0 0.01137215 CUSCN)
 (383.81 30.0 300.0 15.0 0.01137215 CUSCN)
 (383.81 30.0 300.0 15.0 0.01137215 CUSCN)
 (383.81 30.0 300.0 15.0 0.01137215 CUSCN)
 (383.81 30.0 300.0 15.0 0.01137215 CUSCN)
 (383.81 30.0 300.0 75.0 9.098E-5 CUSCN)
 (383.81 30.0 300.0 75.0 9.098E-5 CUSCN)

CLASSA

(MASS LENGTH TEMP TIME LAMBDA COMPOUND)

(981.45 30.0 323.0 60.0 4.2202E-4 CUZTE)
 (981.45 30.0 323.0 30.0 0.00337616 CUZTE)
 (981.45 30.0 323.0 15.0 0.02700929 CUZTE)
 (981.45 30.0 323.0 15.0 0.02700929 CUZTE)

== - - - Hypotheses - - - =

CLASSA

Cover:

4 [MASS = 981.45]

Relation:

LAMBDA * TIME^3 = 91.1563

CLASSB

Cover:

7 [MASS = 383.81]
Relation:
LAMBDA * TIME^3 = 38.3813

CLASSC
Cover:
42 [MASS = 181.75 +/- 188.5]
Relation:
LAMBDA * TIME^3 = 13.3480

CLASSD
Cover:
14 [MASS = 71.075]
Relation:
LAMBDA * TIME^3 = 5.6862

APPENDIX B9 - Speed of Sound in Air

This experiment uses data that conforms to the equation for the the speed of sound in air. This experiment was run many times, and the values of the uncertainty and constancy parameters were changed each time. It was determined that the higher the constancy and the lower the uncertainty, the higher the precision that can be achieved.

;The speed of sound in air

```
(
(declarations
(T L ((seconds 1)))
(V L ((meters 1) (second -1)))
(P L ())
(FREQ L ())
)
```

```
(parameters
(*debug* true)
(*print-flags* (p e))
)
```

```
(variables
(t v p freq)
)
```

```
(events
(250 316.75 0.75 1500)
(250 316.75 0.75 1000)
(250 316.75 1 1500)
(250 316.75 0.8 1500)
(273 331 1 1500)
(273 331 1 3500)
(273 331 2 2050)
(273 331 3 3000)
(273 331 0.5 2500)
(273 331 0.75 3500)
(273 331 1.25 2000)
(320 358.3617 0.8 6500)
(330 363.918 0.75 3500)
(330 363.918 2 10000)
(330 363.918 1 9500)
)
)
```

== - Parameters - ==

```
*MAXSTAR* = 10
*COVERMODE* = 1C
*UNCERTAINTY* = 0.01
*CONSTANCY* = 0.98
```

```

*INVERSE-SW* = 30
*DIRECT-SW* = 70
*UNTVIGNORE* = NIL
*DEBUG* = TRUE
*PRINT-FLAGS* = (P E)
*FILTER-LEVEL* = 2
*SEARCH-LIMIT* = 3
*MAX-NODES* = 1000
*CLUSTER-ON* = T
*CLUSTER-ORDER* = (NC GC RC)
*BEAM-VALUE* = 0.5
*MINIMUM-LINK-VALUE* = 50.0

*starLEF* = ((MAX-COVERED 0.5) (MIN-WEIGHT 0.0) (MIN-SELECTORS 0.0))

```

=== Class Events ===

```

CLASSA
(T V P FREQ)
(330.0 363.9179 0.75 3500.0)
(330.0 363.9179 1.0 9500.0)
(330.0 363.9179 2.0 10000.0)
(320.0 358.3616 0.8 6500.0)
(250.0 316.75 0.75 1000.0)
(250.0 316.75 0.75 1500.0)
(250.0 316.75 0.8 1500.0)
(250.0 316.75 1.0 1500.0)
(273.0 331.0 0.5 2500.0)
(273.0 331.0 0.75 3500.0)
(273.0 331.0 1.0 1500.0)
(273.0 331.0 1.0 3500.0)
(273.0 331.0 1.25 2000.0)
(273.0 331.0 2.0 2050.0)
(273.0 331.0 3.0 3000.0)

```

=== Hypotheses ===

```

CLASSA
Cover: Relation:
V^2 = 401.3222 * T

```

APPENDIX C1 - Listing of topsed.s SED Script

This is a listing of the *topsed.s* sed script. *topsed.s* calls *topsed.f* to perform the formatting of the Top output for use by ABACUS.2. *topsed.s* provides the user with the following information:

- when processing begins
- when processing is finished
- the name of the output file (formatted for use with ABACUS.2)

```
# Program:    topsed.s {shell script}
# Calls:      topsed.f {file of sed commands}
# Purpose:    To edit output from the Unix Top program for input to
#             ABACUS.2 as a dataset. Top provides statistics such
#             operating system statistics for the system it is running
#             on. Topsed.s preprocesses top output for input into
#             ABACUS.2. ABACUS.2 requires that the dataset be in
#             a Lisp readable format.
# Author:     Bret Michael
# Created:    May 8, 1989
# Last Modified: May 22, 1989
# Location:    ~jmichael/SUN_AB2/useful_tools
#
#
# preprocess file for use by ABACUS.2
# Comment: The following sed commands are to some degree order dependent.
#          Check to make sure that the behavior of this program is correct
#          each time you modify this file.
#
#          The ordering used below is to edit out all of the non-numeric
#          data which is generated by top, remove all extra spaces, and
#          then to append all of the lines that make up an event (i.e.,
#          an observation). The last operation performed is to enclose
#          the each event in parentheses.
#
# begin preprocessing of top output file for use by ABACUS.2
echo *** Beginning preprocessing of $*
sed -f topsed.f $* > $*.lisp
echo *** Finished processing $*
echo *** Output file is called $*.lisp"
# end of topsed.s shellscript
```

APPENDIX C2 - Listing of topsed.f SED Script

topsed.f is essentially a set of Unix sed commands that remove all extraneous spaces and text from the Top output. That is, all textual annotations are removed, one blank space is left between each variable value, and each event (i.e., line) is enclosed in parentheses. *topsed.f* is called by *topsed.s*.

```
# Program:      topsed.f {file of sed commands}
# Called by:    topsed.s {shell script}
# Purpose:      To edit output from the Unix Top program for input to
#               ABACUS.2 as a dataset. Top provides statistics such
#               operating system statistics for the system it is running
#               on. Topsed.f is the file of sed commands used by the
#               shell script topsed.s.
# Author:       Bret Michael
# Created:      May 8, 1989
# Last Modified: May 18, 1989
# Location:     ~jmmichael/SUN_AB2/useful_tools
#
#
# preprocess file for use by ABACUS.2
# Comment: The following sed commands are to some degree order dependent.
#          Check to make sure that the behavior of this program is correct
#          each time you modify this file.
#
#          The ordering used below is to edit out all of the non-numeric
#          data which is generated by top, remove all extra spaces, and
#          then to append all of the lines that make up an event (i.e.,
#          an observation). The last operation performed is to enclose
#          the each event in parentheses.
#
# begin preprocessing of top output file for use by ABACUS.2
{
s/last pid://
s/load//
s/averages://
s/processes://
s/sleeping//
s/running//
s/stopped//
s/Cpu states://
s/user//
s/nice//
s/system//
s/idle//
s/Memory://
s/K//g
s/real//
s/virtual//
```

```

s/free//
s/zombie//
s/s//g
s:/_/_/g
s/%//g
s/,//g
s/(//g
s/)//g
s/$/ /
s/^.*; //
N
s/$/ /
N
N
s/$/) /
s/0/g
s/ */ /g
s/^(/
s/ ) /)/g
}
# end of topsed.f sed commands

```

APPENDIX C3 - Listing of Dataset Generation Program

Unlike the network experiment which used data gathered from an actual network, most of the experiments were conducted using contrived data generated by the user with the aid of utility programs. The function `createdataset` is an example of the type of data generation aid that was used in this study. This particular version of the function was used to create data sets for the illumination of surfaces experiments. In addition, the datasets that were generated with the function below were further modified by the user in order to intentionally introduce some degree of irregularity (e.g., error) into the data.

```
;;      Program:   createillumdataset
;;      Created:   April 10, 1989
;;      Last modified: May 15, 1989
;;      Input file: inputdata
;;      Input args: values for I, R, and i
;;      Outputfile: illum.dataset
;;      Output format: formatted dataset illum.dataset
;;      Purpose: To create sample datasets for running experiments.
;;      The data fit the equation:  $E = (I / R * R) \cos i$ . After
;;      generating the data, the output file must be merged with
;;      a file that contains the other data structures that ABACUS.2
;;      expects.
;;
;;
(defun createdataset ()
  (with-open-file (data-stream "inputdata" :direction :input)
    (with-open-file (dataset-stream "illum.dataset" :direction :output)
      (do ((value (read data-stream nil)(read data-stream nil)))
          ((not value))
            (setq illum (float (* (/ (car value) (* (cadr value)(cadr value)))
                                   (cos (caddr value))))))
            (setq newset (cons illum value))
            (print newset dataset-stream)
          )))
  ))))
```

APPENDIX D - Paper Submitted to IJCAI-89 Workshop

The following extended abstract was submitted to the IJCAI-89 Workshop on Knowledge and Discovery in Databases.

EXPERIMENTATION WITH ABACUS.2

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EXTENDED ABSTRACT

ABACUS.2 is a quantitative discovery system that relies on dependency analysis to discover linear and non-linear terms [Greene, 88]. The heuristics used by ABACUS.2 for discovery of terms do not support the identification of exponential or logarithmic dependencies. This lack of breadth of dependency analysis limits the number of domains in which ABACUS.2 would be useful to use. For example, many of the phenomena studied in the biological sciences are characterized by exponential and logarithmic relationships. In order to alleviate this deficiency, an additional heuristic has been added to ABACUS.2. This rule differs from the other dependency heuristics in that it takes into account the rate of change of dependencies among terms, rather than just the change in dependencies. The algorithm is as follows:

If ABACUS.2 is unable to determine an equation form after processing the linear and non-linear rules, then measure the rate of change in the dependencies
 if $\text{amon}^+(x,y)$ increases at an increasing rate,
 create e^x
 if $\text{amon}^-(x,y)$ decreases at an increasing rate,
 create $\ln(x)$

where $\text{amon}^+(x,y)$ says x is approximately positively monotonic to y , and $\text{amon}^-(x,y)$ says x is approximately negatively monotonic to y . Experimentation with ABACUS.2 using this new rule is currently underway, and a rule for identifying the form $\log_a N$ is being developed.

In addition to trying to discover logarithmic and exponential terms, experiments are being conducted to determine how the various parameters used by ABACUS.2 affect the system's ability to discover equations. ABACUS.2 is parameter-driven. That is, the user must specify the which variables should not be required to be held constant when the dependency is calculated, the level of uncertainty (i.e., used to account for noisy data and round-off error), the number of total events which must evaluate to a constant for an equation to be considered acceptable, and so on. The feasibility of letting ABACUS.2 guide the discovery process is being explored. The motivation for this is two-fold. This approach will aid the user in using ABACUS.2 (e.g., the user can rely on the system to some degree to set the parameters), and will enhance the performance of ABACUS.2 (e.g., reducing the search space when working with large databases of experimental data). The two research questions that are currently being addressed are how much knowledge does the system need about the parameters and domain in which the data was derived, and how should this knowledge be represented? For example, in order for the system to be able to dynamically modify the level of uncertainty, how much knowledge does ABACUS.2 need about the characteristics of this parameter, and the relationships between this parameter and the other parameters? Likewise, what types of domain knowledge would be useful in guiding the discovery process (e.g., if the dataset is from

the field of optics, should the system be aware that many of the regularities among variables in this domain can be described via trigonometric functions, especially sine and cosine)?

ABACUS.2 is written in Common LISP, and the experiments are being conducted on a SUN 3/60 workstation. Sample datasets from the fields of physics, chemistry, biology, and operating systems are being used to perform the experiments.

REFERENCE

[Greene, 88]

Greene, G. H. Quantitative Discovery: Using Dependencies to Discover Non-Linear Terms. Masters thesis, Computer Science Department, University of Illinois at Urbana-Champaign, 1988.

