THE MONK'S PROBLEMS: A PERFORMANCE COMPARISON OF DIFFERENT LEARNING ALGORITHMS

by

91-28

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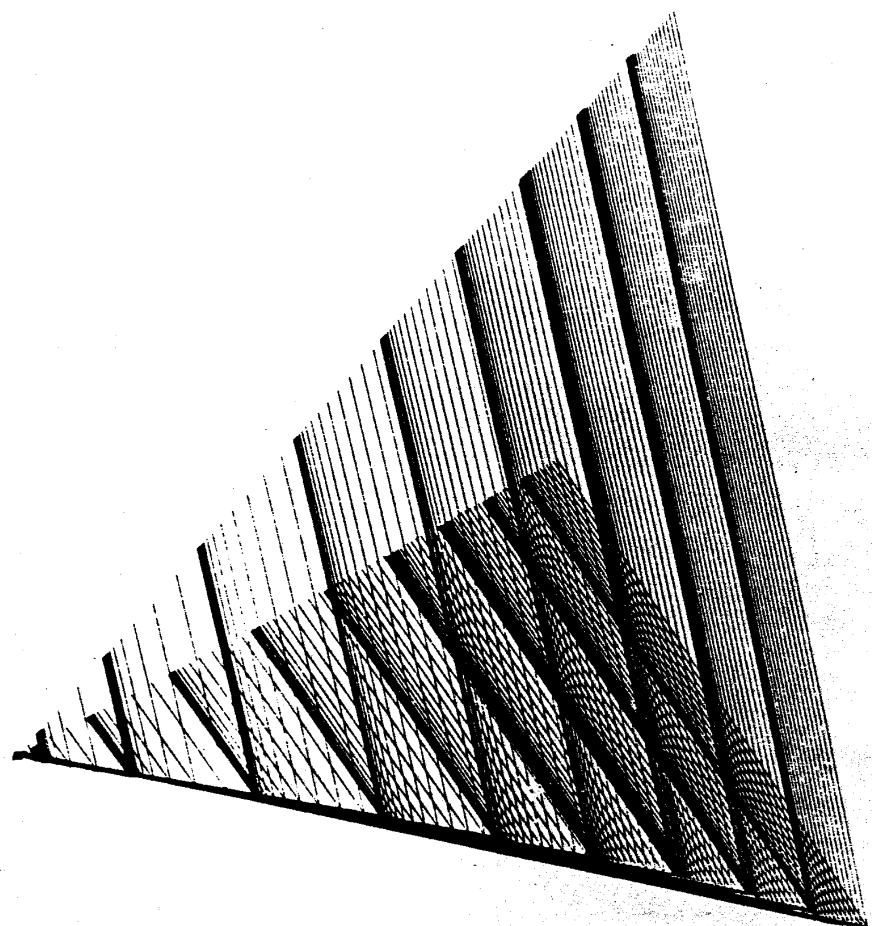
Carnegie Mellon University, Oct. 1991.



The MONK's problems

A Performance Comparison of Different Learning Algorithms

S.B. Thrun, J. Bala, E. Bloedorn, I. Bratko, B. Cestnik, J. Cheng, K. De Jong, S. Dzeroski, S.E. Fahlman, R. Hamann, K. Kaufman, S. Keller, I. Kononenko, J. Kreuziger, R.S. Michalski, T. Mitchell, P. Pachowicz, H. Vafaie, W. Van de Velde, W. Wenzel, J. Wnek and J. Zhang

Carnegie Mellon University, October 1991

Abstract

Once upon a time, in July 1991, the monks of Corsendonk Priory were faced with a school held in their priory, namely the 2nd European Summer School on Machine Learning. After listening more than one week to a wide variety of learning algorithms, they felt rather confused: Which algorithm would be optimal? And which one to avoid? As a consequence of this dilemma, they created a simple task on which all learning algorithms ought to be be compared: the *three MONK's problems*.

This report summarizes the results.

Contents

Results – a short overview

	# 1	# 2	# 3
J. Bala, E. Bloedorn, K. De Jong, K. Kaufman, R.S. Michalski, P. Pachowicz, H. Vafaie, J. Wnek and J. Zhang			
AQ17-DCI	**100%**	99.8%	
AQ17-HCI	**100%**	93.1%	**100%**
AQ17-FCLS		92.6%	97.2%
AQ14-NT			**100%**
AQ15-GA			**100%**
B. Cestnik, I. Kononenko and I. Bratko			
Assistant Professional	**100%**	81.25%	**100%**
S. Dzeroski			
mFOIL	**100%**	69.2%	**100%**
W. Van de Velde			
ID5R	81.7%	61.8%	
IDL	97.2%	66.2%	
ID5R-hat	90.3%	65.7%	
TDIDT	75.7%	66.7%	
J. Kreuziger, R. Hamann and W. Wenzel			
ID3	98.6%	67.9%	94.4%
ID3, no windowing	83.2%	69.1%	95.6%
ID5R	79.7%	69.2%	95.2%
AQR	95.9%	79.7%	87.0%
CN2	**100%**	69.0%	89.1%
CLASSWEB	(see page 102)		
S. Keller			
PRISM	86.3%	72.7%	90.3%
S. Thrun			
Backpropagation	91.7%	**100%**	87.7%
S. Fahlman			
Cascade Correlation	**100%**	**100%**	97.2%

Chapter 1

The MONK's Comparison Of Learning Algorithms – Introduction and Survey

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1.1 The problem

The MONK's problems rely on the an artificial robot domain, in which robots are described by six different attributes [Wnek, Sarma, Wahab and Michalski, 1991]:

x_1: head_shape	$\in$	round, square, octagon
x_2: body_shape	$\in$	round, square, octagon
x_3: is_smiling	$\in$	yes, no
x_4: holding	$\in$	sword, balloon, flag
x_5: jacket_color	$\in$	red, yellow, green, blue
x_6: has_tie	$\in$	yes, no

The learning task is a binary classification task. Each problem is given by a logical description of a class. Robots belong either to this class or not, but instead of providing a complete class description to the learning problem, only a subset of all 432 possible robots with its classification is given. The learning task is then to generalize over these examples and, if the particular learning technique at hand allows this, to derive a simple class description.

- **Problem M_1:**
  (head_shape = body_shape) or (jacket_color = red)

  From 432 possible examples, 124 were randomly selected for the training set. There were no misclassifications.

- **Problem M_2:**
  exactly two of the six attributes have their *first* value.

  (E.g.: body_shape = head_shape = round implies that robot is not smiling, holding no sword, jacket_color is not red and has no tie, since then exactly two (body_shape and head_shape) attributes have their first value) From 432 possible examples, 189 were randomly selected. Again, there was no noise.

- **Problem M_3:**
  (jacket_color is green and holding a sword) or (jacket_color is not blue and body_shape is not octagon)

  From 432 examples, 122 were selected randomly, and among them there were 5% misclassifications, i.e. noise in the training set.

Problem 1 is in standard disjunctive normal form and is supposed to be easy learnable by all symbolic learning algorithms as AQ and Decision Trees. Conversely, problem 2 is similar to parity problems. It combines different attributes in a way which makes it complicated to describe in DNF or CNF using the given attributes only. Problem 3 is again in DNF and serves to evaluate the algorithms under the presence of noise.

1.2 Visualization

All contibutions in this report have two things in common: firstly, they refer to the same problems – the MONK's problems –, and secondly, most results are visualized by a two-dimensional diagram. Due to the difficulties in representing a six-dimensional space on a conventional sheet of paper, the plot is unfolded, as might be found in [Wnek, Sarma, Wahab and Michalski, 1991]. The resulting diagrams of training and testing sets may be found below.

Acknowledgements

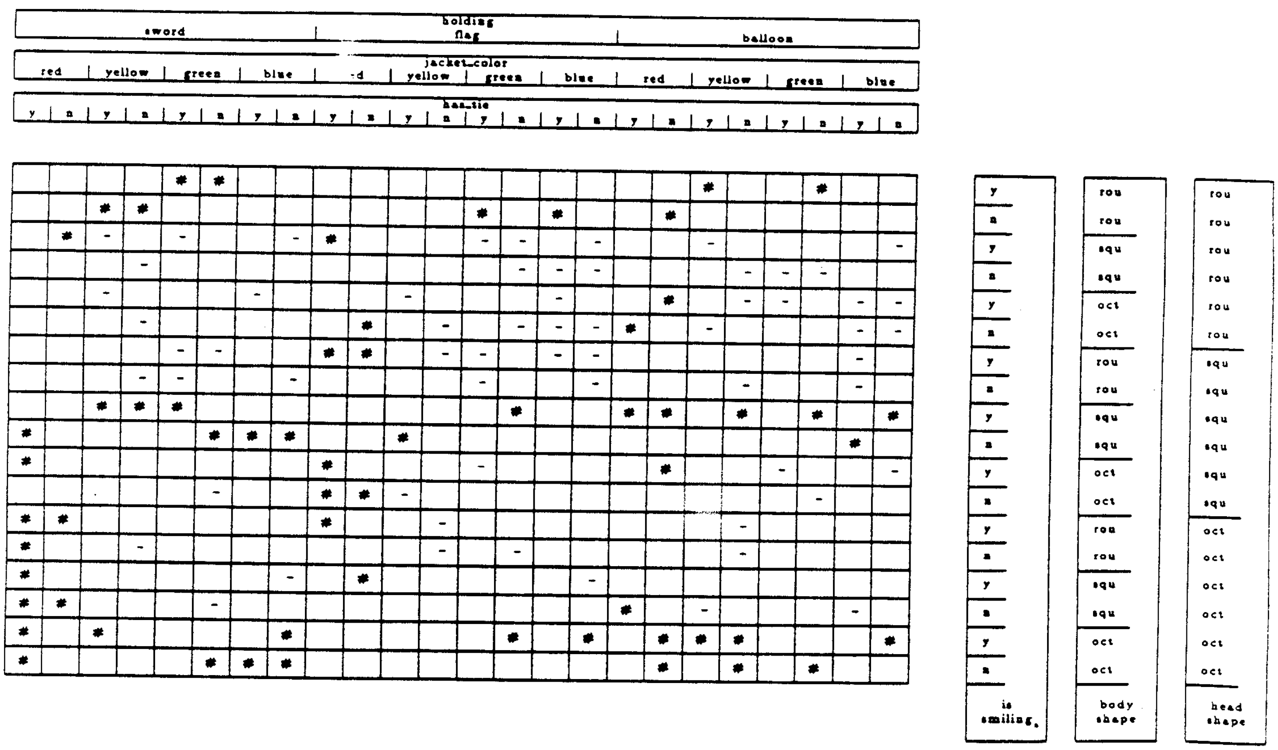
The authors thank Walter Van de Welde for the excellent organization of 2^{nd} European School on Machine Learning, at which this comparison was created. We would also like to thank all participants in this comparison, including Bruno Roger.

References

J. Wnek, J. Sarma, A. Wahab, and R. Michalski: Comparison learning paradigms via diagrammatic Visualization: A case study in single concept learning using symbolic, neural net and genetic algorithm methods, Technical Report, George Mason University, Computer Science Department, 1990

Training set M_1 (124 examples, no noise):

$$(\text{head\_shape} = \text{body\_shape}) \text{ or } (\text{jacket\_color} = \text{red})$$



In all training set diagrams, positive examples are marked by "#" and negative ones by "–". Misclassifications, as in the presence of noise, are indicated by boxes. Correspondingly, in all test sets positive examples are marked by "#", while empty fields indicate negative examples.

In turn, we will plot the results of all learning algorithms in the same way: # indicates that the learning algorithm classifies the entity as a positive member, and a blank as a non-member. However, an additional square will indicate misclassifications, i.e. if the classification obtained by the algorithm is wrong.

Test set M_1:

$$(\text{head\_shape} = \text{body\_shape}) \text{ or } (\text{jacket\_color} = \text{red})$$

---

head shape	body shape	is smiling	sword								flag								balloon							
		jacket_color	red		yellow		green		blue		red		yellow		green		blue		red		yellow		green		blue	
		has_tie	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n
rou	rou	y	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#
rou	rou	n	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#
rou	squ	y	#	#							#	#							#	#						
rou	squ	n	#	#							#	#							#	#						
rou	oct	y	#	#							#	#							#	#						
rou	oct	n	#	#							#	#							#	#						
squ	rou	y	#	#							#	#							#	#						
squ	rou	n	#	#							#	#							#	#						
squ	squ	y	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#
squ	squ	n	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#
squ	oct	y	#	#							#	#							#	#						
squ	oct	n	#	#							#	#							#	#						
oct	rou	y	#	#							#	#							#	#						
oct	rou	n	#	#							#	#							#	#						
oct	squ	y	#	#							#	#							#	#						
oct	squ	n	#	#							#	#							#	#						
oct	oct	y	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#
oct	oct	n	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#

Training set M_2 (189 examples, no noise):

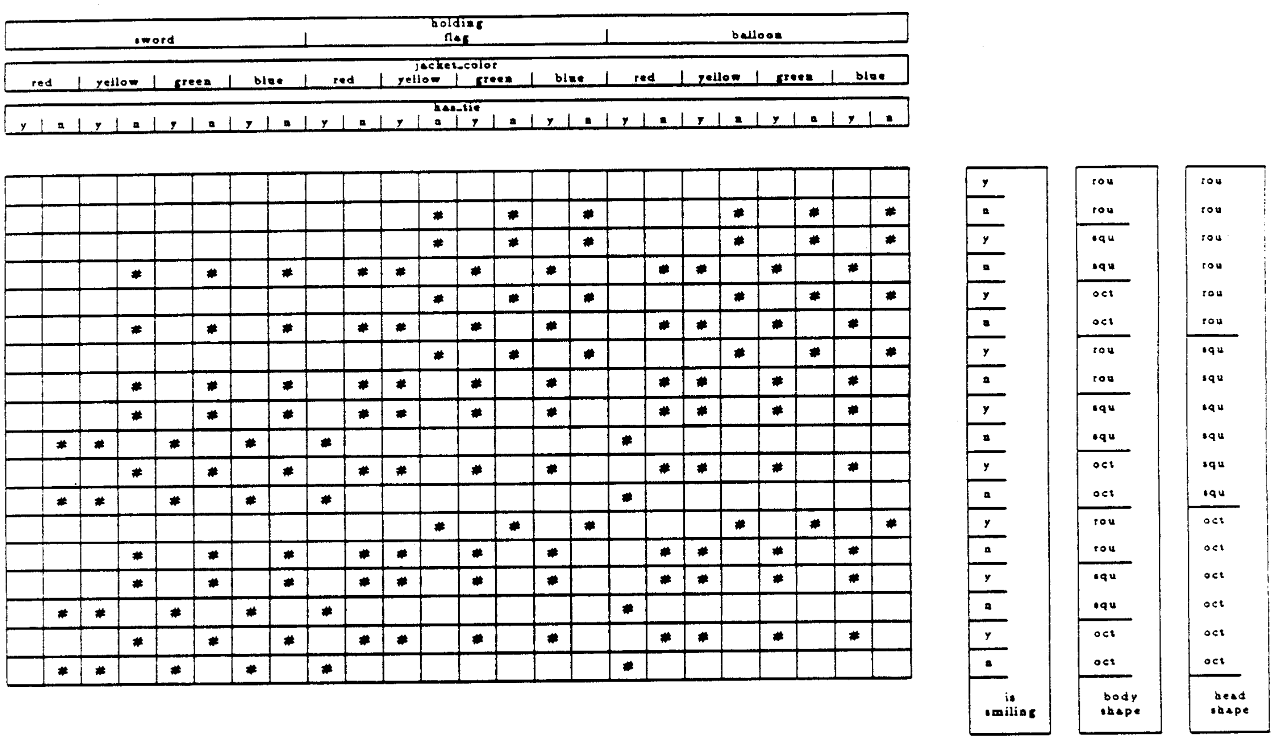
"exactly two of the six attributes have their first value"

---

holding	sword								flag								balloon										
jacket_color	red		yellow		green		blue		red		yellow		green		blue		red		yellow		green		blue				
has_tie	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	is smiling	body shape	head shape
				-			-		-	-	-		-		-				-				-		y	rou	rou
	-	-											-		-	#		-		#					n	rou	rou
		-								-		#	-	#	-		-	-		#	-	#	-	#	y	squ	rou
			-				-						#		#		-	#			#	-	#	-	n	squ	rou
		-		-	-	-					-	#		#	-					#	-			#	y	oct	rou
					-	#	-			#				-		-			#						n	oct	rou
	-			-	-							#						-		#		#	-		y	rou	squ
	-			#			-				#					-	-	#		-		-		-	n	rou	squ
					-			#	-				#								#	-	#		y	squ	squ
	-			-		-	#	-	#			-	-				#		-	-				-	n	squ	squ
	-	-				#			-				#	-		-		#	#				#		y	oct	squ
		#	#		#			-	#		-			-							-	-		-	n	oct	squ
							-			-		#		#	-	#	-	-		#		#			y	rou	oct
	-			#	-	#	-	#			#							#	#	-				-	n	rou	oct
		-		#	-	#				#	#						-		#		#	-			y	squ	oct
	-			-	#	-			#		-	-		-			#					-		-	n	squ	oct
	-		-		-	#								-											y	oct	oct
	-								#		-		-	-			#		-						n	oct	oct

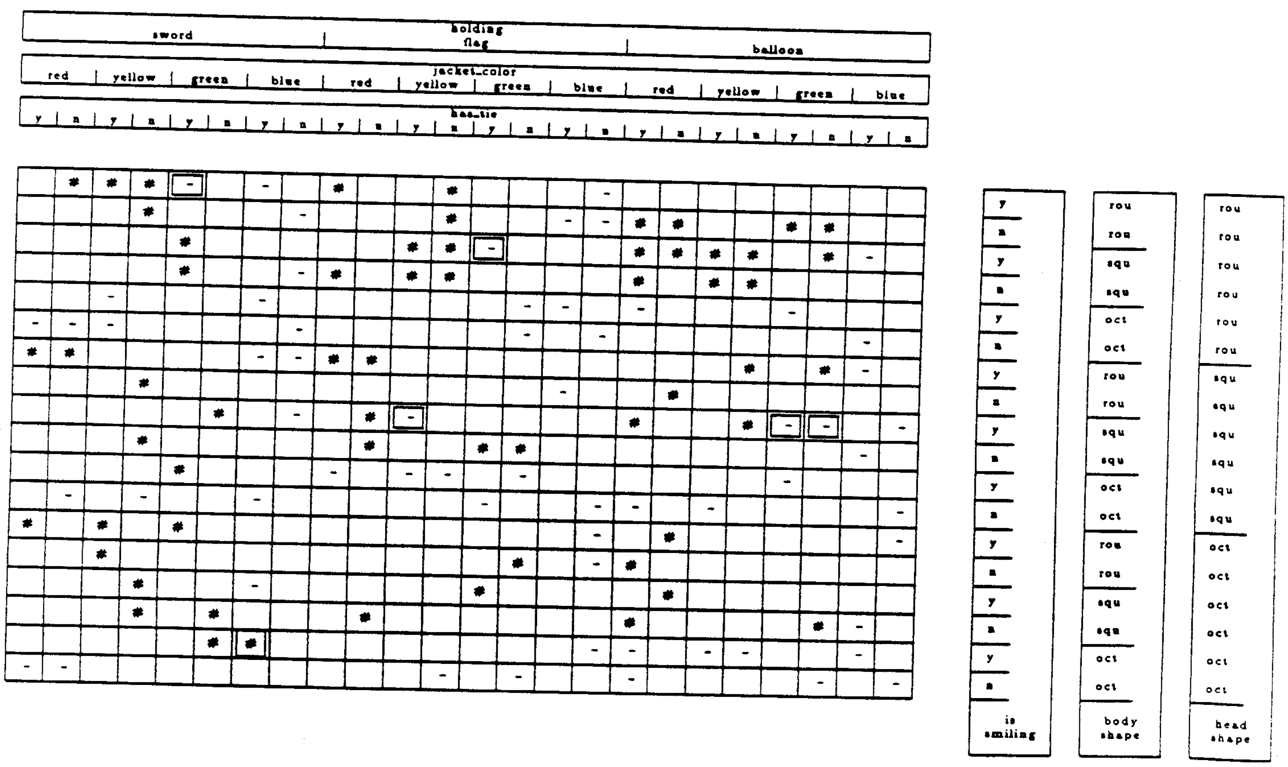
Test set M_2:

"exactly two of the six attributes have their first value"



Training set M_3 (122 examples, 6 misclassifications due to noise):

(jacket_color is green and holding a sword)
or
(jacket_color is not blue and body_shape is not octagon)



Test set M_3:

$$\text{(jacket\_color is green and holding a sword)}$$
$$\text{or}$$
$$\text{(jacket\_color is not blue and body\_shape is not octagon)}$$

---

holding			sword								flag								balloon							
jacket_color			red		yellow		green		blue		red		yellow		green		blue		red		yellow		green		blue	
has_tie			y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n
head shape	**body shape**	**is smiling**																								
rou	rou	y	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#		
rou	rou	n	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#		
rou	squ	y	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#		
rou	squ	n	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#		
rou	oct	y					#	#																		
rou	oct	n					#	#																		
squ	rou	y	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#		
squ	rou	n	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#		
squ	squ	y	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#		
squ	squ	n	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#		
squ	oct	y					#	#																		
squ	oct	n					#	#																		
oct	rou	y	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#		
oct	rou	n	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#		
oct	squ	y	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#		
oct	squ	n	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#		
oct	oct	y					#	#																		
oct	oct	n					#	#																		

Chapter 2

Applying Various AQ Programs to the MONK's Problems: Results and Brief Description of the Methods

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2.1 Introduction

This paper describes briefly results from applying various AQ learning programs to the MONKS' problems. The MONKS' problems are concerned with learning concept descriptions from examples. All examples come from the same event space, which spans 6 multiple-valued attributes (it has the total of 432 possible examples). The sizes of the value sets of the attributes, x1, x2, ..., x6, are 3, 3, 2, 3, 4, and 2, respectively. The problems differ in terms of the type of the target concept to be learned, and in the amount of noise in the data. Here is a brief summary of the data.

Below is a listing of the rules obtained by the various AQ programs (AQ17-DCI, AQ17-HCI, AQ15-GA, AQ15-FCLS or AQ14-NR), and the results of testing them on the testing examples. The testing of the rules was done using the ATEST program that computes a confusion matrix (Reinke, 1984). The program computes the so-called consonance degree between an example and the rules for each class. The output from this program includes numerical evaluations of the the accuracy of the rules based on the percentage of the testing examples correctly classified (by choosing the rule that best fits the example) and the percentage precisely matched by the correct decision rule.

The training and testing sets of examples were provided by the creators of the problems.

- **Problem 1.**

  There were 124 training examples, which represented 30% of the total event space (62 positive and 62 negative). The testing examples were all possible examples (216 positive and 216 negative).

- **Problem 2.**

  There were 189 training examples, which represented 40% of the total event space (105 positive and 64 negative). The testing examples were all possible examples (190 positive and 142 negative).

- **Problem 3.**

  There were 122 training examples, which represented 30% of the total event space (62 positive and 60 negative). The testing examples were all possible examples (204 positive and 228 negative).

2.2 Results for the 1st problem

2.2.1 Rules obtained by AQ17-DCI (one rule is for positive examples, Class 0, and one for negative examples, Class 1):

```
Class 0
  #   cpx
  1   [jacket_color > 1] [head_shape <> body_shape]  (total:62, unique:62)

Class 1
```

```
#   cpx
1   [head_shape=body_shape]  (total:41, unique:33)
2   [jacket_color=1]  (total:29, unique:21)
```

where "cpx" means a rule (a "complex" in VL1), "total" means the total number of examples of the given class covered by the rule, and "unique" means the number of examples covered by that rule only, and not by any other rules.

The results of applying the rules to the testing examples were:

RESULTS	
% FLEX MATCH	100.00
% 100% MATCH	100.00
OVERALL % CORRECT:	100.00

where % FLEX MATCH means the percentage of the correctly classified examples within the total set of testing examples, using a flexible matching function (see Reinke, 1984), and % 100% MATCH means that the percentage of correctly classified examples that matched the rules exactly.

Number of testing events satisfying individual rules in the correct class description:

	RULES R1	R2
CLASS 0	215	
CLASS 1	144	108

2.2.2 Rules obtained by AQ17-HCI

(one rule is for positive examples, Class 0, and one for negative examples, Class 1)

```
Class 0
  #   cpx
  1   [Neg17=false]  (total:62, unique:62)

Class 1
  #   cpx
  1   [Pos16=false]  (total:62, unique:62)
```

where Neg17 and Pos16 are attributes constructed from the original ones, or intermediate ones, as defined below (these rules, as one can check, are logically equivalent to the AQ17-DCI generated rules).

```
c01 <:: [head_shape=1] & [body_shape=2,3] & [jacket_color>1]

c05 <:: [head_shape=2] & [body_shape=1,3] & [jacket_color>1]

c08 <:: [head_shape=3] & [body_shape=1,2] & [jacket_color>1]

c10 <:: [head_shape=1] & [body_shape=1]

c12 <:: [jacket_color=1]

c13 <:: [head_shape=2] & [body_shape=2]

c15 <:: [head_shape=3] & [body_shape=3]

Pos16 <:: [c10=false] & [c12=false] & [c13=false] & [c15=false]

Neg17 <:: [c01=false] & [c05=false] & [c08=false]
```

TEST RESULTS - SUMMARY	
OVERALL % CORRECT:	100.00
OVERALL % CORRECT FLEX MATCH:	100.00
OVERALL % CORRECT 100% MATCH:	100.00

Number of testing events satisfying individual rules in the correct class description:

		RULES
		R 1
CLASS	0	215
CLASS	1	216

Other programs either were either not used on this problem, or generated similar results.

2.3 Results for the 2nd problem

2.3.1 Rules obtained by AQ17-FCLS

(one rule is for positive examples, Class 0, and one for negative examples, Class 1) are listed below. The threshold parameter indicates the minimum percentage of the individual conditions in the rule that must be satisfied. The discovered rules fully encompass Class 0, but they failed to get a complete grasp of the concept of Class 1:

```
Class 0
  #    cpx
  1   [X1 = 1] & [X2 = 1] & [X3 = 1] & [X4 = 1] & [X5 = 1] &
        [X6 = 1]
      with THRESHOLD = 50 %
```

This rule says that three or more variables must be equal to 1.) (Total positive examples covered: 64)

```
  2    [X1 = 2 , 3] & [X2 = 2 , 3] & [X3 = 2] & [X4 = 2 , 3] &
         [X5 = 2 , 3 , 4] & [X6 = 2]
       with THRESHOLD = 83 % (5/6)
```

This rule says that five or six out of six variables must be greater than 1, or equivalently, that at most one variable can be equal to 1. Thus the disjunction of these two rules above means that the number of variables which have value 1 cannot be equal to 2. (Total positive examples covered: 41).

These rules classified 100% all the examples of Class 0.

Class 1.
Since the current program does not have the ability to express the negation of the above two rules, it generated a disjuction of many small rules to cover the other class, and the overall performance using the flexible match was not 100% (in some cases when an example matched equally well the rules for both classes, an incorrect class was chosen.)

```
  #    cpx
  1    [X3 = 1] & [X4 = 2 , 3] & [X5 = 2] & [X6 = 2]
      with THRESHOLD = 100 %
      (total positive examples covered: 8)

  2    [X1 = 2 , 3] & [X2 = 2 , 3] & [X3 = 1] & [X4 = 2 , 3] &
         [X5 = 2 , 3 , 4]  & [X6 = 1]
      with THRESHOLD = 100 %
      (total positive examples covered: 9)

  3    [X1 = 2 , 3] & [X2 = 2 , 2] & [X3 = 2] & [X4 = 2 , 3] &
         [X5 = 2] & [X6 = 2]
      with THRESHOLD = 100 %
      (total positive examples covered: 7)

  4    [X1 = 3] & [X2 = 1] & [X3 = 1] &  & [X4 = 1] &
         [X5 = 3] & [X6 = 2]
      with THRESHOLD = 83 %
      (total positive examples covered: 5)

  5    [X1 = 1] & [X3 = 1] & [X4 = 2 , 3] & [X5 = 3 , 4] &
         [X6 = 2]
```

```
     with THRESHOLD = 100 %
     (total positive examples covered: 5)

6     [X1 = 2 , 3] & [X2 = 1] & [X3 = 2] & [X4 = 1 , 2] &
        [X5 = 2]
     with THRESHOLD = 100 %
     (total positive examples covered: 4)

7     [X1 = 1] & [X2 = 2 , 3] & [X3 = 2] & [X4 = 2 , 3] &
        [X5 = 2 , 3 , 4] & [X6 = 1]
     with THRESHOLD = 100 %
     (total positive examples covered: 5)

8     [X1 = 2 , 3] & [X3 = 2] & [X5 = 1] & [X6 = 2]
     with THRESHOLD = 100 %
     (total positive examples covered: 3)

9     [X1 = 2 , 3] & [X2 = 2 , 3] & [X3 = 2] & [X4 = 1] &
        [X5 = 2 , 3 , 4] & [X6 = 1]
     with THRESHOLD = 100 %
     (total positive examples covered: 4)

10    [X1 = 1 , 3] & [X2 = 1] & [X4 = 1 , 2] & [X5 = 4] &
        [X6 = 2]
     with THRESHOLD = 100 %
     (total positive examples covered: 3)

11    [X1 = 2] & [X2 = 2] & [X3 = 1] & [X4 = 1] &
        [X5 = 2 , 3 , 4] & [X6 = 2]
     with THRESHOLD = 100 %
     (total positive examples covered: 5)

12    [X1 = 1 , 2] & [X2 = 3] & [X4 = 2 , 3] & [X5 = 1] &
        [X6 = 2]
     with THRESHOLD = 100 %
     (total positive examples covered: 2)

13    [X1 = 1] & [X2 = 1] & [X3 = 2] & [X4 = 3] &
        [X5 = 2] & [X6 = 2]
     with THRESHOLD = 100 %
     (total positive examples covered: 1)

14    [X1 = 1] & [X2 = 3] & [X3 = 2] & [X4 = 1] &
        [X5 = 1 , 3] & [X6 = 2]
     with THRESHOLD = 100 %
     (total positive examples covered: 1)

15    [X1 = 1] & [X2 = 2] & [X3 = 2] & [X4 = 2 , 3] &
        [X5 = 1] & [X6 = 2]
     with THRESHOLD = 100 %
     (total positive examples covered: 1)
```

```
16    [X1 = 2] & [X2 = 1] & [X3 = 1] & X4 = 3] &
        [X5 = 2 , 3]
    with THRESHOLD = 100 %
    (total positive examples covered: 2)

17   [X1 = 3] & [X2 = 2] & [X3 = 1] & [X4 = 2] &
        [ X5 = 1 , 2]
    with THRESHOLD = 100 %
    (total positive examples covered: 2)

18   [X1 = 2 , 3] & [X2 = 1] & [X3 = 2] & [X4 = 2 , 3] &
        [X5 = 2 , 3 , 4] & [X6 = 1]
    with THRESHOLD = 100 %
    (total positive examples covered: 3)
```

TEST RESULTS - SUMMARY	
Result summary of Class 0	
Correctly classified events:	268.00
Total number of events in this class:	290
Result summary of Class 1	
Correctly classified events:	132.00
Total number of events in this class:	142
Overall summary of results	
The percentage of correctly classified events:	92.6%
The total number of conditions in the descriptions:	110
The total number of rules in the descriptions:	2 for Class 0 and 18 for Class 1.

2.3.2 Rules obtained by AQ17-DCI

(one rule for positive examples, Class 0 and one for negative examples, Class 1):

```
Class 0
   #    cpx
   1   [#VarEQ(1)=3..5]
   2   [#VarEQ(1)=0,1]

Class 1
   #    cpx
   1   [#VarEQ(1)=2]
```

#VarEQ(x) is a constructed attribute which counts the number of attributes which have the value x.

The results of applying the rules to the testeing examples were:

RESULTS	
% FLEX MATCH	99.77
% 100% MATCH	99.77
OVERALL % CORRECT:	99.77

The less than 100% correct classification on the testing data was due to a testing event which contained all (6) 1's. This example was not in the training data. Our program currently generalizes to the limits of the attribute values seen, not the values possible. This however, can be easily changed.

2.3.3 Rules obtained by AQ17-HCI

(one rule is for positive examples, Class 0, and one for negative examples, Class 1):

```
  Class 0
    #   cpx
    1   [Pos73=true]  (total:90, unique:49)
    2   [c14=false] & [c26=false] & [c53=false] & [c67=false] &
[c72=false] & [Neg74=false]  (total:38, unique:6)
    3   [x4=2,3] & [c6=false] & [c20=false] & [Neg74=false]
  (total:22, unique:5)
    4   [x1=2] & [x6=2] & [c44=false] & [c50=false] & [Neg74=false]
  (total:6, unique:2)
```

```
  Class 1
    #   cpx
    1   [Neg74=true]  (total:43, unique:30)
    2   [x5=2,3,4] & [x6=1] & [c60=true] & [Pos73=false]  (total:17, unique:4)
    3   [x1=2,3] & [x2=2] & [c28=false] & [Pos73=false]  (total:16, unique:7)
    4   [x2=3] & [c48=true] & [c66=true]  (total:4, unique:2)
    5   [x5=3] & [c43=true] & [c52=false] & [c53=false] & [c55=false] &
[c69=true] & [Pos73=false]  (total:4, unique:2)
    6   [x2=3] & [c9=false] & [c10=true] & [c23=true] & [c32=true]
  (total:3, unique:1)
```

Attributes "ci," "Pos73," and "Neg74" were constructed during the learning process.

```
c2 <:: [x5=1,4]

c4 <:: [x2=2,3] & [x3=2]

c5 <:: [x1=2,3] & [x3=2]
```

```
c6 <:: [x1=2,3] & [x2=2,3]
c7 <:: [x4=1,2] & [x5=1,3,4]
c9 <:: [x1=1,3] & [x5=2,3,4]
c10 <:: [x4=1,2] & [x5=2,3,4]
c14 <:: [x5=2,3,4] & [x6=2]
c15 <:: [x3=1] & [x5=2,3,4]
c16 <:: [x4=2,3] & [x6=2]
c17 <:: [x4=2,3] & [x5=2,3,4]
c18 <:: [x3=2] & [x5=2,3,4]
c20 <:: [x5=2,3,4] & [x6=1]
c21 <:: [x2=2,3] & [x4=2,3]
c22 <:: [x3=2] & [x4=1,2]
c23 <:: [x4=1,3] & [x5=2,3,4]
c26 <:: [x1=2,3] & [x5=2,3,4]
c28 <:: [x2=1,3] & [x5=2,3,4]
c32 <:: [x1=2,3] & [x5=1,2,3]
c33 <:: [x1=2,3] & [x6=2]
c37 <:: [x3=2] & [x4=2,3]
c38 <:: [c21=false] & [c37=false]
c39 <:: [c6=true] & [c17=true]
c40 <:: [c5=true] & [c17=true]
c41 <:: [c15=false] & [c28=false]
c42 <:: [x4=2,3] & [c39=false]
c43 <:: [x2=2,3] & [c39=false]
c44 <:: [x4=2,3] & [x5=2,3,4]
c46 <:: [c15=false] & [c39=false]
```

```
c47 <:: [c7=false] & [c39=false]

c48 <:: [x5=1,2,4] & [c7=false]

c49 <:: [c17=false] & [c33=true]

c50 <:: [x2=2,3] & [c22=false]

c52 <:: [x5=2,3,4] & [c14=false]

c53 <:: [x5=2,3,4] & [c21=true]

c55 <:: [x4=1,2] & [c14=false]

c56 <:: [x4=1,3] & [c14=false]

c59 <:: [x5=2,4]

c60 <:: [c38=false] & [c49=false]

c61 <:: [x2=2,3] & [x5=2,3,4]

c65 <:: [c20=false] & [c39=false]

c66 <:: [x5=1,2,3] & [c46=true]

c67 <:: [c38=false] & [c49=true]

c68 <:: [c40=false] & [c55=false]

c69 <:: [c16=false] & [c55=false]

c70 <:: [x5=2,3,4] & [c18=false]

c72 <:: [x5=1,2,3] & [c37=true]

Pos73 <:: [c4=false] & [c16=false] & [c33=false] & [c39=false] & [c40=false]
        v [c15=false] & [c43=false] & [c47=false] & [c68=false]
        v [x2=1,2] & [c21=false] & [c41=true] & [c44=false] & [c65=true]
           & [c67=false]
        v [c33=true] & [c60=true]

Neg74 <:: [c4=false] & [c42=true] & [c56=false] & [c65=true] & [c68=true]
        v [c2=false] & [c4=false] & [c16=false] & [c17=true] & [c26=true]
        v [x3=2] & [x4=2,3] & [c14=false] & [c41=true] & [c43=true]
                 & [c59=false] & [c69=false] & [c70=false]
        v [x6=2] & [c5=true] & [c44=false] & [c61=false]
```

TEST RESULTS - SUMMARY	
OVERALL % CORRECT:	100.00
OVERALL % CORRECT FLEX MATCH:	93.06
OVERALL % CORRECT 100% MATCH:	86.57

Number of testing events satisfying individual complexes in the correct class description:

	RULES					
	R 1	R 2	R 3	R 4	R 5	R 6
CLASS 0	232	84	54	12		
CLASS 1	77	44	32	10	5	4

2.4 Results for the 3rd problem

2.4.1 Rules obtained by AQ14-NT

AQ14-NT is a program for learning from noisy example sets (one rule is for positive examples, Class 0, and one for negative examples, Class 1):

After only two loops of concept-driven filtration of training dataset (with truncation parameter equal to 10%) and repeated learning, we received the following set of rules:

```
Class 0:
[jacket_color=4]   v
[body_shape=3] & [holding=2..3]   v
[body_shape=3] & [jacket_color=1..2]
Class 1:
[body_shape=1..2] & [jacket_color=1..3]   v
[holding=1] & [jacket_color=3]
```

These rules recognized all test data correctly (i.e., on the 100% level).

Since there was supposed to be noise in the data, we are somewhat surprised by such a high degree of recognition. (Details of the testing runs are available.)

2.4.2 Rules obtained by AQ17-FCLS

(one rule is for positive examples, Class 0, and one for negative examples, Class 1). The threshold parameter indicates the minimum percentage of selectors in the rule that must fire. This set of rules is intentionally incomplete and inconsistent with the training set; it was generated with a 10% error tolerance:

```
Class 0
  #    cpx
  1    [X1 > 1] & [X2 = 3] & [X5 = 4]
        with THRESHOLD = 67 %

     (Total positive examples covered: 42)


  2    [X1 = 1] & [X2 = 3] & [X5 = 4]
       with  THRESHOLD = 67 %

     (Total positive examples covered: 26)

Class 1
  #    cpx
  1    [X2 = 1 , 2] & [X5 = 1 , 2 , 3]
       with  THRESHOLD = 100 %

     (Total positive examples covered: 57)
```

TEST RESULTS - SUMMARY	
Result summary of Class 0	
correctly classified events:	204.00
total number of events in this class:	204
Result summary of Class 1	
correctly classified events:	216.00
total number of events in this class:	228
Result Summary	
The percentage of correctly classified events:	97.2%
The total number of conditions in the descriptions:	8
The total number of rules in the descriptions:	2 for Class 0 and 1 for Class 1

2.4.3 Rules obtained by AQ15-GA

Below are the rules obtained by AQ15-GA (one rule is for positive examples, Class 0, and one for negative examples, Class 1). A genetic algorithm determined that 3 attributes (body_shape, holding, and jacket_color) were the most meaningful. Using these, the rules discovered were as follows:

```
Class 0
  1    [jacket_color=4]
  2    [body_shape=3] & [jacket_color=1..2]
  3    [body_shape=2..3] & [holding=2..3] & [jacket_color=3]
  4    [body_shape=1] & [holding=1] & [jacket_color=3]
  5    [body_shape=2] & [holding=2] & [jacket_color=2] : 1.00: 0.00


Class 1
  1    [body_shape=1..2] & [jacket_color=1..3]
  2    [body_shape=3] & [holding=1] & [jacket_color=3..4] : 1.00: 0.00
```

TEST RESULTS - SUMMARY	
OVERALL % CORRECT:	100.00
OVERALL % CORRECT FLEX MATCH:	100.00
OVERALL % CORRECT 100% MATCH:	100.00

2.4.4 Results obtained by AQ17-HCI:

```
Class 0
  #    cpx
  1    [Pos1=true]  (total:49, unique:49)
  2    [x2=2,3] & [x4=2,3] & [x5=3]  (total:11, unique:11)
  3    [x2=1] & [x4=1] & [x5=3]  (total:1, unique:1)
  4    [x2=2] & [x4=2] & [x5=2]  (total:1, unique:1)


Class 1
  #    cpx
  1    [Neg2=true]  (total:57, unique:57)
  2    [x2=3] & [x4=1] & [x5=3,4]  (total:3, unique:3)
```

where Pos1 and Neg2 are attributes constructed from the original ones (Wnek & Michalski, 1991)

```
Pos1 <:: [x5=4]
       v [x2=3] & [x5=1,2,4]

Neg2 <:: [x2=1,2] & [x5=1,2,3]
```

TEST RESULTS - SUMMARY	
OVERALL % CORRECT:	100.00
OVERALL % CORRECT FLEX MATCH:	100.00
OVERALL % CORRECT 100% MATCH:	86.11

Number of testing events satisfying individual complexes in the correct class description:

	RULES			
	R 1	R 2	R 3	R 4
CLASS 0	180	24	0	0
CLASS 1	216	12		

2.5 A brief description of the programs and algorithms

2.5.1 AQ17-DCI (Data-driven constructive induction)

This program is based on the classical AQ algorithm, but it includes procedures for constructive induction that generates all kinds of derived attributes. The mechanism of constructive induction is done in the following way (Bloedorn and Michalski, 1991):

1. Identify all linear type attributes.
2. Repeat steps 3 through 5 for each possible attribute combination.
3. Repeat steps 4 and 5 for each constructive induction operator The operators include: addition, multiplication, logical comparison of attribute values, etc.
4. Calculate the values of this attribute pair for the given constructive induction operator.
5. Evaluate the discriminatory power of this newly constructed attribute using the Quality Function (QF) described below. If the attribute is above some threshold, then store it, else discard it.
6. Exit.

After selecting the possible candidates, the algorithm generates possible combinations of attributes and the constructive induction operators applicable to the given attributes. After each new attribute's values are calculated, an evaluation function, QF, is used to judge its quality before adding it to the attribute set.

2.5.2 AQ17-FCLS (Flexible concept learning)

This method (Zhang and Michalski, 1991) combines both symbolic and numeric representations in generating a concept description. The program is oriented toward learning flexible concepts, i.e, imprecise and context dependent. To describe such concepts it creates two-tiered descriptions, which consist of a Basic Concept Representation (BCR) and an Inferential Concept Interpretation (ICI) to handle exceptions. In the program, the BCR is in the form of rules, and the ICI is in the form of a weighted evaluation function which sums up the contributions of individual conditions in a rule, and compares it with a THRESHOLD. The learning program learns both the rules and an appropriate value for the THRESHOLD.

Each rule of a concept description is learned in two steps, the first step is similar to the STAR algorithm in AQ that generates a general rule, and the second step optimizes the rule by specializing it and adjusting the accuracy threshold..

2.5.3 AQ17- HCI (Hypothesis-driven constructive induction)

The AQ17-HCI (Hypothesis-Driven Constructive Induction) represents a module (a method) employed in the AQ17 attribute- based learning system. The module implements a new iterative constructive induction capability in which new attributes are generated based on the analysis of the hypotheses produced in the previous iteration (Wnek and Michalski, 1991). Input to the HCI module consists of the example set and a set of rules (in this case generated by the AQ15 program) The rules are then evaluated according to a rule quality criterion, and the rules that score the best for each decision class are combined into new attributes. These attributes are incorporated into the set of training examples, and the learning process is repeated. The process continues until a stopping criterion is satisfied. The method is a special implementation of the idea of the "survival of the fittest," and therefore can be viewed as a combination of symbolic learning with a form of genetic algorithm-based learning.

A brief description of the HCI algorithm follows:

1. Induce rules for each decision class using a subset of the complete set of available training examples.
2. Analyze the rules in order to identify any irrelevant variables.
3. For each decision class, generate a new attribute that corresponds to a subset of the highest quality rules.
4. Modify the training examples by adding the newly constructed attributes and removing the ones found to be irrelevant.
5. Induce rules from this modified training set.
6. Test these rules against the remainder of the training set. If the performance is not satisfactory, return to step 1. Otherwise, use this attribute set to induce rules from the entire training set.

In these examples, the induction in steps 1, 5 and 6 was performed the learning algorithm, implemented in the AQ15 program.

2.5.4 AQ14 - NT (noise-tolerant learning from engineering data)

The learning method was specially designed to learn from noisy engineering data of complex and unknown distribution of attributes (Pachowicz and Bala, 1991a and 1991b). The acquisition of concept descriptions is performed in the following two phases:

- **Phase 1:**
  Concept-driven closed-loop filtration of training data, where a single loop of gradual noise removal from the training dataset is composed of the following three stages:

1. Concept acquisition from available training dataset (incorporating the AQ14 learning program),

2. Truncation of concept descriptions through the removal of the less significant concept components covering a given small proportion of the training data, and

3. Filtration of the training dataset - a new training dataset is created by passing the training examples that are covered by truncated concept descriptions.

- **Phase 2:**
  Acquisition of concept descriptions from filtered training data incorporating the AQ14 learning program.

2.5.5 AQ15-GA (AQ15 with attribute selection by a genetic algorithm)

In this approach we use genetic algorithms in conjunction with AQ15. Genetic algorithms are used to explore the space of all subsets of a given attribute set. Each of the selected attribute subsets is evaluated (its fitness measured) by invoking AQ15 and measuring the recognition rate of the rules produced.

The evaluation procedure as shown is divided into three main steps. After an attribute subset is selected, the initial training data, consisting of the entire set of attribute vectors and class assignments corresponding to examples from each of the given classes, is reduced. This is done by removing the values for attributes that were eliminated from the original attribute vector. The second step is to apply a classification process (AQ15) to the new reduced training data. The decision rules that AQ15 generates for each of the given classes in the training data are then used for classification. The last step is to use the rules produced by the AQ algorithm in order to evaluate the classification and hence, recognition with respect to the test data.

In order to use genetic algorithms as the search procedure, it is necessary to define a fitness function which properly assesses the decision rules generated by the AQ algorithm. The fitness function takes as an input a set of attribute or attribute definitions, a set of decision rules created by the AQ algorithm, and a collection of testing examples defining the attribute values for each example. The fitness function then views the AQ- generated rules as a form of class description that, when applied to a vector of attribute or attribute values, will evaluate to a number. It is evaluated for every attribute subset by applying the following steps: For every testing example a match score is evaluated for all the classification rules generated by the AQ algorithm, in order to find the rule(s) with the highest or best match. At the end of this process, if there is more than one rule having the highest match score, one rule will be selected based on the chosen conflict resolution process. This rule then represents the classification for the given testing example. If this is the appropriate classification, then the testing example has been recognized correctly. After all the testing examples have been classified, the overall fitness function will be evaluated by adding the weighted sum of the match score of all of the correct recognitions and subtracting the weighted sum of the match score of all of the incorrect recognitions.

2.6 The AQ Algorithm

All the above programs use AQ as the basic induction algorithm. Here is a brief description of the AQ algorithm:

1. Select a seed example from the set of training examples for a given decision class.
2. Using the extend against operator, generate a set of alternative most general rules (a star) that cover the seed example, but do not cover any negative examples of the class.
3. Select the "best" rule from the star, and remove the examples covered by this rule from from the set of positive examples yet to be covered.
4. If this set is not empty, select a new seed from it and go to step 2. Otherwise, if another decision class still requires rules to be learned, return to step 1.

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Chapter 3

The Assistant Professional Inductive Learning System: MONK's Problems

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3.1 Results

I have tested Assistant Professional inductive learning system (Cestnik, Kononenko and Bratko, 1987), which is a successor of ID3, with the latest modifications described in (Cestnik and Bratko, 1991). The system generates binary decision tree. The tests were conducted on IBM PS II, model 60.

I have named the domains as follows: FIRST, SECOND and THIRD. Here are the results of the measurements of classification accuracy.

	classification	accuracy on testing sample
FIRST	100.00 %	(432 of 432)
SECOND	81.25 %	(351 of 432)
THIRD	100.00 %	(432 of 432)

On the first and the third domain Assistant Professional was able to find a perfect domain model. However, in the second domain the constructed tree is very large and its performance is relatively poor. In an extensive study of the domain (testing sample) we were able to determine (with a help of our "neural nets") the correct model which is the following:

```
A robot is O.K.
    if exactly two attributes (out of 6) are equal to 1.
```

This concept is extremely complicated for a system that learns decision trees in an attribute-value logic formalism.

References

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Constructed decision trees in the three domains:

Decision Tree From Domain: FIRST
Pruned with m= 0.00

Number of Nodes : 15
Number of Leaves: 8
Number of Nulls : 0

```
| A5:Jacket_color [124]
| [red]
|    | V1-yes           [29]
| [yellow, green, blue]
|    | A1:Head_shape [95]
|    | [round, square]
|    |    | A2:Body_shape [71]
|    |    | [square]
|    |    |    | A1:Head_shape [27]
|    |    |    | [round]
|    |    |    |    | V2-no           [15]
|    |    |    | [square]
|    |    |    |    | V1-yes          [12]
|    |    | [round, octagon]
|    |    |    | A2:Body_shape [44]
|    |    |    | [round]
|    |    |    |    | A1:Head_shape [22]
|    |    |    |    | [round]
|    |    |    |    |    | V1-yes          [8]
|    |    |    |    | [square]
|    |    |    |    |    | V2-no           [14]
|    |    |    | [octagon]
|    |    |    |    | V2-no           [22]
|    | [octagon]
|    |    | A2:Body_shape [24]
|    |    | [octagon]
|    |    |    | V1-yes          [13]
|    |    | [round, square]
|    |    |    | V2-no           [11]
```

Decision Tree From Domain: SECOND
Pruned with m= 1.00

Number of Nodes : 113
Number of Leaves: 57
Number of Nulls : 1

```
| A4:Holding [169]
| [sword]
|    | A5:Jacket_color [54]
|    | [red]
|    |    | A2:Body_shape [15]
|    |    | [round, square]
|    |    |    | V2-no           [9]
|    |    | [octagon]
|    |    |    | A3:Is_smiling [6]
|    |    |    | [yes]
|    |    |    |    | V2-no           [4]
|    |    |    | [no]
|    |    |    |    | A1:Head_shape [2]
|    |    |    |    | [round]
|    |    |    |    |    | NULL LEAF:
|    |    |    |    |    | V1-yes          [46.0%]
|    |    |    |    |    | V2-no           [54.0%]
|    |    |    |    | [square]
|    |    |    |    |    | V1-yes          [1]
|    |    |    |    | [octagon]
|    |    |    |    |    | V2-no           [1]
|    | [yellow, green, blue]
|    |    | A1:Head_shape [39]
|    |    | [round]
|    |    |    | A5:Jacket_color [10]
|    |    |    | [green]
|    |    |    |    | A6:Has_tie [4]
|    |    |    |    | [no]
|    |    |    |    |    | A3:Is_smiling [2]
|    |    |    |    |    | [yes]
|    |    |    |    |    |    | V2-no           [1]
|    |    |    |    |    | [no]
|    |    |    |    |    |    | V1-yes          [1]
|    |    |    |    | [yes]
|    |    |    |    |    | V2-no           [2]
|    |    |    | [yellow, blue]
|    |    |    |    | V2-no           [6]
|    |    | [square, octagon]
|    |    |    | A6:Has_tie [29]
|    |    |    | [no]
|    |    |    |    | A3:Is_smiling [16]
```

```
|    |    |    |    |    | [yes]
|    |    |    |    |    |    | A2:Body_shape [6]
|    |    |    |    |    |    | [round]
|    |    |    |    |    |    |    | V2-no           [1]
|    |    |    |    |    |    | [square, octagon]
|    |    |    |    |    |    |    | V1-yes          [5]
|    |    |    |    |    | [no]
|    |    |    |    |    |    | A2:Body_shape [10]
|    |    |    |    |    |    | [round]
|    |    |    |    |    |    |    | V1-yes          [4]
|    |    |    |    |    |    | [square, octagon]
|    |    |    |    |    |    |    | V2-no           [6]
|    |    |    |    | [yes]
|    |    |    |    |    | A3:Is_smiling [13]
|    |    |    |    |    | [yes]
|    |    |    |    |    |    | V2-no           [6]
|    |    |    |    |    | [no]
|    |    |    |    |    |    | A2:Body_shape [7]
|    |    |    |    |    |    | [square, octagon]
|    |    |    |    |    |    |    | V1-yes          [4]
|    |    |    |    |    |    | [round]
|    |    |    |    |    |    |    | V2-no           [3]
| [balloon, flag]
|    | A5:Jacket_color [115]
|    | [green, blue]
|    |    | A3:Is_smiling [58]
|    |    | [yes]
|    |    |    | A6:Has_tie [31]
|    |    |    | [yes]
|    |    |    |    | A1:Head_shape [17]
|    |    |    |    | [round]
|    |    |    |    |    | V2-no           [9]
|    |    |    |    | [square, octagon]
|    |    |    |    |    | A2:Body_shape [8]
|    |    |    |    |    | [round]
|    |    |    |    |    |    | V2-no           [2]
|    |    |    |    |    | [square, octagon]
|    |    |    |    |    |    | V1-yes          [6]
|    |    |    | [no]
|    |    |    |    | A1:Head_shape [14]
|    |    |    |    | [round]
|    |    |    |    |    | V1-yes          [5]
|    |    |    |    | [square, octagon]
|    |    |    |    |    | A2:Body_shape [9]
|    |    |    |    |    | [round]
|    |    |    |    |    |    | V1-yes          [4]
|    |    |    |    |    | [square, octagon]
|    |    |    |    |    |    | V2-no           [5]
|    |    | [no]
|    |    |    | A1:Head_shape [27]
|    |    |    | [round]
```

```
|    |    |    |    | A2:Body_shape [11]
|    |    |    |    | [round, square]
|    |    |    |    |    | A2:Body_shape [9]
|    |    |    |    |    | [square]
|    |    |    |    |    |    | A6:Has_tie [6]
|    |    |    |    |    |    | [yes]
|    |    |    |    |    |    |    | V1-yes          [4]
|    |    |    |    |    |    | [no]
|    |    |    |    |    |    |    | V2-no           [2]
|    |    |    |    |    | [round]
|    |    |    |    |    |    | A6:Has_tie [3]
|    |    |    |    |    |    | [yes]
|    |    |    |    |    |    |    | V2-no           [2]
|    |    |    |    |    |    | [no]
|    |    |    |    |    |    |    | V1-yes          [1]
|    |    |    |    | [octagon]
|    |    |    |    |    | V2-no           [2]
|    |    |    | [square, octagon]
|    |    |    |    | V2-no           [16]
|    | [red, yellow]
|    |    | A2:Body_shape [57]
|    |    | [round, square]
|    |    |    | A1:Head_shape [43]
|    |    |    | [round]
|    |    |    |    | A6:Has_tie [13]
|    |    |    |    | [yes]
|    |    |    |    |    | V2-no           [5]
|    |    |    |    | [no]
|    |    |    |    |    | A5:Jacket_color [8]
|    |    |    |    |    | [red]
|    |    |    |    |    |    | A3:Is_smiling [5]
|    |    |    |    |    |    | [yes]
|    |    |    |    |    |    |    | V2-no           [3]
|    |    |    |    |    |    | [no]
|    |    |    |    |    |    |    | A2:Body_shape [2]
|    |    |    |    |    |    |    | [round]
|    |    |    |    |    |    |    |    | V2-no           [1]
|    |    |    |    |    |    |    | [square]
|    |    |    |    |    |    |    |    | V1-yes          [1]
|    |    |    |    |    | [yellow]
|    |    |    |    |    |    | V1-yes          [3]
|    |    |    | [square, octagon]
|    |    |    |    | A4:Holding [30]
|    |    |    |    | [balloon]
|    |    |    |    |    | A2:Body_shape [13]
|    |    |    |    |    | [round]
|    |    |    |    |    |    | A5:Jacket_color [5]
|    |    |    |    |    |    | [red]
|    |    |    |    |    |    |    | V2-no           [1]
|    |    |    |    |    |    | [yellow]
|    |    |    |    |    |    |    | V1-yes          [4]
```

```
|    |    |    |    |    | [square]
|    |    |    |    |    |    | A5:Jacket_color [8]
|    |    |    |    |    |    | [red]
|    |    |    |    |    |    |    | A3:Is_smiling [4]
|    |    |    |    |    |    |    | [yes]
|    |    |    |    |    |    |    |    | A1:Head_shape [2]
|    |    |    |    |    |    |    |    | [square]
|    |    |    |    |    |    |    |    |    | V2-no           [1]
|    |    |    |    |    |    |    |    | [octagon]
|    |    |    |    |    |    |    |    |    | V1-yes          [1]
|    |    |    |    |    |    |    | [no]
|    |    |    |    |    |    |    |    | V1-yes          [2]
|    |    |    |    |    |    | [yellow]
|    |    |    |    |    |    |    | A3:Is_smiling [4]
|    |    |    |    |    |    |    | [yes]
|    |    |    |    |    |    |    |    | V1-yes          [1]
|    |    |    |    |    |    |    | [no]
|    |    |    |    |    |    |    |    | V2-no           [3]
|    |    |    |    | [flag]
|    |    |    |    |    | A1:Head_shape [17]
|    |    |    |    |    | [square]
|    |    |    |    |    |    | A5:Jacket_color [8]
|    |    |    |    |    |    | [red]
|    |    |    |    |    |    |    | A3:Is_smiling [4]
|    |    |    |    |    |    |    | [yes]
|    |    |    |    |    |    |    |    | V2-no           [1]
|    |    |    |    |    |    |    | [no]
|    |    |    |    |    |    |    |    | A2:Body_shape [3]
|    |    |    |    |    |    |    |    | [round]
|    |    |    |    |    |    |    |    |    | A6:Has_tie [2]
|    |    |    |    |    |    |    |    |    | [yes]
|    |    |    |    |    |    |    |    |    |    | V2-no           [1]
|    |    |    |    |    |    |    |    |    | [no]
|    |    |    |    |    |    |    |    |    |    | V1-yes          [1]
|    |    |    |    |    |    |    |    | [square]
|    |    |    |    |    |    |    |    |    | V1-yes          [1]
|    |    |    |    |    |    | [yellow]
|    |    |    |    |    |    |    | A3:Is_smiling [4]
|    |    |    |    |    |    |    | [yes]
|    |    |    |    |    |    |    |    | V1-yes          [1]
|    |    |    |    |    |    |    | [no]
|    |    |    |    |    |    |    |    | V2-no           [3]
|    |    |    |    |    | [octagon]
|    |    |    |    |    |    | A3:Is_smiling [9]
|    |    |    |    |    |    | [yes]
|    |    |    |    |    |    |    | A5:Jacket_color [5]
|    |    |    |    |    |    |    | [red]
|    |    |    |    |    |    |    |    | V2-no           [3]
|    |    |    |    |    |    |    | [yellow]
|    |    |    |    |    |    |    |    | V1-yes          [2]
|    |    |    |    |    |    | [no]
```

```
|    |    |    |    |    |    |    | A5:Jacket_color [4]
|    |    |    |    |    |    |    | [yellow]
|    |    |    |    |    |    |    |    | A6:Has_tie [2]
|    |    |    |    |    |    |    |    | [yes]
|    |    |    |    |    |    |    |    |    | V1-yes          [1]
|    |    |    |    |    |    |    |    | [no]
|    |    |    |    |    |    |    |    |    | V2-no           [1]
|    |    |    |    |    |    |    | [red]
|    |    |    |    |    |    |    |    | V1-yes          [2]
|    |    | [octagon]
|    |    |    | A6:Has_tie [14]
|    |    |    | [yes]
|    |    |    |    | A4:Holding [10]
|    |    |    |    | [balloon]
|    |    |    |    |    | A5:Jacket_color [6]
|    |    |    |    |    | [yellow]
|    |    |    |    |    |    | V2-no           [3]
|    |    |    |    |    | [red]
|    |    |    |    |    |    | A3:Is_smiling [3]
|    |    |    |    |    |    | [yes]
|    |    |    |    |    |    |    | V2-no           [1]
|    |    |    |    |    |    | [no]
|    |    |    |    |    |    |    | V1-yes          [2]
|    |    |    |    | [flag]
|    |    |    |    |    | A1:Head_shape [4]
|    |    |    |    |    | [round, square]
|    |    |    |    |    |    | V1-yes          [2]
|    |    |    |    |    | [octagon]
|    |    |    |    |    |    | A5:Jacket_color [2]
|    |    |    |    |    |    | [yellow]
|    |    |    |    |    |    |    | V2-no           [1]
|    |    |    |    |    |    | [red]
|    |    |    |    |    |    |    | V1-yes          [1]
|    |    |    | [no]
|    |    |    |    | V1-yes          [4]
```

Decision Tree From Domain: THIRD
Pruned with m= 3.00

Number of Nodes : 9
Number of Leaves: 5
Number of Nulls : 0

```
| A2:Body_shape [122]
| [octagon]
|     | A4:Holding [41]
|     | [sword]
|     |     | A5:Jacket_color [14]
|     |     | [green]
|     |     |     | V1-yes          [2]
|     |     | [red, yellow, blue]
|     |     |     | V1-yes          [1]
|     |     |     | V2-no           [11]
|     | [balloon, flag]
|     |     | V2-no           [27]
| [round, square]
|     | A5:Jacket_color [81]
|     | [blue]
|     |     | V2-no           [19]
|     | [red, yellow, green]
|     |     | V1-yes          [57]
|     |     | V2-no           [5]
```

Chapter 4

mFOIL on the MONK's Problems

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4.0.1 Description

The learning system considered in this summary is named mFOIL and learns prolog clauses. The basic structure of mFOIL is similar to FOIL (Quinlan 1990), but the search heuristics and stopping criteria employed are different. They are adapted to learning from imperfect (noisy) data. Instead of the entropy (information gain) heuristic, error estimates such as Laplacian and m-estimate (Cest- nik 1990) are used as search heuristics. The system is described in my masters thesis which will be available by the end of Sept. (Dzeroski 1991) mFOIL is implemented in Quintus Prolog 2.5.1 on Sun SPARC Station 1 (cca. 600 lines of code)

I run the system with different search heuristics (Laplacian or m-estimate with different values of m: higher values of m direct the search towards and allow only more 'reliable' clauses, i.e., clauses that cover more examples). This did not influence the results on the first training set, but had some effect on the results on the second and the third set. Below are given the rules obtained together with the corresponding search heuristics

The bad results on the second set are due to the small number of examples for each of the disjuncts and the bias in mFOIL which favors shorter rules.

References

Quinlan (1990) Learning logical definitions from relations. ML journal. Dzeroski (1991) Handling noise in Inductive Logic Programming. MSc Thesis. Dept. of EE & CS, University of Ljubljana, Slovenia, Yugoslavia. Cestnik (1990) Estimating probabilities: A crucial task in ML. ECAI 90

4.1 Set 1

Heuristics used in mFOIL: Laplace, m=0, 0.01, 0.5, 1, 2, 3, 4, 8, 16, 32, 64
Induction time: cca 1 min
Accuracy: 100 %

```
robot(A,B,C,D,E,F) :-
        true,
        A=B.
robot(A,B,C,D,E,F) :-
        true,
        E=red.
```

4.2 Set 2

Heuristic used in mFOIL: m=3
Induction time: cca 10 min
Accuracy: 69.21 %

```
robot(A,B,C,D,E,F) :-
        (((true,
        E=yellow),
        not C=no),
        not D=sword),
        F=no.
robot(A,B,C,D,E,F) :-
        (((true,
        D=flag),
        B=octagon),
        C=yes),
        not E=green.
robot(A,B,C,D,E,F) :-
        ((((true,
        C=no),
        E=red),
        not D=sword),
        not B=round),
        not A=round.
robot(A,B,C,D,E,F) :-
        (((true,
        E=yellow),
```

```
        B=round),
        not C=yes),
        not D=flag.
robot(A,B,C,D,E,F) :-
        ((true,
        B=square),
        C=yes),
        E=yellow.
robot(A,B,C,D,E,F) :-
        (((true,
        E=green),
        B=round),
        not F=yes),
        not A=square.
robot(A,B,C,D,E,F) :-
        ((((true,
        E=green),
        not C=no),
        F=no),
        A=round),
        not D=sword.
robot(A,B,C,D,E,F) :-
        (((true,
        B=square),
        E=blue),
        C=yes),
        not A=round.
robot(A,B,C,D,E,F) :-
        (((true,
        not C=yes),
        A=round),
        E=yellow),
        not D=sword.
robot(A,B,C,D,E,F) :-
        ((((true,
        E=green),
        D=sword),
        F=no),
        C=yes),
        not A=round.
robot(A,B,C,D,E,F) :-
        ((((true,
        E=green),
        not F=no),
        B=square),
        not C=yes),
        not A=square.
robot(A,B,C,D,E,F) :-
        (((true,
        not C=yes),
        E=red),
```

```
        F=no),
        not A=round.
robot(A,B,C,D,E,F) :-
        ((((true,
        E=green),
        A=square),
        not C=no),
        not D=sword),
        not F=no.
robot(A,B,C,D,E,F) :-
        ((((true,
        E=blue),
        B=square),
        not F=no),
        not C=yes),
        not D=sword.
robot(A,B,C,D,E,F) :-
        (((true,
        E=blue),
        F=no),
        not C=no),
        not A=square.
robot(A,B,C,D,E,F) :-
        (((true,
        F=no),
        E=red),
        not C=yes),
        not B=round.
robot(A,B,C,D,E,F) :-
        ((((true,
        D=sword),
        C=no),
        B=octagon),
        A=square),
        F=yes.
robot(A,B,C,D,E,F) :-
        ((((true,
        B=round),
        F=no),
        E=blue),
        not D=flag),
        not A=square.
robot(A,B,C,D,E,F) :-
        ((((true,
        A=octagon),
        D=flag),
        not F=no),
        not E=red),
        not B=octagon.
```

4.3 Set 3

Heuristic used in mFOIL: m=64
Induction time: cca 1 min
Accuracy: 100%

```
robot(A,B,C,D,E,F) :-
        (true,
        not B=octagon),
        not E=blue.
robot(A,B,C,D,E,F) :-
        ((true,
        E=green),
        D=sword),
        B=octagon.
```

Chapter 5

Comparison of Decision Tree-Based Learning Algorithms on the MONK's Problems

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5.1 IDL: A Brief Introduction

5.1.1 Introduction

IDL is an algorithm for the incremental induction of decision trees. Incremental learning methods are useful when examples become available on a regular basis but good hypotheses are needed anytime, possibly for a performance task. Incrementality is, however, not the primary motivation for this research. More importantly, IDL is specifically designed to find small decision trees. There are various reasons to prefer smaller trees. One reason is efficiency: the fewer decision nodes in a tree, the more efficient an instance can be classified with it. This is, however, a weak argument since cost and frequency of test execution should be taken into account, so that the most cost-effective tree is not necessarily also the smallest one [Nunez 88; Tan and Schlimmer 89]. Another reason to prefer small trees is comprehensibility: small trees tend to be easier to understand. Comprehensibility, however, also depends on the form of the tree. For example Arbab and Michy (85) argue that linear trees are easier to understand. Perhaps the strongest argument for small trees is the relation between tree complexity and classification accuracy [Breiman, Friedman, Olshen and Stone 84; Quinlan 86; Mingers 89a,b; Utgoff 90]. Pearl [78] showed that the complexity of a hypothesis for explaining data is related to the likelihood that it actually explains it. A learning algorithm with a bias towards simplicity is likely to find more accurate hypotheses as well. This heuristic of Occam's Razor has been employed and justified by many authors both empirically [Clark and Niblett 89; Fisher and Schlimmer 88; Iba, Wogulis and Langley 88] and theoretically [Blumer, Ehrenfeucht, Haussler and Warmuth 87].

Complex trees are sometimes unavoidable. For example, an accurate tree for a concept exhibiting the parity problemII has an exponential number of nodes [Seshu 89] and trees for boolean disjunctive normal form concepts contain duplicated subtrees when only using ground attributes as tests [Pagallo and Haussler 89]. Also, different heuristics in otherwise similar algorithms may lead to significant variations in tree size [Mingers 89a]. The induced trees may nonetheless be more complex than strictly necessary. For example, finding the smallest trees for the six-multiplexer concept [Barto 85; Wilson 87] is well known to be far beyond all classical decision tree induction algorithms [Quinlan 88]. So, even when a small tree exists, state of the art decision tree algorithms may fail to find, or even come close to it. IDL on the other hand finds small trees which are often optimal in size. For example, it has no problem inducing a best tree for the 6-multiplexer while requiring fewer examples and less computation than the other algorithms. The problem of induding optimal decision trees is, however, NP-hard [Hyafil and Rivest 76; Hancock 89]. A practical algorithm is necessarily based on strong heuristic guidance and is guaranteed to fail on at least some induction tasks.

To appreciate the novelty of the approach taken in IDL, it is useful to take a look at the relationship with its predecessors, non-incremental top-down induction of decision trees like ID3 [Quinlan 83,86] and the incremental algorithms ID4 [Schlimmer and Fisher 86], ID5 [Utgoff 88a] and ID5R [Utgoff 90]. Top-down induction performs a general-to-specific hill-climbing search, guided by statistical heuristics and without backtracking. The incremental versions, for which a statistics-based best split is always tentative, are designed to recover with minimal loss of training effort from deviations from the search path which ID3 would follow given the same examples E. More sophisticated representations and search operators allow these algorithms to simulate a backtracking top-down search in a hill climbing search [Langley, Gennari and Iba 87; Fisher 87]. However, these algorithms do not contribute any new ideas to improve the complexity or accuracy of learned decision trees. IDL uses the same search operators to construct a small and accurate tree which is not necessarily ID3-equivalent but topologically minimal. In a topologically minimal tree only a minimal number of tests is required to classify objects. IDL is guided by statistics in a top-down search for an accurate tree. At the same time it looks for smaller trees in a bottom-up fashion. Here it is guided, not by statistics, but by tree topological considerations. In effect, IDL simulates a bi-directional search.

5.1.2 Related Work

ID4 [Schlimmer & Fisher 1986], ID5 [Utgoff 88a] and ID5R [Utgoff 89] are three recently developed algorithms for incremental induction of decision trees. The relation with IDL was briefly explained in the introduction. In [Van de Velde 89] it was conjectured that IDL finds a topologically minimal tree if it exists. Elomaa and Kivinen [90] showed, however, how IDL may fail to find the optimal tree for the 3-multiplexer. The multi-multiplexer concept also disproves this conjecture. Their algorithm IDL' nevertheless successfully postprocesses trees and removes irrelevant attributes. Related experiments are reported on in [Van de Velde 90]. These experiments use a version of IDL which is more eager to apply the statistical selection criterion. This has the advantage that any consistent tree can be taken as an initial hypothesis, no matter how it was generated.

Others have explicitly addressed the problem of suboptimality in tree-size. Pruning techniques [Quinlan 87; Fisher and Schlimmer 88; Mingers 89b] avoid overfitting and reduce complexity, often while increasing accuracy. In a multiplexer-like concept the problem occurs at the top: a TDIDT-like algorithm will choose a wrong top-level attribute and there is no way to prune this away. Quinlan [88] proposes to transform a tree into a set of rules which are subsequently simplified. Every possible classification path is interpreted as a rule. Each of the conditions in the rule is removed in turn and classification accuracy of the rule set is tested. If this is improved, then the condition is permanently removed. This process has been shown to be capable of strong optimization at the expense of introducing a different representation. More sophisticated rule simplification techniques have been studied by many authors [Michalski 87; Clark and Niblett 89; Zhang and Michalski 89]. They use statistical measures to balance the importance and typicality of patterns. The techniques of pruning, tree transformation, and rule tweaking can be viewed along a continuum of increasing liberty to manipulate the representation of patterns. IDL is somewhere in the middle: it manipulates several rules at once and is capable of both introducing and deleting tests in a rule. Also note that IDL is incremental, is not motivated by noise, works with one representation, and uses tree structure information in addition to statistics.

Other researchers reduce tree complexity by allowing different tests than the primitive ones, for example boolean combinations [Breiman, Friedman, Olhsen, Stone 84; Clark and Niblett 89; Pagallo and Haussler 89; Seshu 89] or linear threshold units [Utgoff 88b; Utgoff and Brodley 90]. Of these, FRINGE [Pagallo and Haussler 89] is closest in spirit to IDL. It was developed to overcome the problem of replicated subtrees when learning Disjunctive Normal Form concepts. Such concepts usually have no decision tree representation without replications when the primitive attributes are used. FRINGE examines the fringe (2 bottom levels) of a complete tree to find replicated partial paths. The conjunction of two attributes or their negation is added as first class attribute and a new tree is built. This process iterates until no more changes occur. In comparison, note that IDL is incremental, does not change representation bias and tackles the replication problem for concepts which do have a representation without replication. Utgoff and Brodley's method [90] is also incremental.

Wilson [87] used multiplexer concepts to test his classifier system, called Boole. Quinlan [88] noted the extremely slow convergence rate and obtains much better results when using C4, a TDIDT like algorithm, and postprocessing to rules (see above). Bonelli, Parodi, Sen and Wilson [90] describe NewBoole, a new version of Boole which converges significantly faster to accurate results. It still requires around 800 examples to find an (almost) accurate hypothesis, and around 5000 examples to find the minimal set of rules. The same authors also used neural nets of different sizes to learn the same concept. They report convergence after 1600 cycles for a reasonable net (6:20-20-10-10:1). On the 11-multiplexer NewBoole requires around 4000 examples to converge, a neural net around 8000.

Selective training goes back to the windowing technique in ID3 [Quinlan 83]. Wirth and Catlett [88] discuss related techniques and note that the benefit of windowing is limited. Utgoff [89] shows that a window size of one (i.e., ID5R-hat) results in improved training. The idea is not really applicable in IDL, because it still does much work after the tree has become fully accurate.

5.1.3 Conclusion

IDL represents a new approach to the incremental induction of decision trees. It uses a similar representation as ID4 [Schlimmer and Fisher 86] and the same set of search operators, (splitting, pruning and transposition) as ID5(R) [Utgoff 88a,90]. It was argued that a decision tree represents a target concept by virtue of representing a specialization of it. The task of induction is to find a tree such that this specialization is as close as possible to the target concept. Search for a good decision tree can be understood as search in concept space, mediated by decision tree manipulations. The role of the three operations was reconsidered, as well as the heuristics to guide their application. A statistical selection measure, based on a metric on concept space [Lopez de Mantaras 90] is used to guide the expansion of a tree. Tree topological considerations, based on a notion of topological relevance, guide the transposition of nodes to generate oportunities for pruning. IDL uses these heuristics to simulate a bi-directional search for a tree which is topologically minimal. Such a tree minimizes the number of tests needed for classification, and is therefore small. Experiments show that IDL finds small trees, and often optimal ones.

A number of things need to be investigated further. A major open issue is to characterize the concepts for which IDL finds a topologically minimal tree. It is not understood, for example, what makes the 3-multiplexer so different from the 6-multiplexer concept to justify the occasional failure of IDL on the former. Also, the large standard deviations on the mushroom domain are not well understood. It is disappointing that IDL could not find drastically better trees on natural domains, like it did for the multiplexers. Are there no natural data sets for multiplexer-like concepts? Since IDL occasionally fails to find an optimal tree an average case analysis, as outlined by Pazanni and Sarrett [90] would be more useful than a worst-case one. Integration of IDL with constructive induction techniques seems a promising line of research. Situations in which IDL keeps on switching the levels of attributes could be used as an indication that a new attribute may be useful. The behavior of IDL in the presence of noise has not been studied. The integration of techniques developed for top-down algorithms [Mingers 89b] should be investigated.

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5.2 Experimental Results

I have done some of the experiments for the comparison of the algorithms. The runs on the first data-set are complete, except for the timing information. The runs for the second example are in progress and I will send them later today. I will not do the third example since I surender to noise. Nevertheless I think you will agree that in the class of decision tree algorithms, the performance of IDL is quite impressive.

Here is what I did. I ran several algorithms on the training-set and tested them on the test-set. If the algorrithm is non-incremental I used a run on the complete training set. If the algorithm is incremental I ran it with 500 examples randomly selected from the training set. Testing is always on the full test set. All results are averaged over 10 runs.

I used the following algorithms:

```
TDIDT:  plain old ID3 with information gain as selection measure,
no pruning.
ID5R:  the incremental version of ID3 produced by Utgoff. Information
gain is the selection measure. No pruning.
IDL: IDL as described in an unpublished paper, very similar to the
algorithm described in IML-90
ID5R-hat: ID5R with example filter. Trains only if the example is
misclassified by the current hypothsis. No pruning.
```

I send the results in several files. In seperate mails I will provide the following information:

```
TDIDT: the tree
size and accuracy of the tree
the concept described by it

ID5R, IDL, ID5R-hat:
data on size and accuracy as it evolves with training
a typical tree and its size and accuracy
the concept described by that typical tree
```

The evolving data for the incremental algorithms allow to produce the learning curves for each of the algorithms. I produced graphs with Exel and will send them by mail if I do not succeed making a postscript version of it.

About the results:

IDL is clearly the best. It produces the smallest trees with by far the best accuracy of all. It is also worth noticing that the standard deviations for IDL are very small, and that the concepts described by the trees that IDL produces are the same. This means that search in concept space is finished, but IDL can not decide on the best representation. So it limit-cycles between 3 different trees, all small and equally accurate (the only difference is in the order of testing the three relevant attributer). This illustrates how the use of not only statistical information bt also tree-topological one makes the algorithm unsensitive to sampling differences (small disjuncts or sparse sampling are no big problem either). Here are the data for all 10 trees to show this:

MONKS-1	IDL used	IDL nodes	IDL leaves	IDL accuracy
500	500	42	29	97.22222
500	500	36	26	97.22222
500	500	42	29	97.22222
500	500	40	27	97.22222
500	500	40	27	97.22222
500	500	40	28	97.22222
500	500	36	26	97.22222
500	500	42	29	97.22222
500	500	40	27	97.22222
500	500	42	29	97.22222

On the other hand ID5R produces larger and less accurate trees with enormous standard deviations as shown by data for the 10 trees that ID5R produces:

MONKS-1	ID5R used	ID5R nodes	ID5R leaves	ID5R accuracy
500	500	75	48	81.94444
500	500	64	40	81.71296
500	500	50	32	90.97222
500	500	61	40	87.73148
500	500	70	43	77.31481
500	500	40	27	97.22222
500	500	73	45	77.546295
500	500	78	50	84.02778
500	500	74	46	80.32407
500	500	59	37	86.34259

As expected ID5R-hat does somewhat better than ID5R. Here are the data for the 10 trees to give an idea of the deviations.

MONKS-1	ID5R-hat used	ID5R-hat nodes	ID5R-hat leaves	ID5R-hat accuracy
500	51	56	36	85.416664
500	62	68	43	79.861115
500	52	40	27	97.22222
500	49	40	27	97.22222
500	53	40	27	97.22222
500	52	51	33	92.361115
500	50	39	26	94.44444
500	48	40	27	97.22222

```
500 58 49 32 90.27778
500 53 40 27 97.22222
```

I sent a number of files with the results of TDIDT, IDL, ID5R and ID5R-hat on the second monk's concept. The results are averaged only over 5 runs this time.

The effect I seem to get is that IDL does not get beyond its initial phase of building up a large tree. In other words, it does not get anyway near to collapsing it. The fact that it grows larger than for ID5R is not anomalous, but normally this is followed by a rapid collaps to a smaller form (see MONKS-1 this effect). This concept seems to be too difficult for trees to handle anyway...

Here are the 5 individual results for IDL:

```
MONKS-2 IDL used IDL nodes IDL leaves IDL accuracy
500 500 176 111 74.30556
500 500 170 104 65.046295
500 500 180 114 73.84259
500 500 197 112 68.05556
500 500 184 111 61.34259
```

Here are the 5 individual results for ID5R:

```
MONKS-2 ID5R used ID5R nodes ID5R leaves ID5R accuracy
500 500 145 93 64.12037
500 500 153 91 64.583336
500 500 173 104 65.74074
500 500 171 102 65.27778
500 500 165 95 61.805557
```

Here are the 5 individual results for ID5R-hat:

```
MONKS-2 ID5R-HAT used ID5R-HAT nodes ID5R-HAT leaves ID5R-HAT accuracy
500 113 130 77 63.425926
500 115 131 82 65.74074
500 118 133 80 64.81481
500 120 133 84 62.5
500 115 138 83 62.73148
```

IDL finds larger trees, slightly more accurate. ID5R and ID5R-HAT find trees that are comparable in accuracy to the TDIDT tree (66.666664smaller.

5.2.1 ID5R on test set 1

```
DESCRIPTION OF THE TREE:

;; Typical tree found by id5r trained on first monks's training set
;; 500 examples (random from full training set)
;; 64 nodes
;; 40 leaves
;; 81.71296 accuracy on test set


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```

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     BODY_SHAPE = 1 : <0>...
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     BODY_SHAPE = 3 : <0>...
  HEAD_SHAPE = 3 :
     BODY_SHAPE = 2 : <0>...
     BODY_SHAPE = 3 : <1>...
```

5.2.2 IDL on test set 1

```
DESCRIPTION OF THE TREE:

;; Typical tree found by idl trained on first monks's training set
;; 500 examples (random from full training set)
;; 36 nodes
;; 26 leaves
;; 97.22222 accuracy on test set

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      JACKET_COLOR = 3 : <0>...
      JACKET_COLOR = 4 : <0>...
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```

5.2.3 ID5R-HAT on test set 1

```
DESCRIPTION OF THE TREE:

;; Tree found by id5r-hat trained on first monks's training set
;; 58 examples used out of 500 (random from full training set)
;; 49 nodes
;; 32 leaves
;; 90.27778 accuracy on test set


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```

5.2.4 TDIDT on test set 1

```
DESCRIPTION OF THE TREE:

;; Tree found by tdidt trained on first monks's training set
;; 124 examples (full training set)
;; 86 nodes
;; 52 leaves
;; 75.69444 accuracy on test set

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       IS_SMILING = 2 :
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    HOLDING = 3 : <0>
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```

5.2.5 ID5R on test set 2

```
DESCRIPTION OF THE TREE:

;; Typical tree found by id5r trained on second monks's training set
;; 500 examples (random from full training set)
;; 165 nodes
;; 95 leaves
;; 61.805557 accuracy on test set


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```

```
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            IS_SMILING = 2 : <1>...
         HOLDING = 2 : <1>...
         HOLDING = 3 :
            IS_SMILING = 1 : <1>...
            IS_SMILING = 2 : <0>...
      HEAD_SHAPE = 3 :
         HOLDING = 1 : <1>...
         HOLDING = 2 : <1>...
         HOLDING = 3 :
            IS_SMILING = 1 : <1>
            IS_SMILING = 2 :
               HAS_TIE = 1 : <1>
               HAS_TIE = 2 : <0>
   BODY_SHAPE = 2 :
      IS_SMILING = 1 : <1>...
      IS_SMILING = 2 : <0>...
   BODY_SHAPE = 3 :
      HEAD_SHAPE = 1 :
         HOLDING = 2 : <0>...
         HOLDING = 3 : <1>...
      HEAD_SHAPE = 2 :
         HOLDING = 1 : <1>...
         HOLDING = 2 : <0>...
         HOLDING = 3 : <1>...
      HEAD_SHAPE = 3 : <0>...
JACKET_COLOR = 3 :
   HEAD_SHAPE = 1 :
      BODY_SHAPE = 1 : <0>...
      BODY_SHAPE = 2 :
         HOLDING = 2 :
            HAS_TIE = 1 :
               IS_SMILING = 1 : <0>
               IS_SMILING = 2 : <1>
            HAS_TIE = 2 : <1>...
         HOLDING = 3 :
            IS_SMILING = 1 : <0>...
            IS_SMILING = 2 :
               HAS_TIE = 1 : <1>
               HAS_TIE = 2 : <0>
      BODY_SHAPE = 3 :
         HAS_TIE = 1 : <0>...
         HAS_TIE = 2 :
            HOLDING = 1 :
               IS_SMILING = 1 : <0>
               IS_SMILING = 2 : <1>
            HOLDING = 2 : <0>...
   HEAD_SHAPE = 2 :
```

```
      IS_SMILING = 1 :
         BODY_SHAPE = 1 :
            HAS_TIE = 1 : <0>
            HAS_TIE = 2 : <1>...
         BODY_SHAPE = 2 :
            HOLDING = 1 : <0>...
            HOLDING = 2 : <1>...
            HOLDING = 3 :
               HAS_TIE = 1 : <1>
               HAS_TIE = 2 : <0>
         BODY_SHAPE = 3 :
            HOLDING = 1 : <1>...
            HOLDING = 2 :
               HAS_TIE = 1 : <1>
               HAS_TIE = 2 : <0>
      IS_SMILING = 2 :
         HOLDING = 1 :
            HAS_TIE = 1 : <1>...
            HAS_TIE = 2 : <0>...
         HOLDING = 2 : <0>...
         HOLDING = 3 : <0>...
   HEAD_SHAPE = 3 :
      HOLDING = 1 :
         HAS_TIE = 1 :
            BODY_SHAPE = 1 : <0>...
            BODY_SHAPE = 2 :
               IS_SMILING = 1 : <0>
               IS_SMILING = 2 : <1>
            BODY_SHAPE = 3 : <0>...
         HAS_TIE = 2 :
            IS_SMILING = 1 : <1>...
            IS_SMILING = 2 :
               BODY_SHAPE = 1 : <1>
               BODY_SHAPE = 2 : <0>
      HOLDING = 2 :
         BODY_SHAPE = 1 : <1>...
         BODY_SHAPE = 2 : <0>...
         BODY_SHAPE = 3 : <0>...
      HOLDING = 3 :
         HAS_TIE = 1 : <1>...
         HAS_TIE = 2 :
            BODY_SHAPE = 1 : <1>...
            BODY_SHAPE = 2 : <0>...
JACKET_COLOR = 4 :
   BODY_SHAPE = 1 :
      HAS_TIE = 1 : <0>...
      HAS_TIE = 2 :
         HEAD_SHAPE = 1 : <1>...
         HEAD_SHAPE = 2 : <0>...
         HEAD_SHAPE = 3 :
            IS_SMILING = 1 : <1>...
```

```
         IS_SMILING = 2 : <0>...
BODY_SHAPE = 2 :
   HOLDING = 1 :
      IS_SMILING = 1 : <1>...
      IS_SMILING = 2 :
         HEAD_SHAPE = 1 : <0>...
         HEAD_SHAPE = 2 :
            HAS_TIE = 1 : <1>
            HAS_TIE = 2 : <0>
   HOLDING = 2 : <1>...
   HOLDING = 3 :
      HAS_TIE = 1 :
         IS_SMILING = 1 :
            HEAD_SHAPE = 1 : <0>
            HEAD_SHAPE = 2 : <1>
         IS_SMILING = 2 : <1>...
      HAS_TIE = 2 :
         IS_SMILING = 1 : <1>...
         IS_SMILING = 2 : <0>...
BODY_SHAPE = 3 :
   IS_SMILING = 1 :
      HOLDING = 2 : <0>...
      HOLDING = 3 : <1>...
   IS_SMILING = 2 : <0>...
```

5.2.6 IDL on test set 2

```
DESCRIPTION OF THE TREE:

;; Typical tree found by idl trained on second monks's training set
;; 500 examples (random from full training set)
;; 170 nodes
;; 107 leaves
;; 66.203705 accuracy on test set


IS_SMILING = 1 :
   HAS_TIE = 1 :
      JACKET_COLOR = 1 : <0>...
      JACKET_COLOR = 2 :
         BODY_SHAPE = 1 : <0>...
         BODY_SHAPE = 2 : <1>...
         BODY_SHAPE = 3 :
            HEAD_SHAPE = 1 : <0>...
            HEAD_SHAPE = 2 : <1>...
            HEAD_SHAPE = 3 : <0>...
      JACKET_COLOR = 3 :
         BODY_SHAPE = 1 : <0>...
         BODY_SHAPE = 2 :
            HEAD_SHAPE = 1 : <0>...
            HEAD_SHAPE = 2 :
               HOLDING = 1 : <0>
               HOLDING = 2 : <1>
               HOLDING = 3 : <1>
            HEAD_SHAPE = 3 :
               HOLDING = 1 : <0>
               HOLDING = 3 : <1>
         BODY_SHAPE = 3 :
            HEAD_SHAPE = 1 : <0>...
            HEAD_SHAPE = 2 : <1>...
            HEAD_SHAPE = 3 : <0>...
      JACKET_COLOR = 4 :
         BODY_SHAPE = 1 : <0>...
         BODY_SHAPE = 2 :
            HEAD_SHAPE = 1 : <0>...
            HEAD_SHAPE = 2 : <1>...
         BODY_SHAPE = 3 :
            HEAD_SHAPE = 1 : <0>...
            HEAD_SHAPE = 2 : <1>...
   HAS_TIE = 2 :
      JACKET_COLOR = 1 :
         BODY_SHAPE = 1 : <0>...
         BODY_SHAPE = 2 :
            HEAD_SHAPE = 1 : <0>...
            HEAD_SHAPE = 3 :
```

```
               HOLDING = 1 : <0>
               HOLDING = 2 : <1>
         BODY_SHAPE = 3 :
            HEAD_SHAPE = 1 : <0>...
            HEAD_SHAPE = 2 :
               HOLDING = 1 : <0>
               HOLDING = 3 : <1>
      JACKET_COLOR = 2 :
         BODY_SHAPE = 1 :
            HEAD_SHAPE = 1 : <0>...
            HEAD_SHAPE = 2 :
               HOLDING = 1 : <0>
               HOLDING = 2 : <1>
               HOLDING = 3 : <1>
            HEAD_SHAPE = 3 : <1>...
         BODY_SHAPE = 2 : <1>...
         BODY_SHAPE = 3 :
            HOLDING = 1 : <0>...
            HOLDING = 2 : <1>...
            HOLDING = 3 : <1>...
      JACKET_COLOR = 3 :
         BODY_SHAPE = 1 : <1>...
         BODY_SHAPE = 2 :
            HEAD_SHAPE = 1 : <1>...
            HEAD_SHAPE = 2 : <0>...
            HEAD_SHAPE = 3 :
               HOLDING = 1 : <1>
               HOLDING = 3 : <0>
         BODY_SHAPE = 3 :
            HEAD_SHAPE = 1 :
               HOLDING = 1 : <0>
               HOLDING = 2 : <1>
            HEAD_SHAPE = 2 :
               HOLDING = 1 : <1>
               HOLDING = 2 : <0>
            HEAD_SHAPE = 3 :
               HOLDING = 1 : <1>
               HOLDING = 2 : <0>
      JACKET_COLOR = 4 :
         BODY_SHAPE = 1 : <1>...
         BODY_SHAPE = 2 : <1>...
         BODY_SHAPE = 3 :
            HEAD_SHAPE = 1 : <1>...
            HEAD_SHAPE = 2 : <0>...
IS_SMILING = 2 :
   BODY_SHAPE = 1 :
      HAS_TIE = 1 :
         JACKET_COLOR = 1 : <0>...
         JACKET_COLOR = 2 : <1>...
         JACKET_COLOR = 3 : <0>...
         JACKET_COLOR = 4 : <0>...
```

```
   HAS_TIE = 2 :
      JACKET_COLOR = 1 :
         HEAD_SHAPE = 1 : <0>...
         HEAD_SHAPE = 2 : <1>...
         HEAD_SHAPE = 3 : <1>...
      JACKET_COLOR = 2 :
         HEAD_SHAPE = 1 : <1>...
         HEAD_SHAPE = 2 :
            HOLDING = 1 : <1>
            HOLDING = 3 : <0>
         HEAD_SHAPE = 3 :
            HOLDING = 1 : <1>
            HOLDING = 3 : <0>
      JACKET_COLOR = 3 :
         HEAD_SHAPE = 2 : <0>...
         HEAD_SHAPE = 3 : <1>...
      JACKET_COLOR = 4 :
         HEAD_SHAPE = 1 : <1>...
         HEAD_SHAPE = 2 : <0>...
         HEAD_SHAPE = 3 :
            HOLDING = 1 : <1>
            HOLDING = 3 : <0>
BODY_SHAPE = 2 :
   HAS_TIE = 1 :
      JACKET_COLOR = 1 :
         HEAD_SHAPE = 1 : <0>...
         HEAD_SHAPE = 2 :
            HOLDING = 1 : <0>
            HOLDING = 2 : <1>
            HOLDING = 3 : <1>
         HEAD_SHAPE = 3 :
            HOLDING = 1 : <0>
            HOLDING = 3 : <1>
      JACKET_COLOR = 2 : <0>...
      JACKET_COLOR = 3 :
         HEAD_SHAPE = 1 : <1>...
         HEAD_SHAPE = 2 : <0>...
         HEAD_SHAPE = 3 : <1>...
      JACKET_COLOR = 4 :
         HEAD_SHAPE = 1 :
            HOLDING = 1 : <0>
            HOLDING = 2 : <1>
            HOLDING = 3 : <1>
         HEAD_SHAPE = 2 : <1>...
   HAS_TIE = 2 :
      JACKET_COLOR = 1 : <1>...
      JACKET_COLOR = 2 : <0>...
      JACKET_COLOR = 3 : <0>...
      JACKET_COLOR = 4 : <0>...
BODY_SHAPE = 3 :
   HAS_TIE = 1 :
```

```
   JACKET_COLOR = 1 :
      HEAD_SHAPE = 2 : <1>...
      HEAD_SHAPE = 3 :
         HOLDING = 1 : <0>
         HOLDING = 2 : <1>
         HOLDING = 3 : <1>
   JACKET_COLOR = 2 :
      HEAD_SHAPE = 1 : <1>...
      HEAD_SHAPE = 2 :
         HOLDING = 1 : <1>
         HOLDING = 2 : <0>
      HEAD_SHAPE = 3 : <0>...
   JACKET_COLOR = 3 :
      HEAD_SHAPE = 1 : <0>...
      HEAD_SHAPE = 2 :
         HOLDING = 1 : <1>
         HOLDING = 3 : <0>
      HEAD_SHAPE = 3 : <0>...
   JACKET_COLOR = 4 : <0>...
HAS_TIE = 2 :
   JACKET_COLOR = 1 : <1>...
   JACKET_COLOR = 3 :
      HEAD_SHAPE = 1 :
         HOLDING = 1 : <1>
         HOLDING = 2 : <0>
      HEAD_SHAPE = 2 : <0>...
      HEAD_SHAPE = 3 : <0>...
   JACKET_COLOR = 4 : <0>...
```

5.2.7 TDIDT on test set 2

```
DESCRIPTION OF THE TREE:

;; The tree found by TDIDT trained on second monks's training set
;; 169 examples (full training set)
;; 159 nodes
;; 95 leaves
;; 66.666664 accuracy on test set


JACKET_COLOR = 1 :
   IS_SMILING = 1 :
      HAS_TIE = 1 : <0>
      HAS_TIE = 2 :
         HEAD_SHAPE = 1 : <0>
         HEAD_SHAPE = 2 :
            HOLDING = 1 : <0>
            HOLDING = 3 :
               BODY_SHAPE = 1 : <0>
               BODY_SHAPE = 3 : <1>
         HEAD_SHAPE = 3 :
            HOLDING = 1 : <0>
            HOLDING = 2 :
               BODY_SHAPE = 1 : <0>
               BODY_SHAPE = 2 : <1>
            HOLDING = 3 : <0>
   IS_SMILING = 2 :
      HOLDING = 1 :
         HAS_TIE = 1 : <0>
         HAS_TIE = 2 :
            BODY_SHAPE = 1 : <0>
            BODY_SHAPE = 3 : <1>
      HOLDING = 2 : <1>
      HOLDING = 3 :
         HEAD_SHAPE = 1 :
            HAS_TIE = 1 : <0>
            HAS_TIE = 2 :
               BODY_SHAPE = 1 : <0>
               BODY_SHAPE = 2 : <1>
         HEAD_SHAPE = 2 :
            HAS_TIE = 1 :
               BODY_SHAPE = 1 : <0>
               BODY_SHAPE = 2 : <1>
            HAS_TIE = 2 : <1>
         HEAD_SHAPE = 3 : <1>
JACKET_COLOR = 2 :
   IS_SMILING = 1 :
      HOLDING = 1 :
```

```
        BODY_SHAPE = 1 : <0>
        BODY_SHAPE = 2 : <1>
        BODY_SHAPE = 3 : <0>
      HOLDING = 2 :
        HAS_TIE = 1 :
          BODY_SHAPE = 1 : <0>
          BODY_SHAPE = 2 : <1>
          BODY_SHAPE = 3 : <0>
        HAS_TIE = 2 : <1>
      HOLDING = 3 :
        HAS_TIE = 1 :
          BODY_SHAPE = 1 : <0>
          BODY_SHAPE = 2 : <1>
          BODY_SHAPE = 3 : <1>
        HAS_TIE = 2 : <1>
    IS_SMILING = 2 :
      BODY_SHAPE = 1 :
        HOLDING = 1 : <1>
        HOLDING = 2 : <1>
        HOLDING = 3 :
          HEAD_SHAPE = 1 : <1>
          HEAD_SHAPE = 2 : <0>
          HEAD_SHAPE = 3 :
            HAS_TIE = 1 : <1>
            HAS_TIE = 2 : <0>
      BODY_SHAPE = 2 : <0>
      BODY_SHAPE = 3 :
        HEAD_SHAPE = 1 : <1>
        HEAD_SHAPE = 2 :
          HOLDING = 1 : <1>
          HOLDING = 2 : <0>
        HEAD_SHAPE = 3 : <0>
JACKET_COLOR = 3 :
  IS_SMILING = 1 :
    HAS_TIE = 1 :
      HEAD_SHAPE = 1 : <0>
      HEAD_SHAPE = 2 :
        HOLDING = 1 : <0>
        HOLDING = 2 : <1>
        HOLDING = 3 : <1>
      HEAD_SHAPE = 3 :
        HOLDING = 1 : <0>
        HOLDING = 3 : <1>
    HAS_TIE = 2 :
      BODY_SHAPE = 1 : <1>
      BODY_SHAPE = 2 :
        HEAD_SHAPE = 1 : <1>
        HEAD_SHAPE = 2 : <0>
        HEAD_SHAPE = 3 :
          HOLDING = 1 : <1>
          HOLDING = 3 : <0>
```

```
        BODY_SHAPE = 3 :
           HOLDING = 1 :
              HEAD_SHAPE = 1 : <0>
              HEAD_SHAPE = 2 : <1>
              HEAD_SHAPE = 3 : <1>
           HOLDING = 2 :
              HEAD_SHAPE = 1 : <1>
              HEAD_SHAPE = 2 : <0>
              HEAD_SHAPE = 3 : <0>
  IS_SMILING = 2 :
     HOLDING = 1 :
        BODY_SHAPE = 1 :
           HAS_TIE = 1 : <0>
           HAS_TIE = 2 : <1>
        BODY_SHAPE = 2 :
           HAS_TIE = 1 : <1>
           HAS_TIE = 2 : <0>
        BODY_SHAPE = 3 :
           HAS_TIE = 1 :
              HEAD_SHAPE = 1 : <0>
              HEAD_SHAPE = 2 : <1>
           HAS_TIE = 2 : <1>
     HOLDING = 2 :
        HEAD_SHAPE = 1 :
           BODY_SHAPE = 1 : <0>
           BODY_SHAPE = 2 : <1>
           BODY_SHAPE = 3 : <0>
        HEAD_SHAPE = 2 : <0>
        HEAD_SHAPE = 3 : <0>
     HOLDING = 3 :
        HAS_TIE = 1 :
           BODY_SHAPE = 2 : <1>
           BODY_SHAPE = 3 : <0>
        HAS_TIE = 2 : <0>
JACKET_COLOR = 4 :
  BODY_SHAPE = 1 :
     HAS_TIE = 1 : <0>
     HAS_TIE = 2 :
        HOLDING = 1 : <1>
        HOLDING = 2 :
           HEAD_SHAPE = 1 : <1>
           HEAD_SHAPE = 2 : <0>
           HEAD_SHAPE = 3 : <1>
        HOLDING = 3 : <0>
  BODY_SHAPE = 2 :
     HEAD_SHAPE = 1 :
        HOLDING = 1 : <0>
        HOLDING = 2 :
           IS_SMILING = 1 : <0>
           IS_SMILING = 2 : <1>
        HOLDING = 3 :
```

```
         HAS_TIE = 1 :
            IS_SMILING = 1 : <0>
            IS_SMILING = 2 : <1>
         HAS_TIE = 2 :
            IS_SMILING = 1 : <1>
            IS_SMILING = 2 : <0>
   HEAD_SHAPE = 2 :
      HAS_TIE = 1 : <1>
      HAS_TIE = 2 :
         IS_SMILING = 1 : <1>
         IS_SMILING = 2 : <0>
   HEAD_SHAPE = 3 : <0>
BODY_SHAPE = 3 :
   HOLDING = 1 : <0>
   HOLDING = 2 : <0>
   HOLDING = 3 :
      IS_SMILING = 1 : <1>
      IS_SMILING = 2 : <0>
```

5.2.8 TDIDT on test set 1

```
DESCRIPTION OF THE TREE:

;; Tree found by tdidt trained on first monks's training set
;; 124 examples (full training set)
;; 86 nodes
;; 52 leaves
;; 75.69444 accuracy on test set

JACKET_COLOR = 1 : <1>
JACKET_COLOR = 2 :
   HOLDING = 1 :
      HEAD_SHAPE = 1 :
         BODY_SHAPE = 1 : <1>
         BODY_SHAPE = 2 : <0>
         BODY_SHAPE = 3 : <0>
      HEAD_SHAPE = 2 :
         IS_SMILING = 1 : <1>
         IS_SMILING = 2 : <0>
      HEAD_SHAPE = 3 :
         HAS_TIE = 1 : <1>
         HAS_TIE = 2 : <0>
   HOLDING = 2 :
      BODY_SHAPE = 1 : <0>
      BODY_SHAPE = 2 : <1>
      BODY_SHAPE = 3 : <0>
   HOLDING = 3 :
      IS_SMILING = 1 :
         HEAD_SHAPE = 1 :
            BODY_SHAPE = 1 : <1>
            BODY_SHAPE = 2 : <0>
            BODY_SHAPE = 3 : <0>
         HEAD_SHAPE = 2 : <1>
         HEAD_SHAPE = 3 :
            BODY_SHAPE = 1 : <0>
            BODY_SHAPE = 3 : <1>
      IS_SMILING = 2 :
         BODY_SHAPE = 1 : <0>
         BODY_SHAPE = 2 : <0>
         BODY_SHAPE = 3 :
            HAS_TIE = 1 : <0>
            HAS_TIE = 2 : <1>
JACKET_COLOR = 3 :
   HAS_TIE = 1 :
      HOLDING = 1 :
         IS_SMILING = 1 :
            BODY_SHAPE = 1 :
               HEAD_SHAPE = 1 : <1>
               HEAD_SHAPE = 2 : <0>
```

```
            BODY_SHAPE = 2 :
               HEAD_SHAPE = 1 : <0>
               HEAD_SHAPE = 2 : <1>
         IS_SMILING = 2 : <0>
      HOLDING = 2 :
         IS_SMILING = 1 : <0>
         IS_SMILING = 2 :
            HEAD_SHAPE = 1 : <1>
            HEAD_SHAPE = 2 : <0>
      HOLDING = 3 : <0>
   HAS_TIE = 2 :
      IS_SMILING = 1 :
         HOLDING = 1 :
            HEAD_SHAPE = 1 : <1>
            HEAD_SHAPE = 2 : <0>
         HOLDING = 2 :
            HEAD_SHAPE = 1 : <0>
            HEAD_SHAPE = 2 : <1>
            HEAD_SHAPE = 3 : <1>
         HOLDING = 3 : <1>
      IS_SMILING = 2 :
         HOLDING = 1 :
            BODY_SHAPE = 2 :
               HEAD_SHAPE = 2 : <1>
               HEAD_SHAPE = 3 : <0>
            BODY_SHAPE = 3 :
               HEAD_SHAPE = 2 : <0>
               HEAD_SHAPE = 3 : <1>
         HOLDING = 2 : <0>
         HOLDING = 3 :
            HEAD_SHAPE = 1 : <0>
            HEAD_SHAPE = 2 : <0>
            HEAD_SHAPE = 3 : <1>
JACKET_COLOR = 4 :
   HEAD_SHAPE = 1 :
      BODY_SHAPE = 1 : <1>
      BODY_SHAPE = 2 : <0>
      BODY_SHAPE = 3 : <0>
   HEAD_SHAPE = 2 :
      BODY_SHAPE = 1 : <0>
      BODY_SHAPE = 2 : <1>
      BODY_SHAPE = 3 : <0>
   HEAD_SHAPE = 3 :
      BODY_SHAPE = 2 : <0>
      BODY_SHAPE = 3 : <1>(1 1 1 1 1 1 -> 1)
```

5.2.9 ID5R-HAT on test set 2

```
DESCRIPTION OF THE TREE:

;; Typical tree found by id5r-hat trained on second monks's training set
;; 115 examples used out of 500 (random from full training set)
;; 131 nodes
;; 82 leaves
;; 65.74074 accuracy on test set

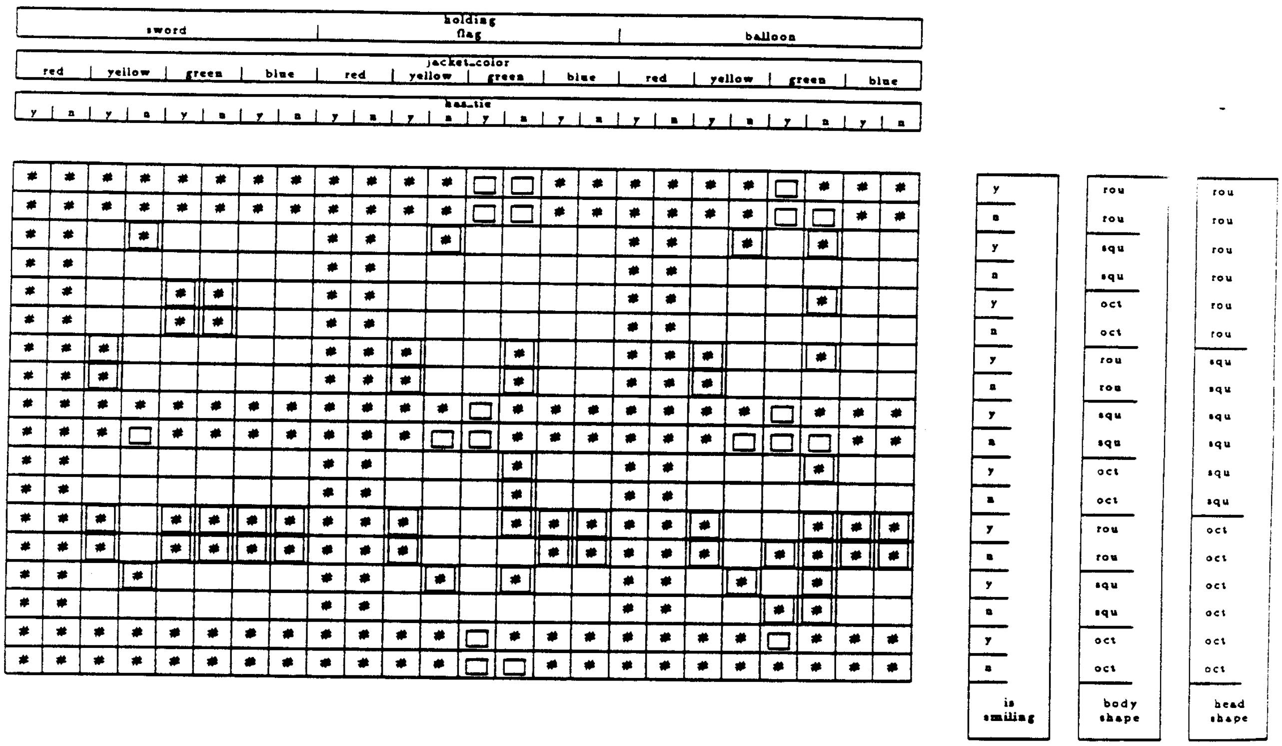

HOLDING = 1 :
   HAS_TIE = 1 :
      JACKET_COLOR = 1 : <0>...
      JACKET_COLOR = 2 :
         HEAD_SHAPE = 1 : <0>...
         HEAD_SHAPE = 2 : <1>...
         HEAD_SHAPE = 3 : <0>...
      JACKET_COLOR = 3 :
         BODY_SHAPE = 1 : <0>...
         BODY_SHAPE = 2 :
            IS_SMILING = 1 : <0>...
            IS_SMILING = 2 : <1>...
         BODY_SHAPE = 3 : <0>...
      JACKET_COLOR = 4 : <0>...
   HAS_TIE = 2 :
      BODY_SHAPE = 1 :
         IS_SMILING = 1 : <0>...
         IS_SMILING = 2 : <1>...
      BODY_SHAPE = 2 :
         IS_SMILING = 1 :
            JACKET_COLOR = 1 : <0>...
            JACKET_COLOR = 2 : <1>...
            JACKET_COLOR = 4 : <1>...
         IS_SMILING = 2 : <0>...
      BODY_SHAPE = 3 :
         JACKET_COLOR = 2 : <0>
         JACKET_COLOR = 1 :
            IS_SMILING = 1 : <0>...
            IS_SMILING = 2 : <1>...
         JACKET_COLOR = 3 :
            HEAD_SHAPE = 1 :
               IS_SMILING = 1 : <0>
               IS_SMILING = 2 : <1>
            HEAD_SHAPE = 2 : <1>...
            HEAD_SHAPE = 3 : <1>...
         JACKET_COLOR = 4 : <0>...
HOLDING = 2 :
   HEAD_SHAPE = 1 :
      HAS_TIE = 1 :
```

```
         IS_SMILING = 1 : <0>...
         IS_SMILING = 2 :
            BODY_SHAPE = 1 : <0>...
            BODY_SHAPE = 2 : <1>...
      HAS_TIE = 2 :
         JACKET_COLOR = 1 :
            IS_SMILING = 1 : <0>...
            IS_SMILING = 2 : <1>...
         JACKET_COLOR = 2 : <1>...
         JACKET_COLOR = 3 :
            IS_SMILING = 1 : <1>...
            IS_SMILING = 2 : <0>...
         JACKET_COLOR = 4 :
            BODY_SHAPE = 1 : <1>...
            BODY_SHAPE = 3 : <0>...
   HEAD_SHAPE = 2 :
      JACKET_COLOR = 1 :
         IS_SMILING = 1 : <0>...
         IS_SMILING = 2 : <1>...
      JACKET_COLOR = 2 :
         BODY_SHAPE = 1 : <1>...
         BODY_SHAPE = 2 : <0>...
         BODY_SHAPE = 3 : <0>...
      JACKET_COLOR = 3 :
         HAS_TIE = 1 :
            IS_SMILING = 1 : <1>...
            IS_SMILING = 2 : <0>...
         HAS_TIE = 2 : <0>...
      JACKET_COLOR = 4 : <0>...
   HEAD_SHAPE = 3 :
      JACKET_COLOR = 1 :
         BODY_SHAPE = 1 : <0>...
         BODY_SHAPE = 2 : <1>...
         BODY_SHAPE = 3 : <1>...
      JACKET_COLOR = 2 :
         BODY_SHAPE = 1 : <1>...
         BODY_SHAPE = 2 :
            IS_SMILING = 1 : <1>...
            IS_SMILING = 2 : <0>...
         BODY_SHAPE = 3 : <0>...
      JACKET_COLOR = 3 : <0>...
      JACKET_COLOR = 4 : <1>...
HOLDING = 3 :
   IS_SMILING = 1 :
      HEAD_SHAPE = 1 :
         HAS_TIE = 1 : <0>...
         HAS_TIE = 2 : <1>...
      HEAD_SHAPE = 2 :
         BODY_SHAPE = 1 :
            JACKET_COLOR = 1 : <0>...
            JACKET_COLOR = 2 : <1>...
```

```
         JACKET_COLOR = 3 : <1>...
         JACKET_COLOR = 4 : <0>...
      BODY_SHAPE = 2 : <1>...
      BODY_SHAPE = 3 : <1>...
   HEAD_SHAPE = 3 :
      JACKET_COLOR = 1 : <0>...
      JACKET_COLOR = 2 : <1>...
      JACKET_COLOR = 3 :
         HAS_TIE = 1 : <1>...
         HAS_TIE = 2 : <0>...
IS_SMILING = 2 :
   HEAD_SHAPE = 1 :
      BODY_SHAPE = 1 : <1>...
      BODY_SHAPE = 2 :
         HAS_TIE = 1 :
            JACKET_COLOR = 1 : <0>
            JACKET_COLOR = 3 : <1>
            JACKET_COLOR = 4 : <1>
         HAS_TIE = 2 :
            JACKET_COLOR = 1 : <1>
            JACKET_COLOR = 3 : <0>
            JACKET_COLOR = 4 : <0>
      BODY_SHAPE = 3 : <1>...
   HEAD_SHAPE = 2 :
      JACKET_COLOR = 1 :
         HAS_TIE = 1 :
            BODY_SHAPE = 1 : <0>
            BODY_SHAPE = 2 : <1>
         HAS_TIE = 2 : <1>...
      JACKET_COLOR = 2 : <0>...
      JACKET_COLOR = 3 : <0>...
      JACKET_COLOR = 4 : <0>...
   HEAD_SHAPE = 3 :
      JACKET_COLOR = 1 : <1>...
      JACKET_COLOR = 2 :
         HAS_TIE = 1 :
            BODY_SHAPE = 1 : <1>
            BODY_SHAPE = 3 : <0>
         HAS_TIE = 2 : <0>...
      JACKET_COLOR = 4 : <0>...
```

5.3 Classification diagrams

Result of ID5R on test set 1



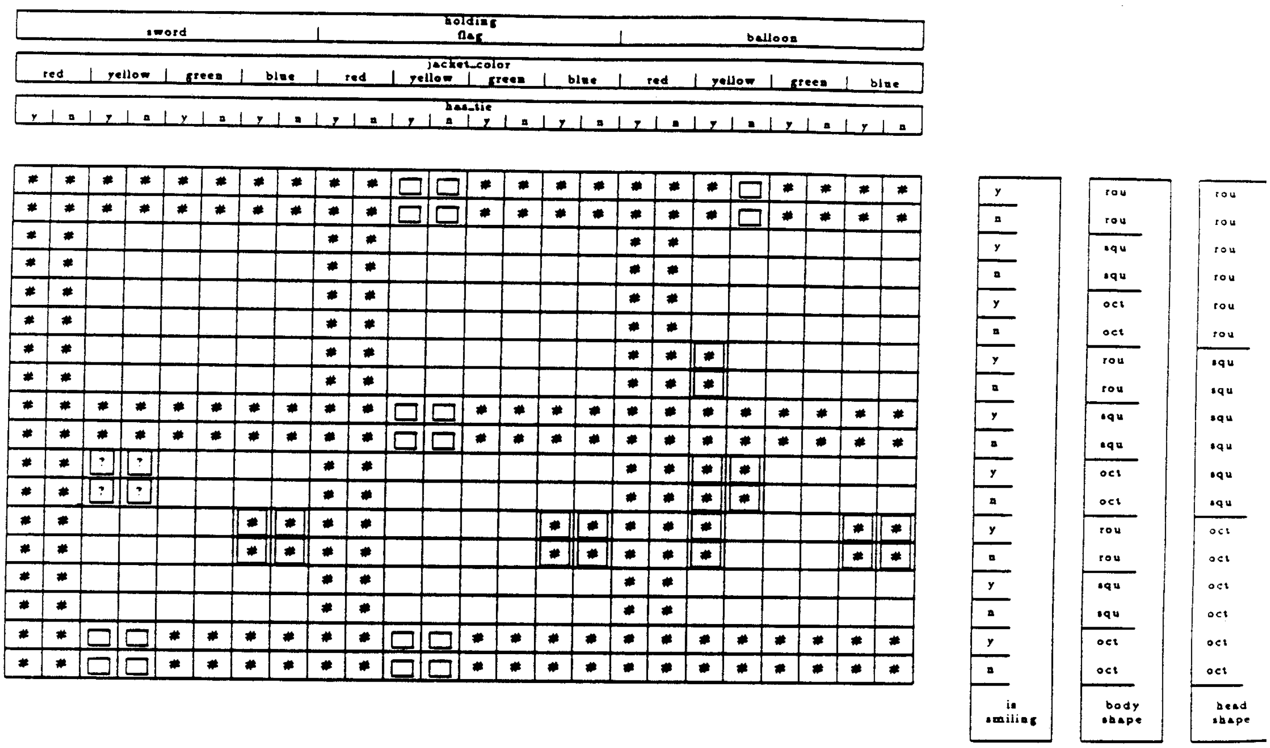
Accuracy: 83.101852

Result of IDL on test set 1 .

holding	sword							flag								balloon										
jacket_color	red		yellow		green		blue		red		yellow		green		blue		red		yellow		green		blue			
has_tie	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	is smiling	body shape	head shape
	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	y	rou	rou
	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	n	rou	rou
	#	#							#	#							#	#							y	squ	rou
	#	#							#	#							#	#							n	squ	rou
	#	#							#	#							#	#							y	oct	rou
	#	#							#	#							#	#							n	oct	rou
	#	#							#	#							#	#							y	rou	squ
	#	#							#	#							#	#							n	rou	squ
	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	y	squ	squ
	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	n	squ	squ
	#	#							#	#							#	#							y	oct	squ
	#	#							#	#							#	#							n	oct	squ
	#	#							#	#							#	#							y	rou	oct
	#	#							#	#							#	#							n	rou	oct
	#	#							#	#							#	#							y	squ	oct
	#	#							#	#							#	#							n	squ	oct
	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	y	oct	oct
	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	n	oct	oct

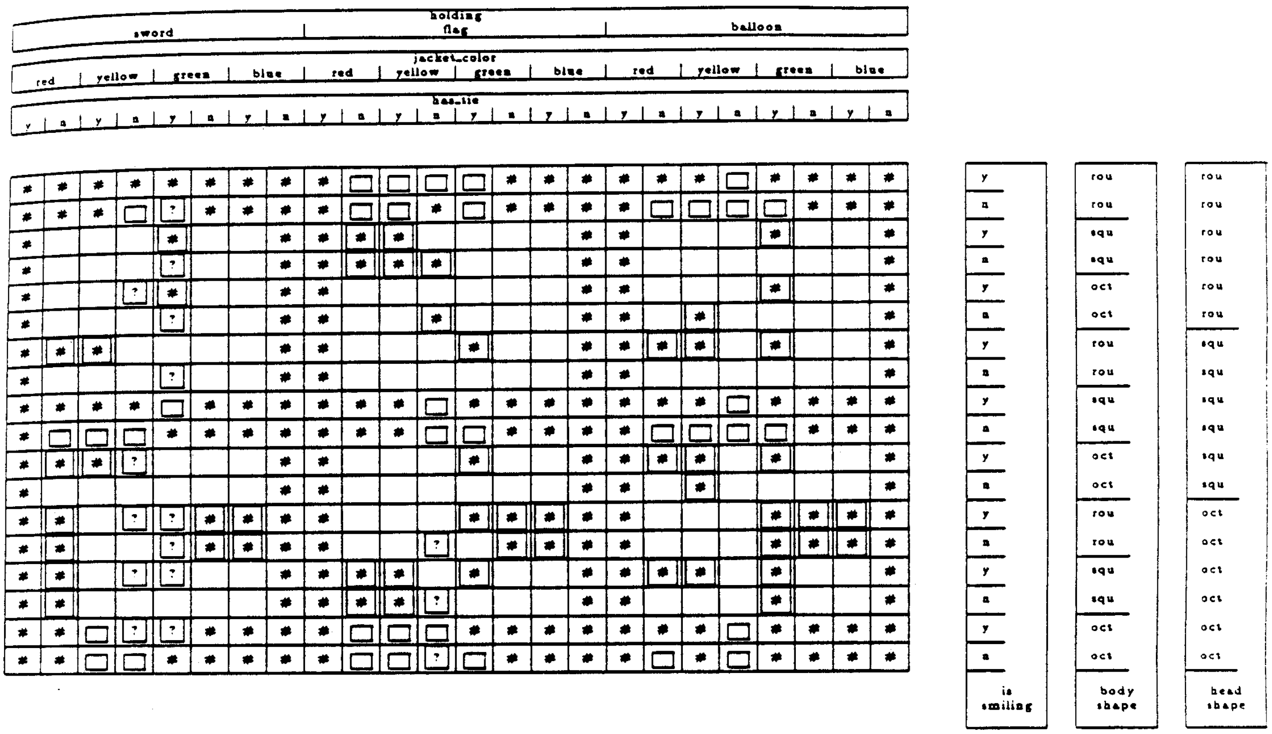
Accuracy: 100.000000

Result of ID5R-HAT on test set 1



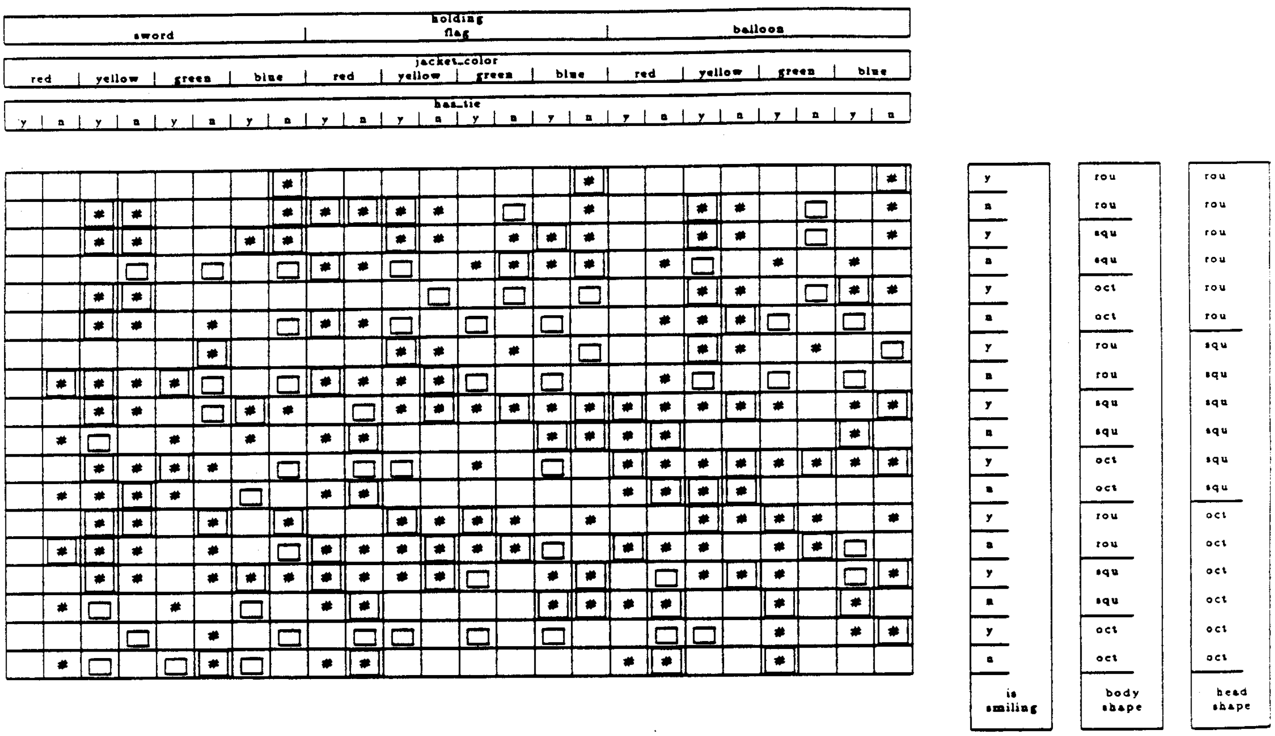
Accuracy: 90.277778

Result of TDIDT-based method on test set 1



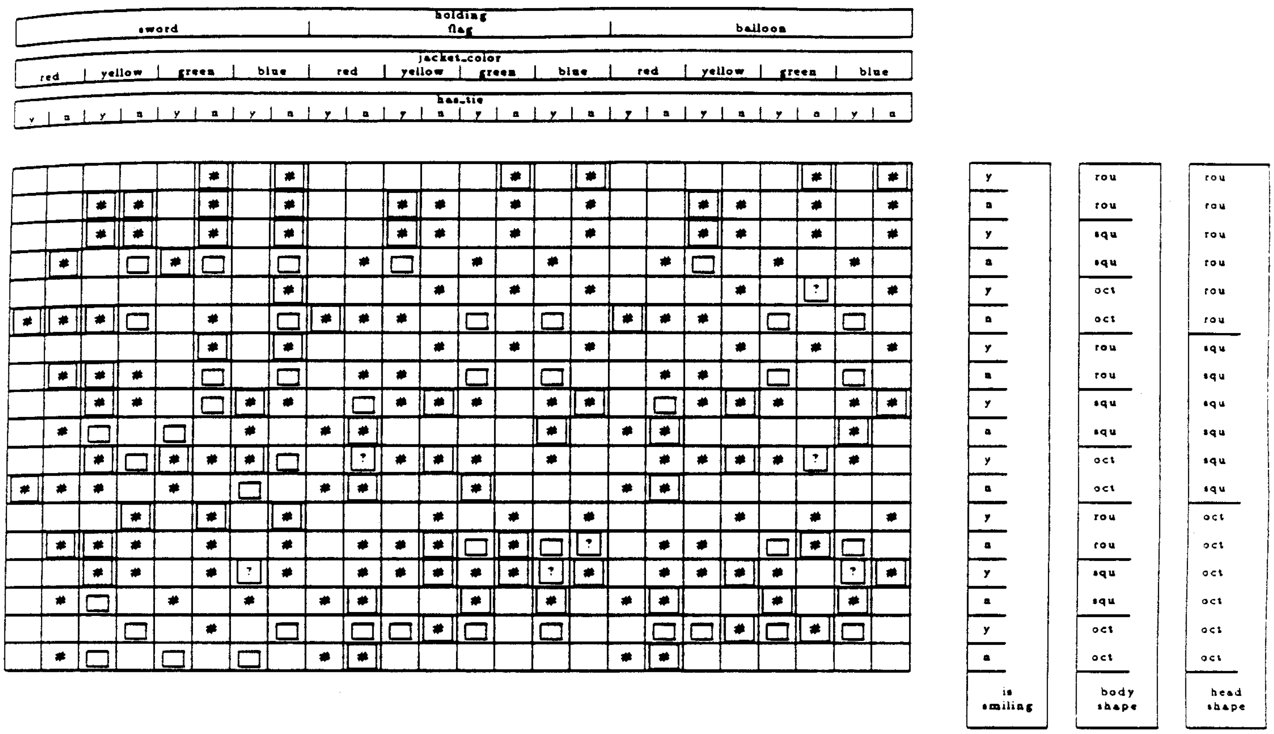
Accuracy: 75.694444

Result of ID5R on test set 2



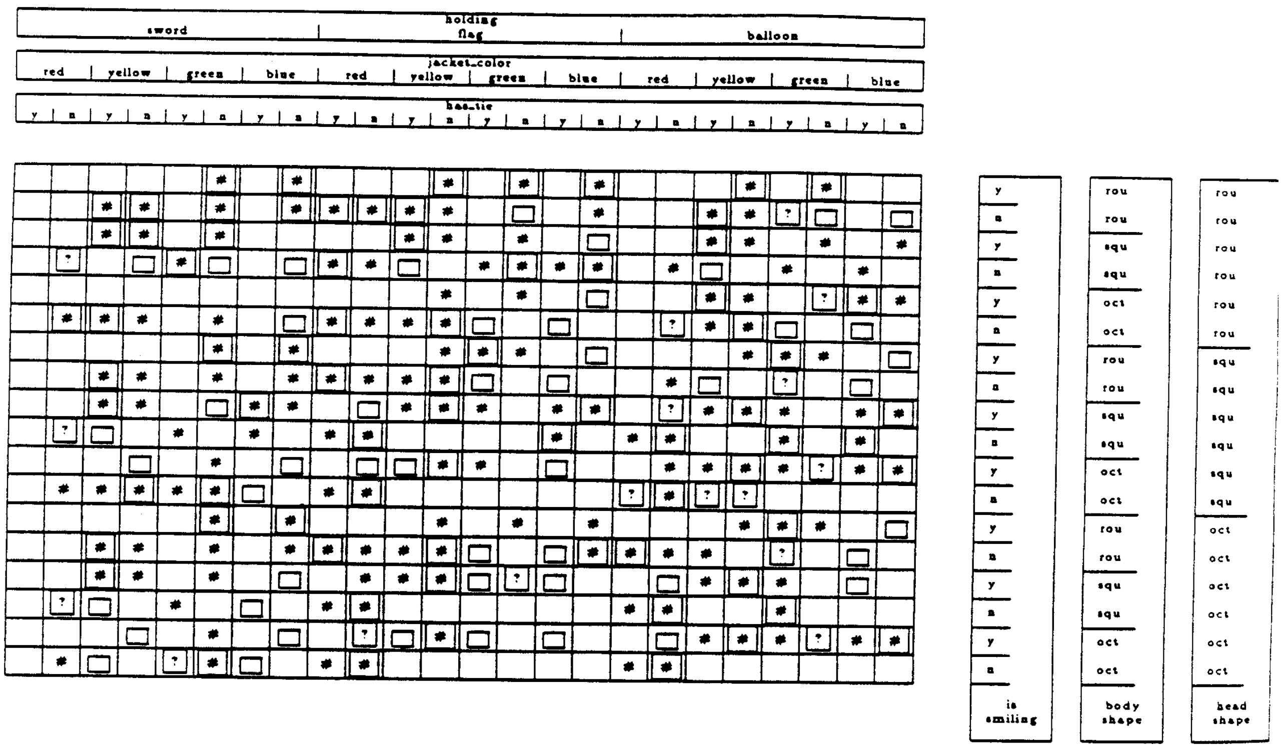
Accuracy: 66.203704

Result of IDL on test set 2



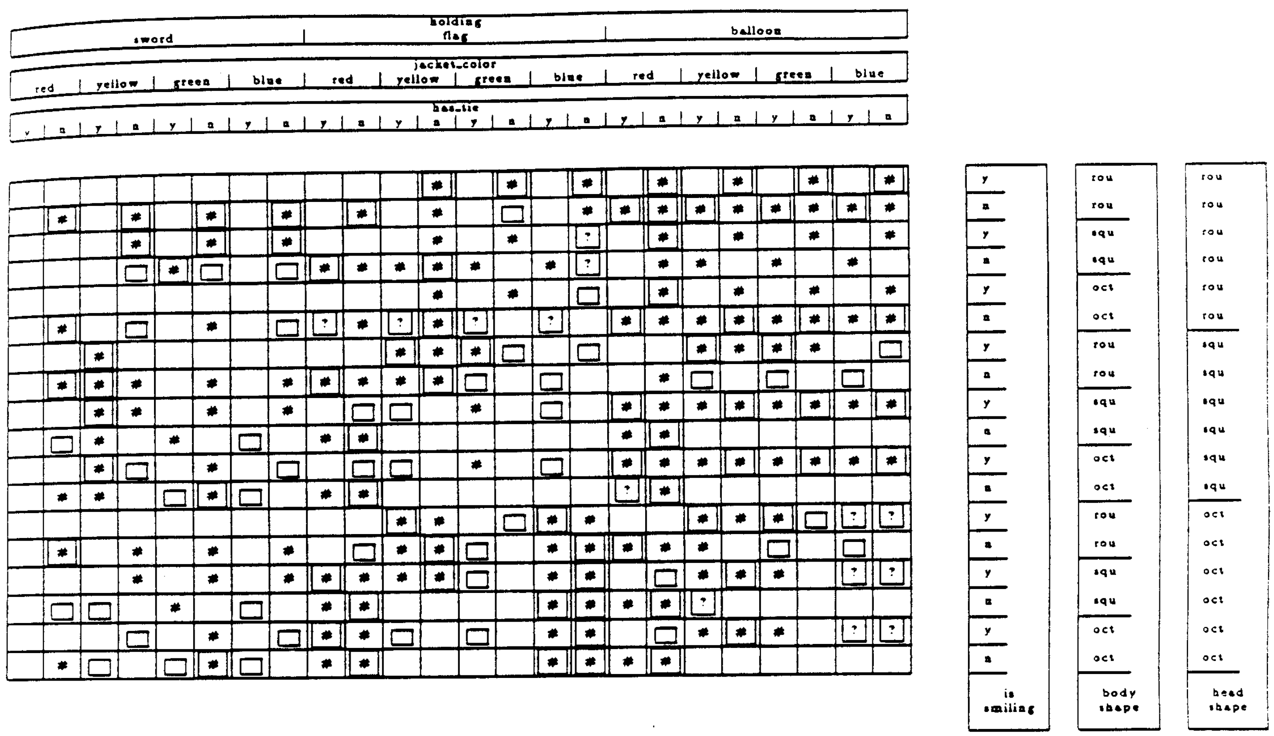
Accuracy: 71.296296

Result of TDIDT on test set 2



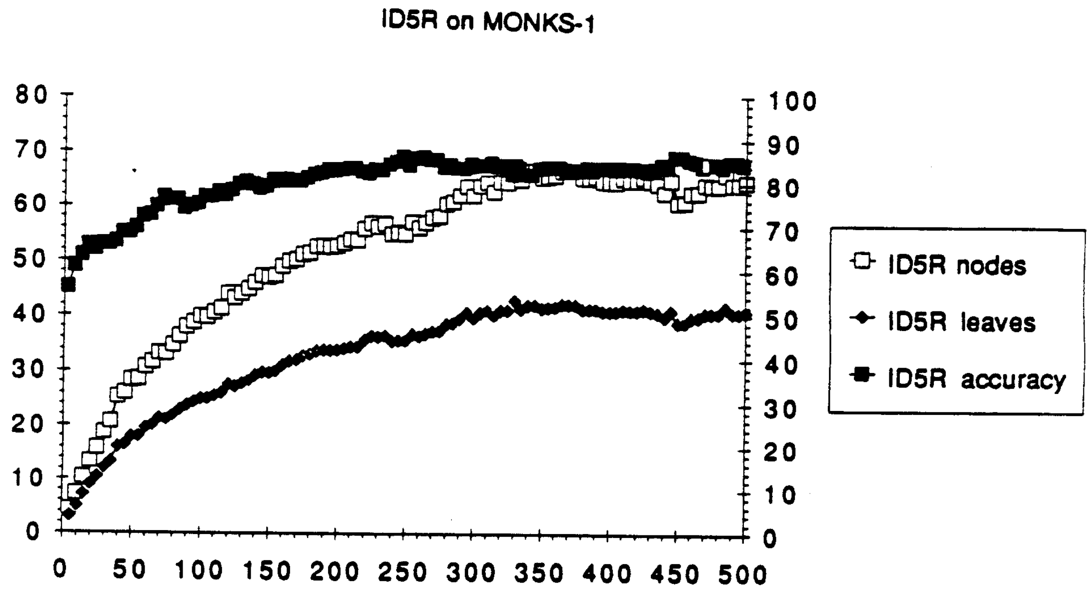
Accuracy: 67.129630

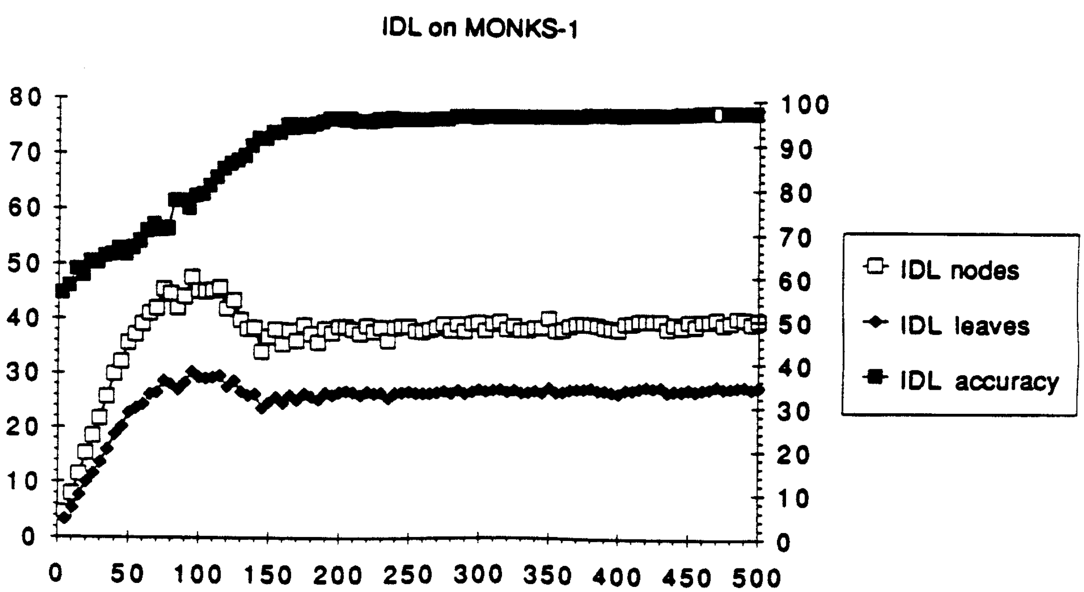
Result of ID5R-HAT on test set 2

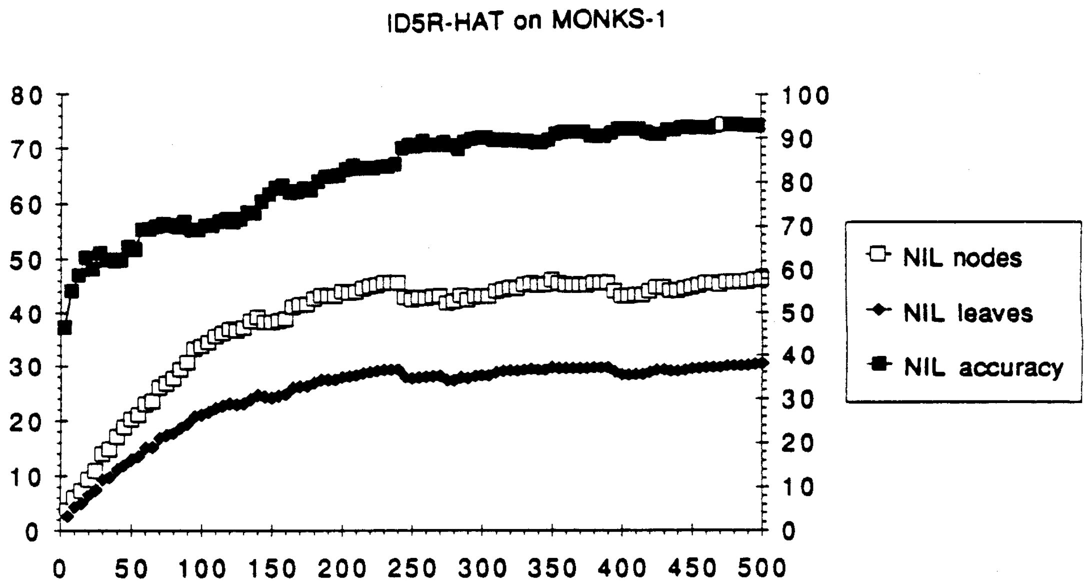


Accuracy: 67.824074

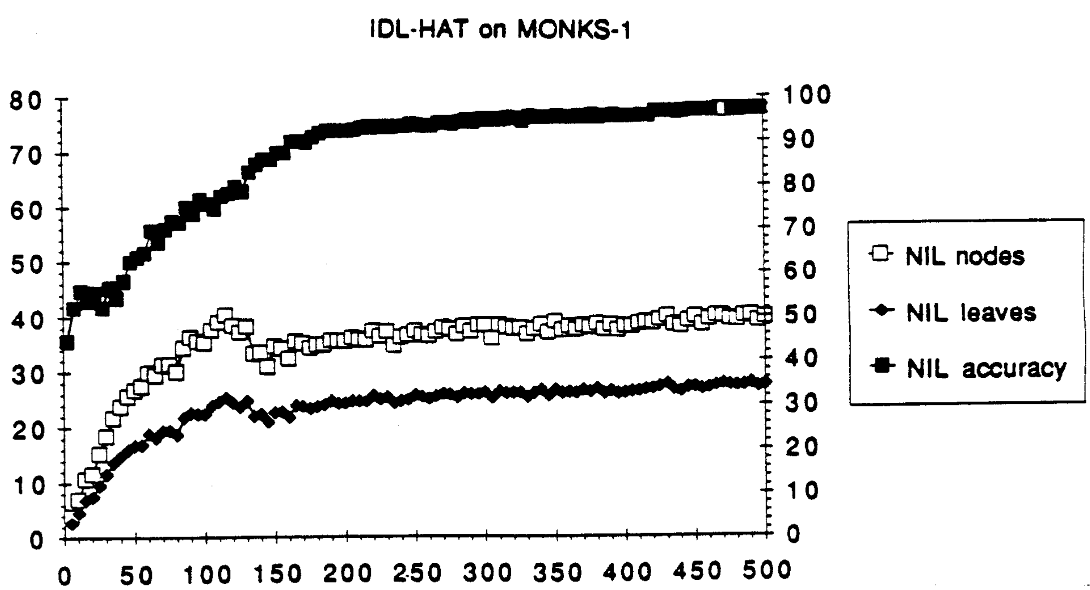
5.4 Learning curves







ID5R-HAT on MONKS-1
80
70
60
50
40
30
20
10
0
100
90
80
70
60
50
40
30
20
10
0
0 50 100 150 200 250 300 350 400 450 500
NIL nodes
NIL leaves
NIL accuracy



IDL-HAT on MONKS-1
80
70
60
50
40
30
20
10
0
100
90
80
70
60
50
40
30
20
10
0
0 50 100 150 200 250 300 350 400 450 500
NIL nodes
NIL leaves
NIL accuracy

Chapter 6

Comparison of Inductive Learning Programs

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6.1 Introduction

At the Institute for Real-Time Computer Control Systems & Robotics a library of inductive machine learning algorithms is being developed. So far this library consists of:

- ID3 - classical decision tree learning algorithm
- ID5R - an incremental decision tree learning algorithm
- AQR - a version of the AQ-rule learning algorithms
- CN2 - rule decision list learning algorithm
- COBWEB - conceptual clustering algorithm for attributes with symbolic values
- CLASSIT - conceptual clustering algorithm for attributes with numerical values
- CLASSWEB - algorithm that integrates COBWEB and CLASSIT. In the following only this algorithm is referred to.

These algorithms have been implemented in a very homogeneous way, i.e. they use the same description for objects that have to be learned, they are called in a similar way and they are all available under one common user interface.

The reason for building up this ML-library is, that our institute is interested in applying machine learning techniques to robotics applications. As a first step we wanted to gain experiences with the classical inductive learning methods in order to find out their capabilities and limitations.

All algorithms base on a common description of the objects to be learned, which consists of a set of attributes, each defined by a name, a domain, a 'noisy-flag' and some additional information for the conceptual clustering algorithm. In addition a symbol which is used for unknown attribute values can be identified. Each algorithm will then be called with a set of examples (classified for ID3, ID5R, AQR and CN2; unclassified for CLASSWEB). As ID5R and CLASSWEB are incremental methods, a former received classifier can also be given as input. Each algorithm results in a classifier which can be used for classifying further given objects. For a better understanding of the results a textual representation of the classifier can be printed on the screen. For decision tree learning algorithms and conceptual clustering also a graphical display is available. For the incremental methods it is also possible to display a trace during classifier generation. The implementation work has been done on a SUN Sparc Station 1+ IN SUN Lucid Common Lisp using CLX and CLUE for only the graphical interface ([HW91]).

6.2 Short description of the algorithms

In this section a very short description of the algorithms will be given. For further details please see the corresponding literature. The representation of examples as attribute-value-pairs, where the set of attributes is given and fixed, is common to all algorithms.

6.2.1 ID3

ID3 is the most popular representative of TDIDT-algorithms (Top Down Induction of Decision Trees). It builds up a decision tree based on the classified training examples ([Qui86]). The internal nodes of a decision tree represent a test based on one specific attribute. For each possible attribute value there is one subtree, which is for itself a decision tree. The leaves of the tree represent class names. For classifying a new object with a built-up decision tree, the value of the attribute at the root of the tree will determine which subtree has to be considered recursively. The recursion will end, if a leaf of the tree is reached. In that case the class name given in that leaf represents the class in which the object has to be classified.

The idea for building up the decision tree is to iteratively find the attribute in the set of attributes of the objects which gives the 'best' partition of the set of training examples. 'Best' is defined in terms of the information gain given by a partition according to the specific attribute.

The basic algorithm has already been extended by Quinlan ([Qui86]) to handle noisy attributes and unknown attribute values. In the implemented algorithm noise is handled by applying chi-square test for stochastic independence to the noisy attribute with respect to the class distribution. Unknown attribute values have to be handled during building of the decision tree and during classification. For building up the decision tree unknown attribute values are taken into account in the calculation of the information gain.

The algorithm as being implemented also uses windowing over the training set, i.e. a subset of the training set is chosen at random and the decision tree is built up by using only these examples. After that all other examples of the training set are classified using this DT. If some of the examples are incorrectly classified, a selection of these will be added to the window and the procedure will start again. Due to the complexity of the given training sets, a lot of iterative steps had to be performed.

6.2.2 ID5R

The ID5R algorithm ([Utg89]) has been developed by P.E. Utgoff as a kind of TDIDT-algorithm which is able to work incrementally, but results finally, i.e. after all training examples, in the same decision tree as ID3. 'Incremental' means that the examples can be given one after another. A very easy solution for the problem of successively given examples would be to generate an ID3 decision tree from scratch with all examples given so far. In contrast to that approach, ID5R always uses the decision tree developed so far for integrating the new example. For that reason the data structure of a node in an ID5R tree has been enlarged by the information necessary to calculate the information gain function of the attributes.

If during insertion of the new example the situation arises that the current test attribute is not the one with the highest information gain, the tree has to be restructured. This is done by investigating all subtrees of the current node by using the new attribute as the test attribute. In a second step the test attribute in the current node is exchanged for the attribute in the subtrees.

In our implementation ID5R does not result in exactly the same tree as ID3, even if all examples are given. First this is caused by the fact that ID5R does not generate NULL-classes, because leaves are only splitted further, if it is really necessary. Second, if there are several attributes with the same information gain and one of these attributes is already used as test attribute, then a restructuring of the tree will not be done. It would be of course also possible to take the first attribute in the list as new test attribute and to restructure the tree accordingly.

6.2.3 AQR

The AQR algorithm is an implementation of the AQ-family, which has been founded by R. Michalski in 1969. AQR is a reconstruction of a straight-forward implementation of the basic AQ algorithm and has been described in [CN89]. The algorithm results in one decision rule for each class. The condition of each rule is called a cover and represents a disjunction of so-called complexes. Each complex for itself is a conjunction of selectors and each selector is a basic attribute test (has the attribute one of a set of values, etc.).

For classifying a new object, each rule is checked to see, whether the condition is completely satisfied, i.e. the example is covered by the rule. If exactly one rule is satisfied, the corresponding class is the classification result. If several rules are applicable, then the most common class of training examples covered by those rules is used as result. If none of the rules can be applied, the class that appeared most often in the training set is used as result.

The decision rules are sequentially built up for the different classes. Starting with an empty cover successively a *seed*, i.e. a positive example which is not covered so far is being selected and a *star* is being generated, which is a set of complexes that cover the seed but no negative examples. From these complexes the one which is the best one according to a user-defined criterion is being chosen and added to the cover as an extra disjunct. The positive examples that are covered by that additional complex are then deleted from the list of examples. In our implementation the best complex is the complex that maximizes the number of positive examples that are covered.

6.2.4 CN2

This algorithm has been developed by P. Clark and T. Niblett ([CN89]). It shall combine the advantages of the families of ID3- and AQ-algorithms. The classifier resulting from that algorithm is an ordered set of if-then-rules (decision list). This means that the representation is very similar to AQ, i.e. if 'complex' then predict 'class', but the rules have to be checked from top to bottom. If none of the rules applies to a new object, again the class that appeared most often in the training set will be taken.

The idea of Clark and Niblett was to enable AQ-like algorithms to handle noisy data by also taking complexes into account that do not fit the positive/negative border accurately. The method is based on the beam-search method as being used in AQ. During each iteration the algorithm searches for a complex that covers a large number of examples of one class and only few examples of other classes. The complexes are evaluated by an evaluation function which determines their predictiveness and reliability. If a good complex has been found, the examples that are covered, are deleted from the set of training examples. The search for a complex can be seen as a general-to-specific search with some pruning. During each iteration a set of the best complexes found so far is being remembered. These are specialized by adding a new conjunctive term or deleting a disjunctive part of one of the selectors. CN2 evaluates all possible specializations of each complex, which may lead to an enormous computational effort.

6.2.5 CLASSWEB

CLASSWEB is a combination of the algorithms COBWEB ([Fis87]) and CLASSIT ([GLF89]). These are methods for conceptual clustering. In contrast to the four algorithms described so far, these use unclassified examples as input and try to find a concept hierarchy for the examples where the similarity in one concept is as high as possible and the similarity between different concepts is as low as possible. While COBWEB only handles nominal values and CLASSIT only numerical ones, our CLASSWEB algorithm is able to handle both types in an integrated way.

For building up a concept hierarchy CLASSWEB uses four different operators to integrate a new example into the already existing concept hierarchy. These are: 1.) classifying the object into an existing class, 2.) creating a new class, 3.) combining two classes into a single class and 4.) dividing a class into several classes. Applied to internal concept nodes these different operators are scored according to category utility and the best one is chosen.

During classification of new objects each object will be incorporated into the tree as if it were a new example. This method also allows to predict unknown attribute values of the new example.

We have also implemented the so-called *cutoff* in CLASSWEB. By that parameter the algorithm does not have to classify each example down to a leaf, but also may decide to stop at some higher level in the hierarchy. Cutoff is a measure whether an example and a concept class are similar enough to stop at that concept node. If cutoff is set to zero, the algorithm behaves exactly like the original COBWEB method.

6.3 Results

The following tables compare the performance of the different algorithms on the three problem sets. The time data given correspond to compiled SUN Lucid Common Lisp 3.0 code on a SUN SPARC station 1+.

6.3.1 Training Time

This following table states the time required for each algorithm on each training set to build up a classifier.

Algorithm	Training Set 1	Training Set 2	Training Set 3
ID3	35.51	154.02	23.04
ID3 no wind.	4.98	7.61	3.74
ID5R	99.20	407.09	78.91
AQR	4.17	9.45	4.00
CN2	4.48	74.04	10.25
CLASSWEB	568.18	731.22	414.10

Time is given in seconds and was averaged over three test runs over each algorithm and each training set.

Remarks:

The ID3-algorithm as implemented uses a 20%-windowing as mentioned above. For the three given problems this leads to a large number of necessary iterations. That's why there are also results given for ID3 without windowing (ID3 no wind.).

The CN2-algorithm uses a user-defined threshold value for doing its noise test. This is set to 0.1.

The cutoff-parameter in CLASSWEB was set to 0.2.

6.3.2 Classifier Results

First we will give some measurements such as number of nodes, leaves, rules and so on, which will reflect the complexity of the resulting algorithms. Afterwards some of the resulting classifiers for the different algorithms and training sets are given.

ID3

Measurement	Training Set 1	Training Set 2	Training Set 3
# nodes	13	66	13
# leaves	28	110	29

ID3 no windowing

Measurement	Training Set 1	Training Set 2	Training Set 3
# nodes	32	64	14
# leaves	62	110	31

ID5R

Measurement	Training Set 1	Training Set 2	Training Set 3
# nodes	34	64	14
# leaves	52	99	28

AQR

Measurement	Training Set 1		Training Set 2		Training Set 3	
	Class 0	Class 1	Class 0	Class 1	Class 0	Class 1
# complexes	30	6	40	43	16	20
# selectors	109	14	147	187	47	67

CN2

Measurement	Training Set 1	Training Set 2	Training Set 3
# rules	10	58	24
# selectors	13	145	38

CLASSWEB

Measurement	Training Set 1	Training Set 2	Training Set 3
# concepts	24	24	14
# nodes	9	8	4
# leaves	15	16	10

Training Set 1

ID3

```
JACKET-COLOR
  1  1
  2
     BODY-SHAPE
        1
           HEAD-SHAPE
              1  1
              2  0
              3  0
        2
           HEAD-SHAPE
```

```
            1  0
            2  1
            3  0
        3
          HEAD-SHAPE
            1  0
            2  0
            3  1
3
    HEAD-SHAPE
        1
          BODY-SHAPE
            1  1
            2  0
            3  0
        2
          BODY-SHAPE
            1  0
            2  1
            3  0
        3
          BODY-SHAPE
            1  0
            2  0
            3  1
4
    HEAD-SHAPE
        1
          BODY-SHAPE
            1  1
            2  0
            3  0
        2
          BODY-SHAPE
            1  0
            2  1
            3  0
        3
          BODY-SHAPE
            1  NULL
            2  0
            3  1
```

AQR

```
BODY-SHAPE = 2  &  JACKET-COLOR = 2  &  HOLDING = 1  &  HEAD-SHAPE = 1  |
HAS-TIE = 1  &  HOLDING = 1  &  BODY-SHAPE = 2  &  HEAD-SHAPE = 1  |
IS-SMILING = 1  &  HEAD-SHAPE = 1  &  JACKET-COLOR = 4  |
BODY-SHAPE = 2  &  HOLDING = 2  &  HEAD-SHAPE = 1  &  JACKET-COLOR = 3  |
JACKET-COLOR = 2  &  HEAD-SHAPE = 1  &  BODY-SHAPE = 2  |
JACKET-COLOR = 4  &  HEAD-SHAPE = 1  &  BODY-SHAPE = 2  |
HOLDING = 3  &  BODY-SHAPE = 2  &  HEAD-SHAPE = 1  |
HOLDING = 1  &  BODY-SHAPE = 3  &  HEAD-SHAPE = 1  |
BODY-SHAPE = 3  &  JACKET-COLOR = 2  &  HEAD-SHAPE = 1  |
HOLDING = 3  &  BODY-SHAPE = 3  &  JACKET-COLOR = 3  &  IS-SMILING = 1
                                                     &  HEAD-SHAPE = 1  |
HAS-TIE = 2  &  IS-SMILING = 2  &  HEAD-SHAPE = 1  &  JACKET-COLOR = 3  |
JACKET-COLOR = 4  &  HEAD-SHAPE = 1  &  BODY-SHAPE = 3  |
HOLDING = 1  &  HEAD-SHAPE = 2  &  BODY-SHAPE = 1  |
HOLDING = 2  &  IS-SMILING = 1  &  BODY-SHAPE = 1  &  JACKET-COLOR = 2  |
```

```
HEAD-SHAPE = 2  &  HOLDING = 2  &  BODY-SHAPE = 1  &  JACKET-COLOR = 3  |
IS-SMILING = 1  &  BODY-SHAPE = 1  &  HEAD-SHAPE = 2  &  JACKET-COLOR = 4  |
HAS-TIE = 2  &  IS-SMILING = 2  &  HOLDING = 2  &  BODY-SHAPE = 1  |
HOLDING = 3  &  BODY-SHAPE = 1  &  HEAD-SHAPE = 2  |
HOLDING = 2  &  BODY-SHAPE = 3  &  HEAD-SHAPE = 2  &  JACKET-COLOR = 3  |
BODY-SHAPE = 3  &  HOLDING = 3  &  JACKET-COLOR = 3  &  HEAD-SHAPE = 2  |
BODY-SHAPE = 3  &  JACKET-COLOR = 4  &  HEAD-SHAPE = 2  |
IS-SMILING = 2  &  HOLDING = 1  &  BODY-SHAPE = 3  &  HEAD-SHAPE = 2  |
IS-SMILING = 2  &  HEAD-SHAPE = 2  &  BODY-SHAPE = 3  &  JACKET-COLOR = 2  |
HAS-TIE = 2  &  HEAD-SHAPE = 3  &  HOLDING = 3  &  BODY-SHAPE = 1  |
JACKET-COLOR = 2  &  BODY-SHAPE = 1  &  HEAD-SHAPE = 3  |
HEAD-SHAPE = 3  &  JACKET-COLOR = 4  &  HOLDING = 1  &  IS-SMILING = 1
                                                    &  BODY-SHAPE = 2  |
HOLDING = 2  &  JACKET-COLOR = 4  &  BODY-SHAPE = 2  |
HEAD-SHAPE = 3  &  JACKET-COLOR = 3  &  BODY-SHAPE = 2  |
HOLDING = 3  &  IS-SMILING = 2  &  BODY-SHAPE = 2  &  JACKET-COLOR = 2  |
HOLDING = 3  &  BODY-SHAPE = 2  &  HEAD-SHAPE = 3  &  JACKET-COLOR = 4
===> CLASS '0' (P['0'] = 1/2)


BODY-SHAPE = 1  &  HEAD-SHAPE = 1  |
JACKET-COLOR = 1  |
IS-SMILING = 1  &  BODY-SHAPE = 2  &  HEAD-SHAPE = 2  |
BODY-SHAPE = 2  &  HEAD-SHAPE = 2  |
HAS-TIE = 1  &  BODY-SHAPE = 3  &  HEAD-SHAPE = 3  |
HAS-TIE = 2  &  HEAD-SHAPE = 3  &  BODY-SHAPE = 3
===> CLASS '1' (P['1'] = 1/2)

DEFAULT ===> CLASS '0' (P['0'] = 1/2)
```

CN2

```
JACKET-COLOR = 1   ===> CLASS '1'
HEAD-SHAPE = 2  &  BODY-SHAPE = 3   ===> CLASS '0'
BODY-SHAPE = 1  &  HEAD-SHAPE = 3   ===> CLASS '0'
BODY-SHAPE = 1  &  HEAD-SHAPE = 2   ===> CLASS '0'
BODY-SHAPE = 1   ===> CLASS '1'
HEAD-SHAPE = 2   ===> CLASS '1'
BODY-SHAPE = 2   ===> CLASS '0'
HEAD-SHAPE = 3   ===> CLASS '1'
HAS-TIE = 2   ===> CLASS '0'
HAS-TIE = 1   ===> CLASS '0'

DEFAULT  ===>  CLASS '0'
```

Accuracy

Algorithm	Training Set 1	Training Set 2	Training Set 3
ID3	100.00	100.00	100.00
ID3 no w.	100.00	100.00	100.00
ID5R	100.00	100.00	100.00
AQR	100.00	100.00	100.00
CN2	100.00	92.90	100.00

Algorithm	Test Set 1	Test Set 2	Test Set 3
ID3	98.56	67.92	94.44
ID3 no w.	83.24	69.12	95.60
ID5R	79.77	69.23	95.28
AQR	95.88	79.63	87.04
CN2	100.00	68.98	89.12

Results of CLASSWEB

The results of CLASSWEB have not been integrated into the tables above, because an accuracy as for the other algorithms cannot be defined. This is due to the fact, that CLASSWEB uses unclassified instances as input and thus does not have any information about classes in the resulting concepts either. For that reason there is only a small probability that CLASSWEB generates concepts that have something to do with the concepts the user expected. Indeed it was not possible to find the structure of the given concepts in the automatically generated concept class. To determine the 'human-machine' concept similarity we built up the concept hierarchies with our CLASSWEB algorithm and then determined the concept for each example. For each concept class we then calculated the number of positive and negative examples covered by that concept. The maximum of both, which represents the amount of human-class-similarity in one concept was summed up and divided by the total number of examples. The resulting value is given in the tables below and can be interpreted as the overall percentage of covering one class by one particular concept. This can not be interpreted as an accuracy.

Training Set 1	Training Set 2	Training Set 3
63.71	64.50	72.13

Test Set 1	Test Set 2	Test Set 3
59.49	67.36	65.28

6.4 Conclusion

It also has to be mentioned that some important capabilities of the algorithms have not been tested and compared by using the given learning problems. These are for example the handling of noise in specific attributes, of costs for determining attribute values and of unknown attribute values in ID3 and ID5R. The incremental nature of ID5R was not really needed in these test cases because all examples were given in advance. The ability to handle unknown attribute values in AQR and CN2 was not used either.

The results of CLASSWEB have to be interpreted carefully, because it differs in nature from the other compared learning algorithms. Possibilities like predicting unknown attribute values by CLASSWEB were not used.

Acknowledgement

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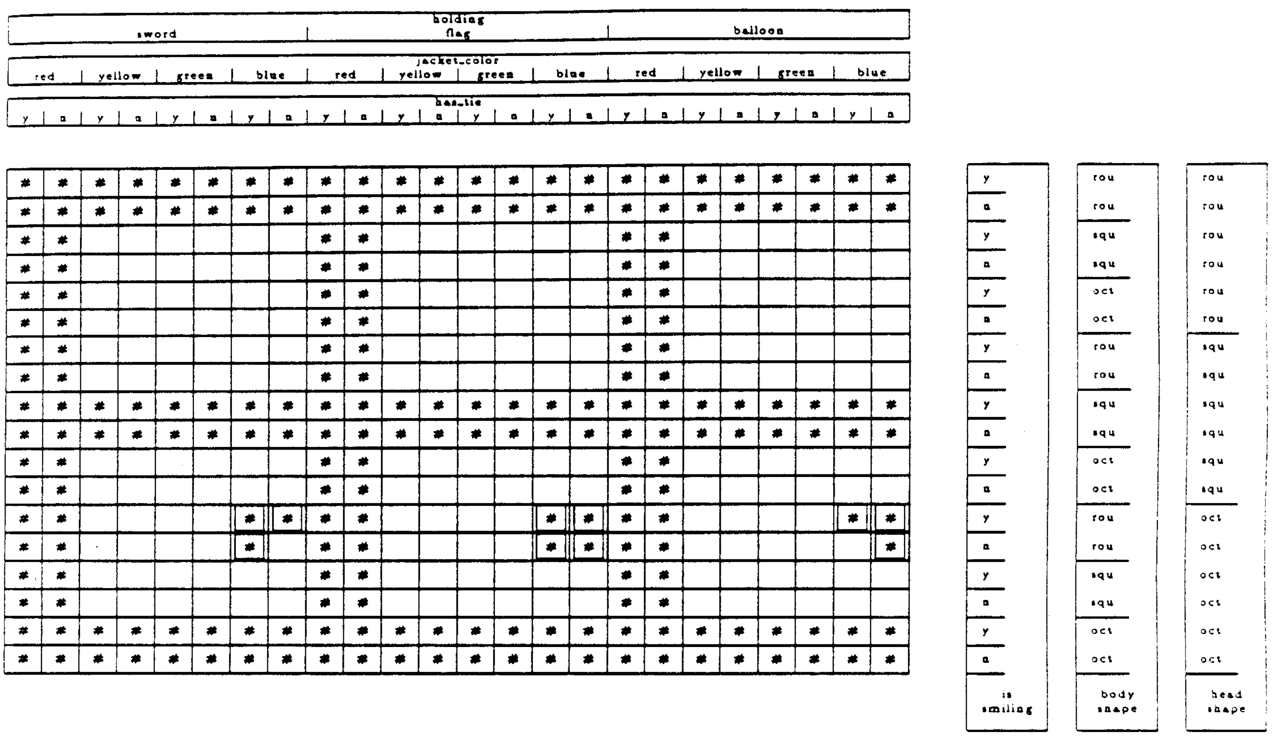
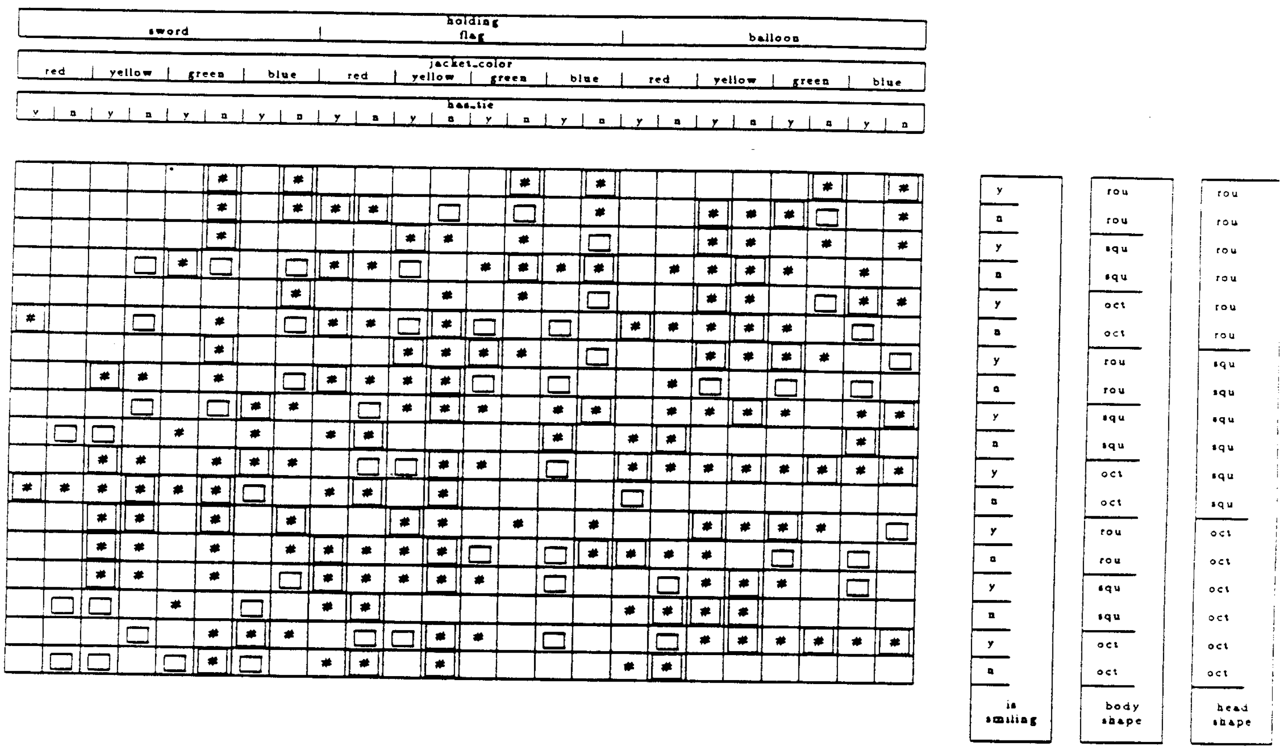
6.5 Classification diagrams

Result of ID3 on test set No. 1



Accuracy: 97.685185

Result of ID3 on test set No. 2



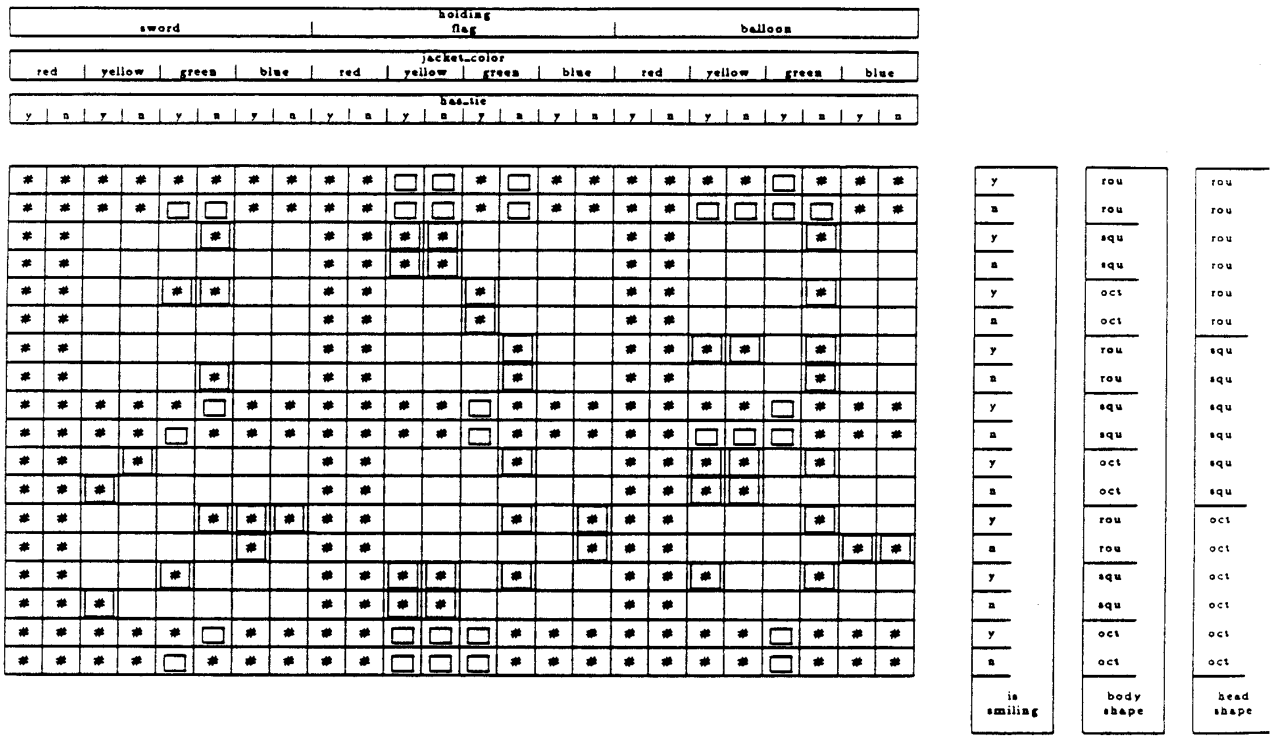
Accuracy: 67.361111

Result of ID3 on test set No. 3

head shape	body shape	is smiling	sword								flag								balloon							
		jacket_color	red		yellow		green		blue		red		yellow		green		blue		red		yellow		green		blue	
		has_tie	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n
rou	rou	y	*	*	*	*	□	□			*	*	*	*	□	□			*	*	*	*	□	□		
rou	rou	n	*	*	*	*	*	*			*	*	*	*	*	*			*	*	*	*	*	*		
rou	squ	y	*	*	*	*	*	*			*	*	*	*	□	□			*	*	*	*	*	*		
rou	squ	n	*	*	*	*	*	*			*	*	*	*	□	□			*	*	*	*	*	*		
rou	oct	y					*	*																		
rou	oct	n					*	*																		
squ	rou	y	*	*	*	*	*	*			*	*	*	*	*	*			*	*	*	*	*	*		
squ	rou	n	*	*	*	*	*	*			*	*	*	*	*	*			*	*	*	*	*	*		
squ	squ	y	*	*	*	*	*	*			*	*	□	□	*	*			*	*	*	*	□	□		
squ	squ	n	*	*	*	*	*	*			*	*	□	□	*	*			*	*	*	*	□	□		
squ	oct	y					*	*																		
squ	oct	n					*	*																		
oct	rou	y	*	*	*	*	*	*			*	*	*	*	*	*			*	*	*	*	*	*		
oct	rou	n	*	*	*	*	*	*			*	*	*	*	*	*			*	*	*	*	*	*		
oct	squ	y	*	*	*	*	*	*			*	*	*	*	*	*			*	*	*	*	*	*		
oct	squ	n	*	*	*	*	*	*			*	*	□	*	*	*			*	*	*	*	*	*		
oct	oct	y					*	*	*	*																
oct	oct	n					*	*	*	*																

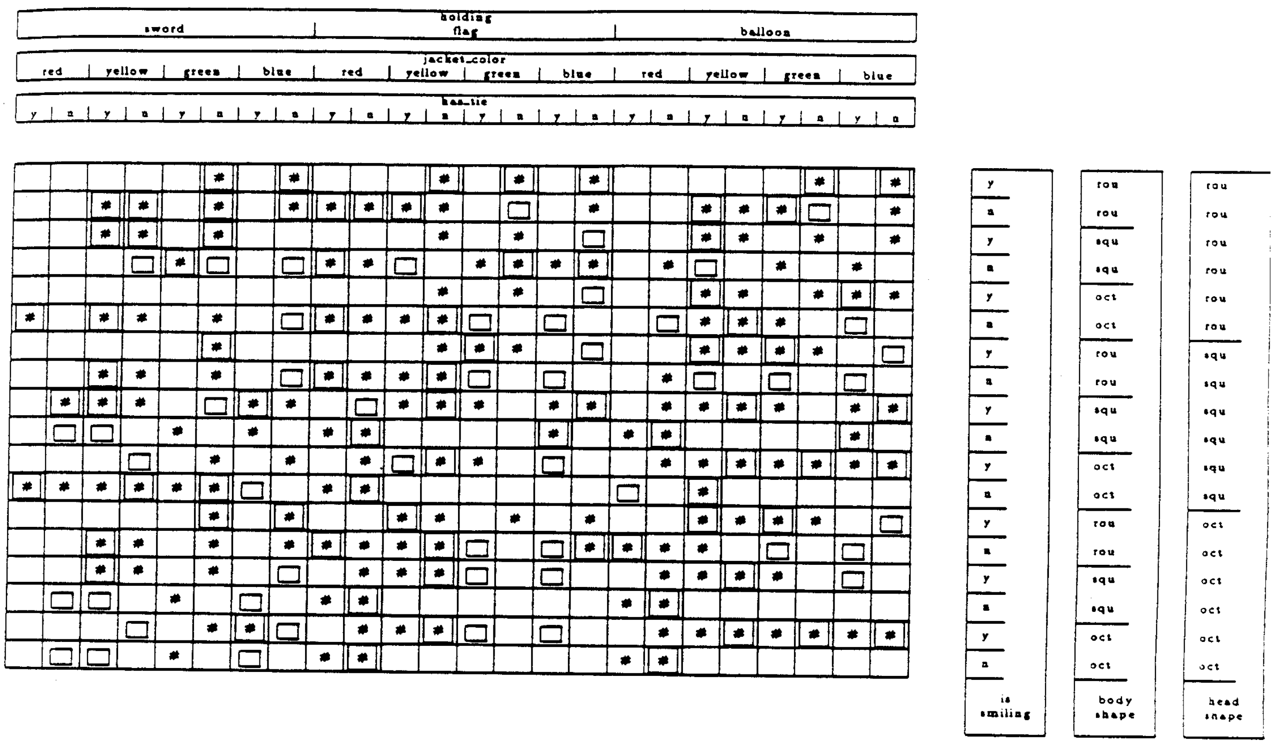
Accuracy: 94.675926

Result of ID3OW on test set No. 1



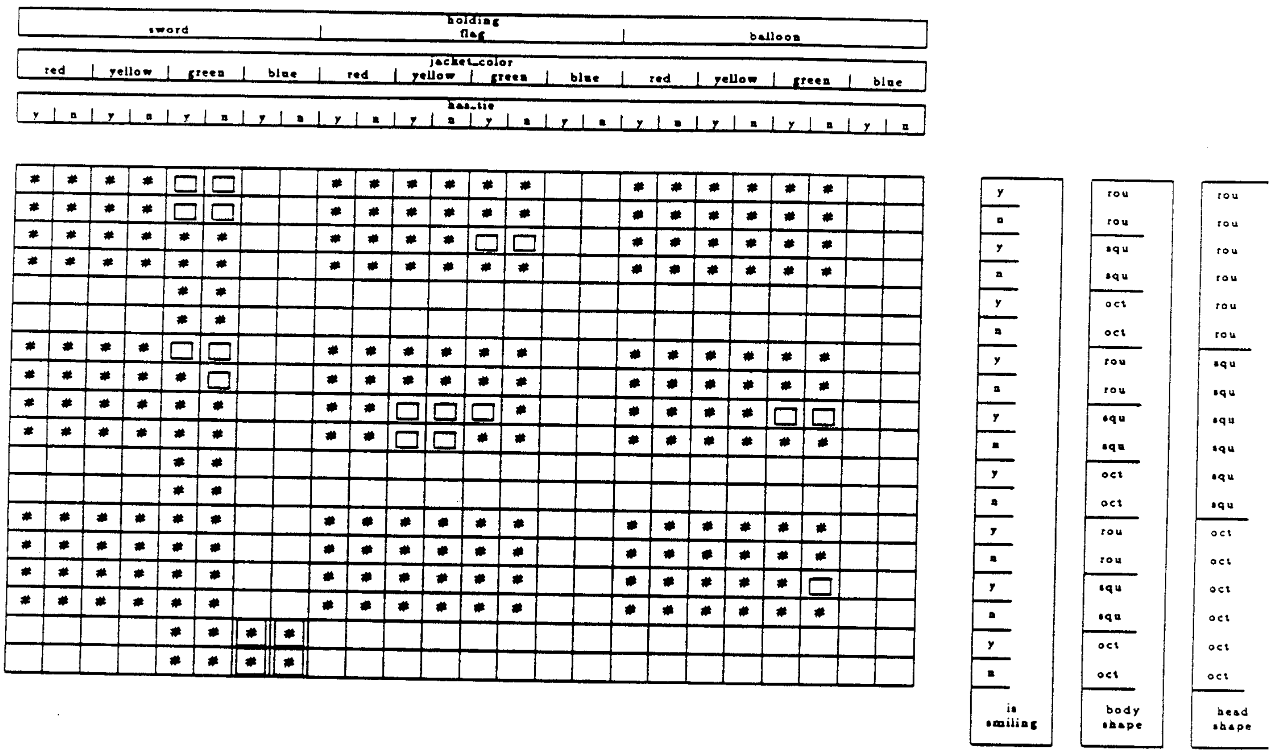
Accuracy: 82.407407

Result of ID3OW on test set No. 2



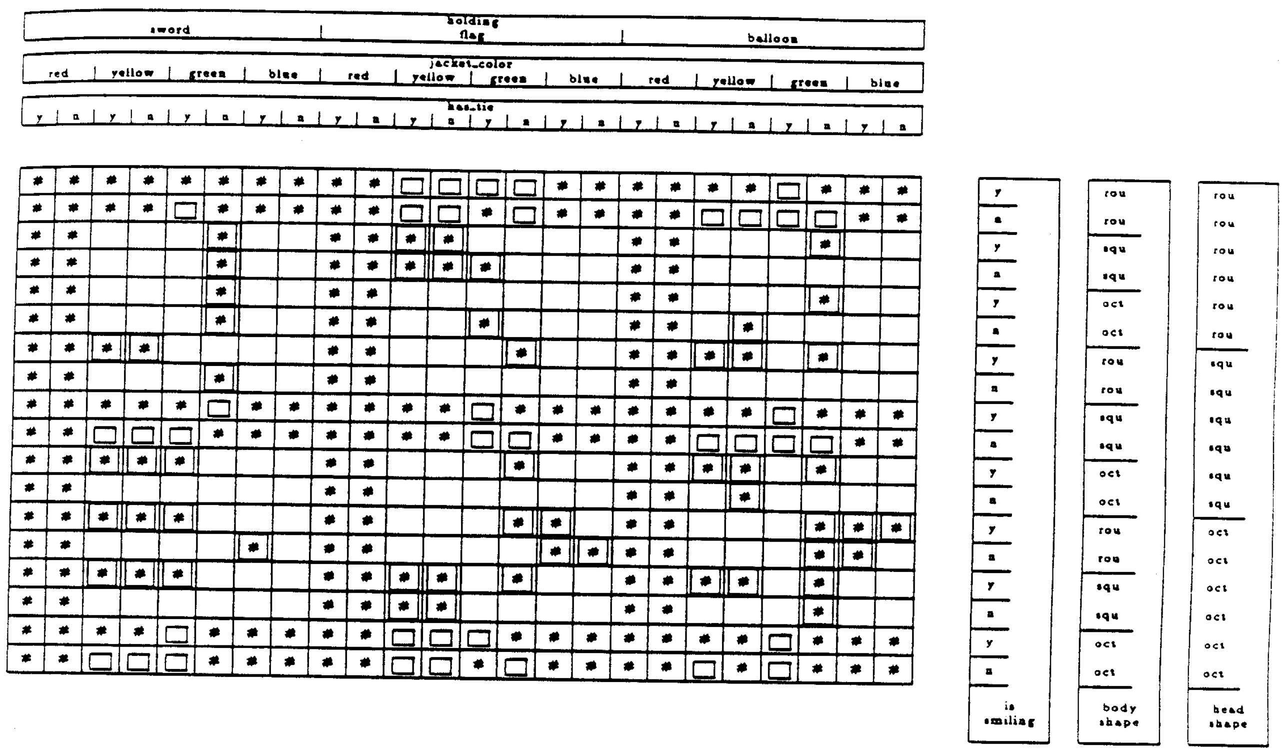
Accuracy: 69.907407

Result of ID3OW on test set No. 3



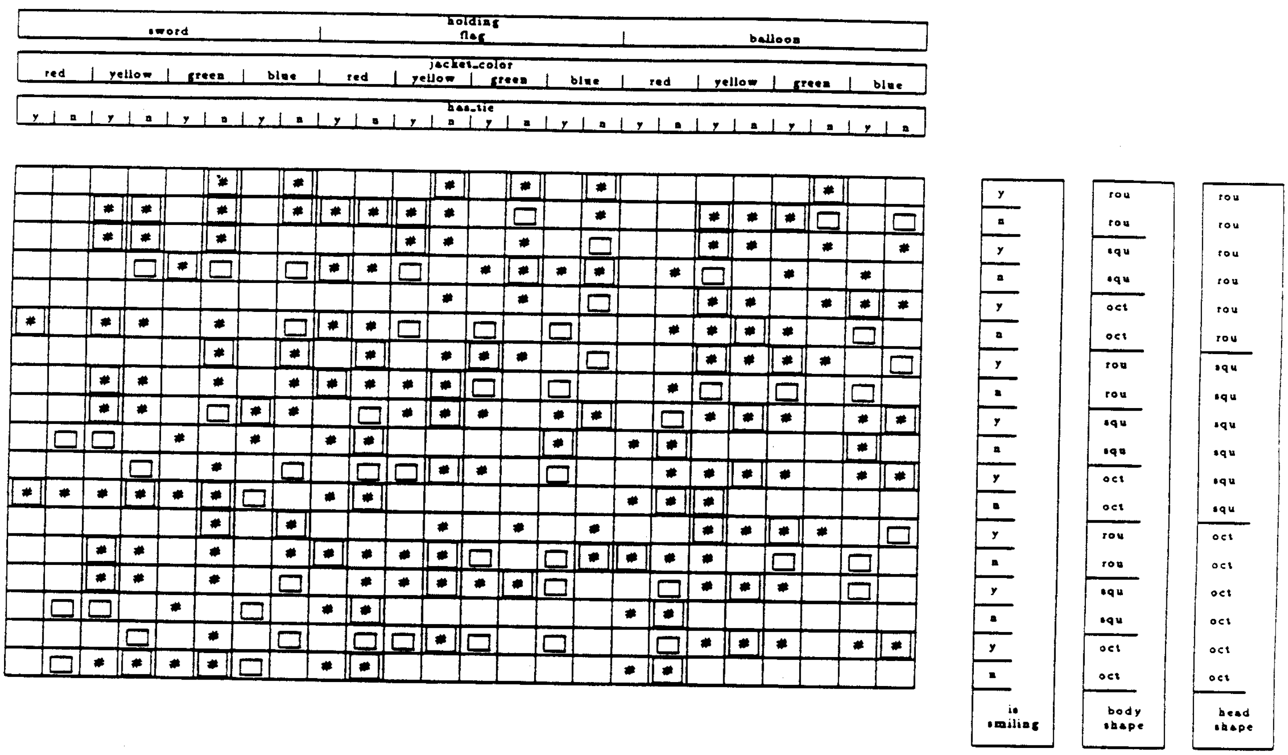
Accuracy: 95.138889

Result of ID5R on test set No. 1



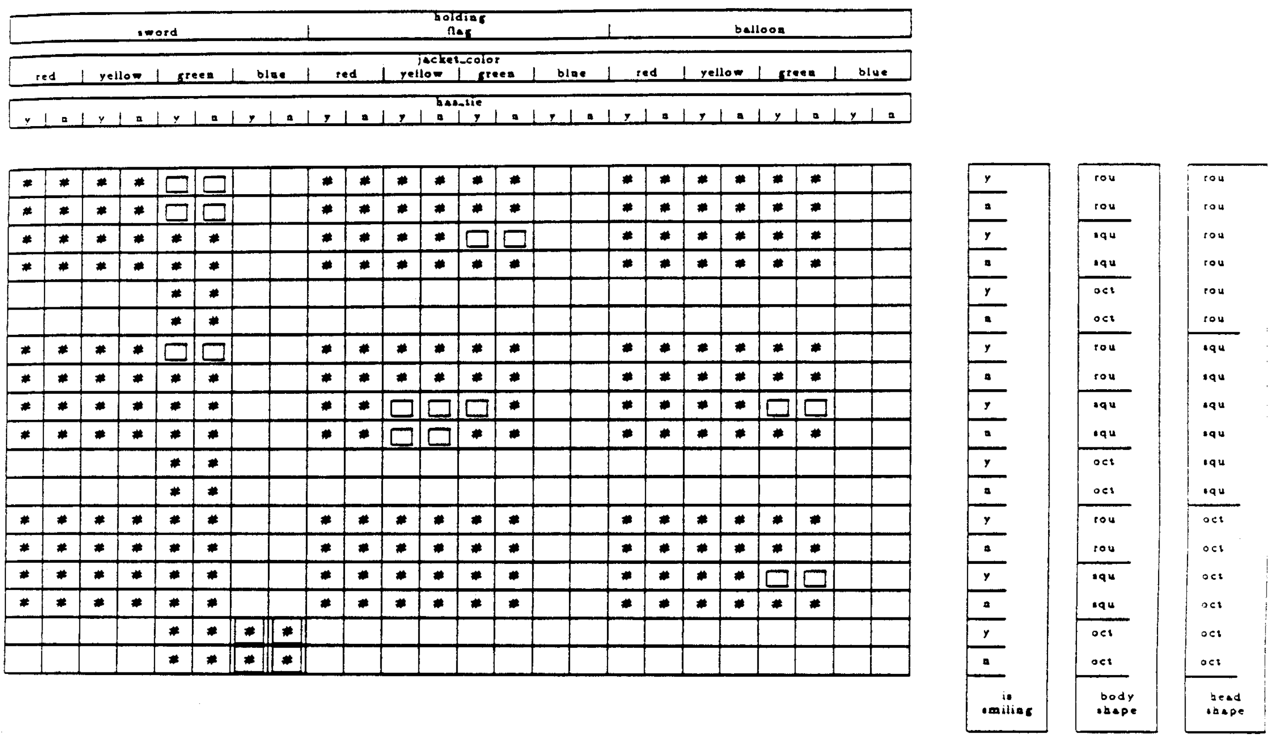
Accuracy: 78.935185

Result of ID5R on test set No. 2



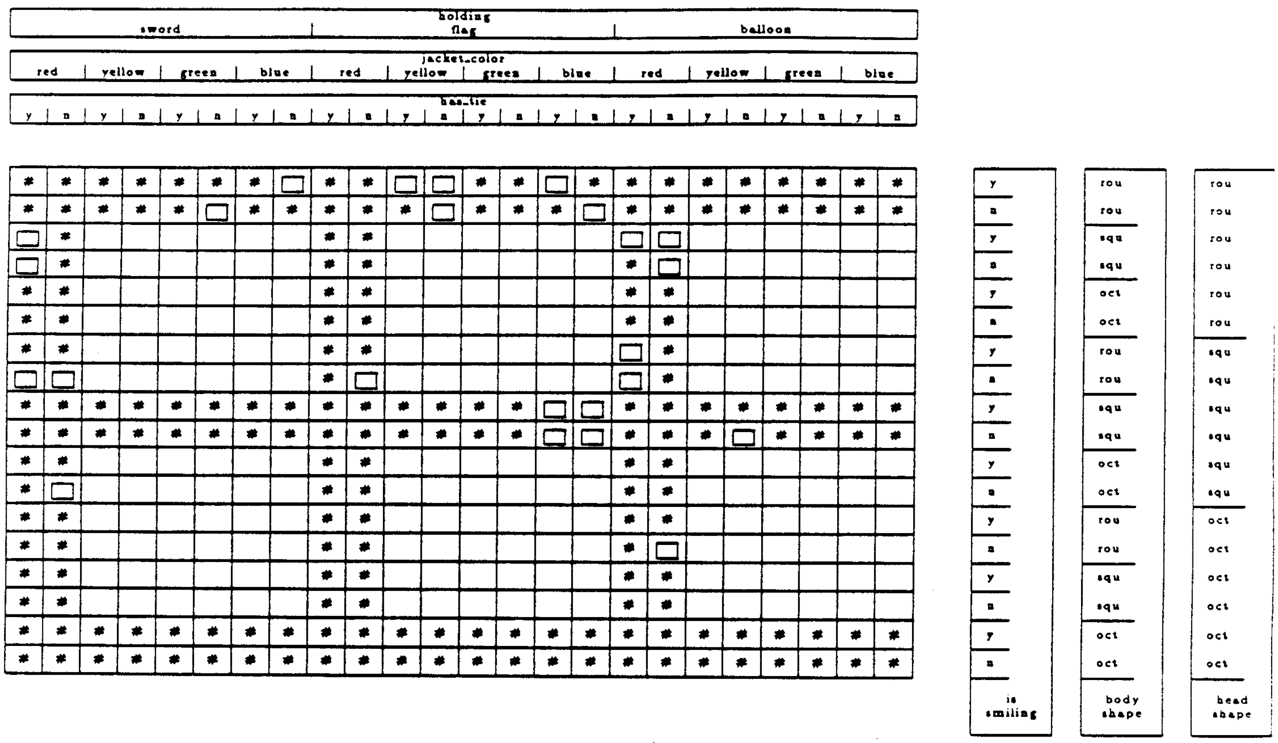
Accuracy: 68.981481

Result of ID5R on test set No. 3



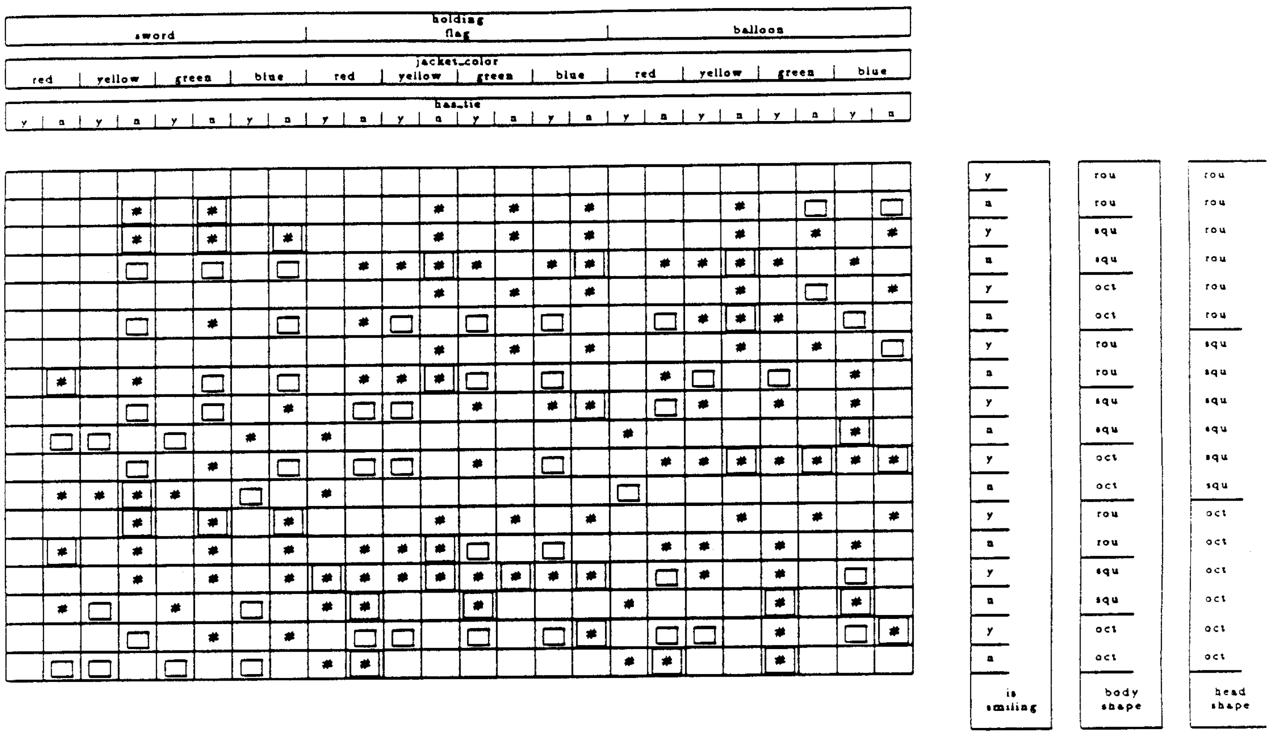
Accuracy: 95.138889

Result of AQR on test set No. 1



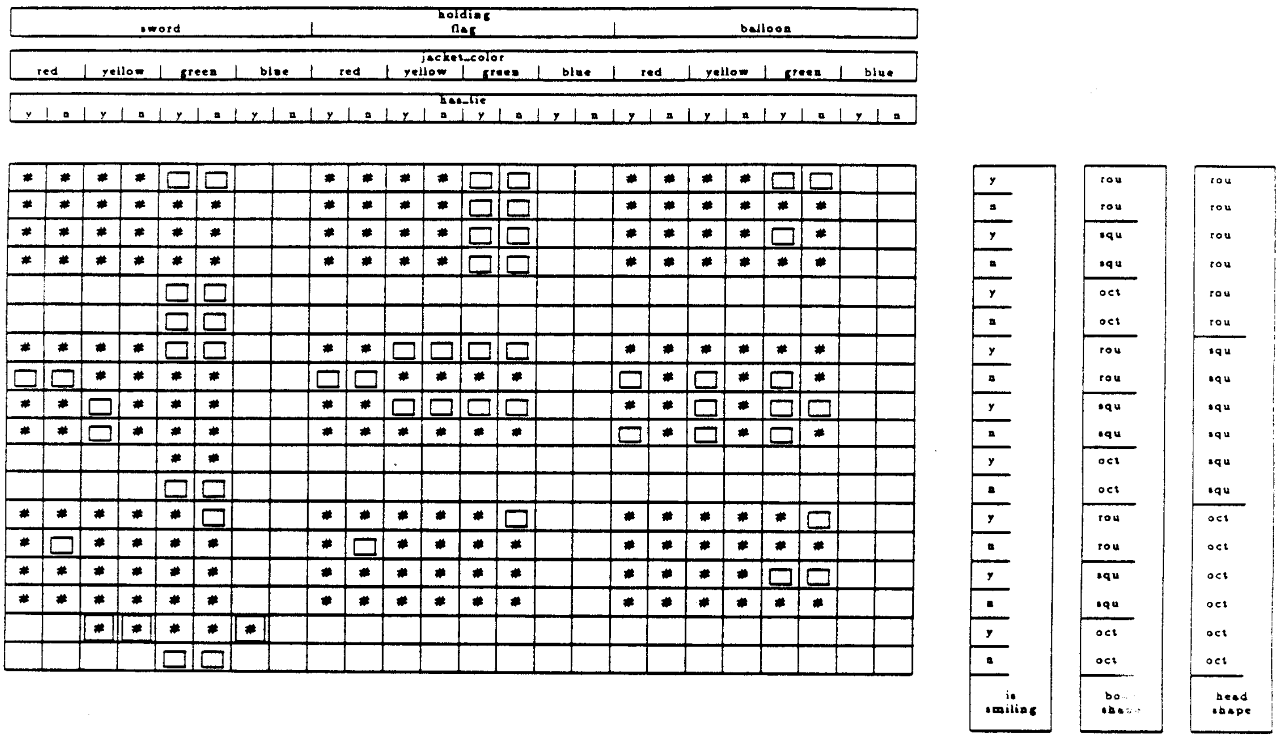
Accuracy: 94.444444

Result of AQR on test set No. 2



Accuracy: 79.629630

Result of AQR on test set No. 3



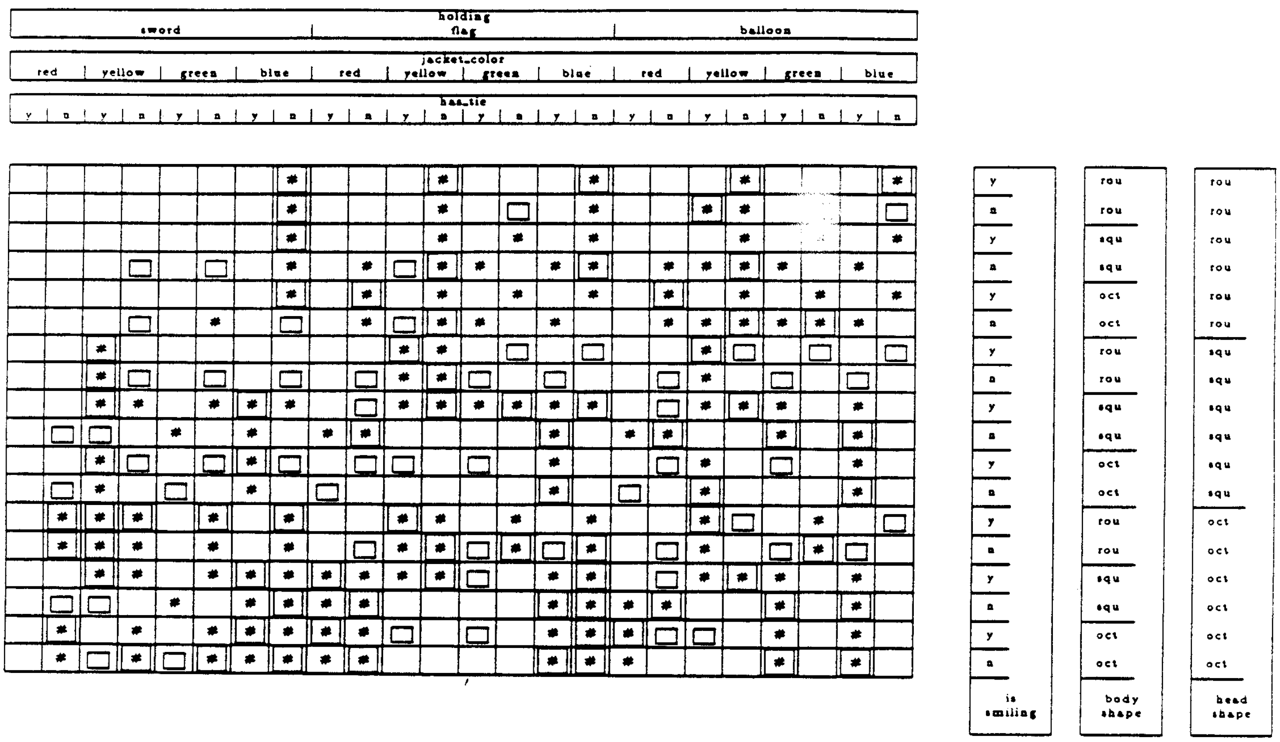
Accuracy: 87.037037

Result of CN2 on test set No. 1

holding			sword								flag								balloon							
jacket_color			red		yellow		green		blue		red		yellow		green		blue		red		yellow		green		blue	
has_tie			y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n
head shape	**body shape**	**is smiling**																								
rou	rou	y	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#
rou	rou	n	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#
rou	squ	y	#	#							#	#							#	#						
rou	squ	n	#	#							#	#							#	#						
rou	oct	y	#	#							#	#							#	#						
rou	oct	n	#	#							#	#							#	#						
squ	rou	y	#	#							#	#							#	#						
squ	rou	n	#	#							#	#							#	#						
squ	squ	y	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#
squ	squ	n	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#
squ	oct	y	#	#							#	#							#	#						
squ	oct	n	#	#							#	#							#	#						
oct	rou	y	#	#							#	#							#	#						
oct	rou	n	#	#							#	#							#	#						
oct	squ	y	#	#							#	#							#	#						
oct	squ	n	#	#							#	#							#	#						
oct	oct	y	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#
oct	oct	n	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#	#

Accuracy: 100.000000

Result of CN2 on test set No. 2



Accuracy: 68.981481

Result of CN2 on test set No. 3

holding	sword								flag								balloon										
jacket_color	red		yellow		green		blue		red		yellow		green		blue		red		yellow		green		blue				
has_tie	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	is smiling	body shape	head shape
	#	#	#	#	□	#			#	#	#	#	□	□			#	#	#	#	□	#			y	rou	rou
	#	#	#	#	□	#			#	#	#	#	□	□			#	#	#	#	#	#			n	rou	rou
	#	#	#	#	#	#			#	#	#	#	□	□			#	#	#	#	□	#			y	squ	rou
	#	#	#	#	#	#			#	#	#	#	□	□			#	#	#	#	#	#			n	squ	rou
					□	#																$\boxed{\#}$			y	oct	rou
					□	#																$\boxed{\#}$			n	oct	rou
	#	#	#	#	#	#			#	#	□	#	#	#			#	#	□	#	□	#			y	rou	squ
	#	#	#	#	#	#			#	#	□	#	#	#			#	#	□	#	#	#			n	rou	squ
	#	#	#	#	#	#			#	#	□	#	#	#			#	#	□	#	□	□			y	squ	squ
	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#			n	squ	squ
					#	#																			y	oct	squ
					#	#																			n	oct	squ
	#	#	#	#	#	#	$\boxed{\#}$		#	#	□	#	#	#	$\boxed{\#}$		#	#	□	#	□	□			y	rou	oct
	#	#	#	#	#	#	$\boxed{\#}$		#	#	□	#	#	#	$\boxed{\#}$		#	#	□	#	□	□			n	rou	oct
	#	#	#	#	#	#			#	#	□	#	#	#			#	#	□	#	□	□			y	squ	oct
	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#			n	squ	oct
					#	#	$\boxed{\#}$						$\boxed{\#}$		$\boxed{\#}$										y	oct	oct
					#	#	$\boxed{\#}$						$\boxed{\#}$		$\boxed{\#}$										n	oct	oct

Accuracy: 89.120370

Chapter 7

Documentation of Prism – an Inductive Learning Algorithm

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7.1 Short Description

PRISM was invented by Jadzia Cendrowska (1987). Based on Quinlan's induction algorithm ID3, PRISM pays attention to maximizing he information gain for a single value of an atribute in contrast to ID3 which tries to minimize the average entropy for an attribute-value pair.

7.2 Introduction

The decision tree output of ID3 algorithm is one of its major weaknesses. Not only can it be incomprehensible and difficult to manipulate, but its use in knowledge based systems frequently demands irrelevant information to be supplied. We argue that the problem lies in the induction algorithm itself and can only be remedied by radically altering the underlying strategy. The resulting algorithm, although based on ID3, uses a different induction strategy to induce rules which are modular in the sense how they are constructed. This approach avoids many of the problems associated with decision trees.

7.3 PRISM: Entropy versus Information Gain

The main cause of the problem described above is either that an attribute is highly relevant to only one classification and irrelevant to the others, or that only one value of the attribute is relevant.

There can be shown that while in the construction process of a decision tree although e.g. the entropy of a distinct branch d1 has been reduced to 0, the entropy of the other branch has actually increased to some higher entropy-measure. Attribute d would be chosen by ID3 because it minimizes the average entropy of the training set, or alternatively, it maximizes the average amount of information contributed by an attribute to the determination of any classifiaction.

In order to eliminate the use of irrelevant values of attributes and attributes which are irrelevant to a classification, an improving algorithm needs to maximize the actual amount of information contributed by knowing the VALUE of the attribute to the determination of a specific classification.

7.3.1 Maximizing the information gain

So, the task of an induction algorithm must be to find the attribute-value pair, ax, which contributes the most information about a specified classification, dn, i.e. for which I(dn l ax) is maximum.

This can be done in the following way: Let S be the data set; first find the ax for which p(dn — ax) is maximum. Lets call the choosen attribute c2 (=attribute c, value 2). Repeat now the process on a subset of S which contains only those instances which have value 2 for attribute c until there are all instances removed.

7.3.2 Trimming the tree

The remaining "branches" are not yet labelled, so the next step in the induction process is to identify the best rule of the set of instances which are not examples of the first rule. This is done by removing from S all instances containing this rule and applying the algorithm to the remaining instances. If this is repeated until there are no instances of class d1 left in S, the result is not a decision tree but a collection of branches. The whole process can then be repeated for each classification in turn, starting with the complete training set, S, each time.

The final output is an unordered collection of modular rules, each rule being as general as possible, thus ensuring that there are no redundant terms.

The following assumptions have been made about the training set:

- the classifications are mutually exclusive
- there is no noise, i.e. each instance is complete and correct
- each instance can be classified uniquely
- no instance is duplicated
- the values of the attributes are discrete
- the training set is complete, i.e. all possible combinations of attribute-value pairs are represented

Given that the assumptions above hold, the algorithm produces a complete set of correct rules.

7.4 The Basic Algorithm

If the training set contains instances of more than one classification, then for each classification, dn, in turn:

Step 1:
calculate the probability of occurence, p(dn — ax), of the classification dn for each attribute-value pair ax,

Step 2:
select the ax for which p(dn — ax) is a maximum and create a subset of the training set comprising all the instances which contain the selected ax,

Step 3:
repeat Steps 1 and 2 for this subset until is contains only instances of class dn. The induced rule is a conjunction of all the attribute-value pairs used in creating the homogeneous subset.

Step 4:
remove all instances covered by this rule from the training set.

Step 5:
repeat Steps 1-4 until all instances of class dn, have been removed.

When the rules for one classification have been induced, the training set is restored to its initial state and the algorithm is applied again to induce a set of rules covering the next classification. As the classifications are considered separately, their order of presentation is immaterial. If all instances are of the same classification then that classification is returned as the rule, and the algorithm terminates.

7.5 The Use of Heuristics

Opting for generality I: If there are two or more rules describing a classification, PRISM tries to induce the most general rule first. Thus PRISM selects that attribute-value pair which has the highest frequency of occurence in the set of instances being considered.

Opting for generality II: When both the information gain offered by two or more attribute-value pairs is the same and the numbers of instances referencing them is the same, PRISM selects the first.

7.6 General Considerations and a Comparison with ID3

A rule will not be induced by PRISM if there are no examples of it in the training set, but this applies to all induction programs. Even human beings cannot be expected to induce rules from non-existent information.

The accuracy of rules induced from an incomplete training set depends on the size of that training set (as with all induction algorithms) but is comparable to the accuracy of a decision tree induced by ID3 from the same training set, despite the gross reduction in number and length of the rules.

The major difference between ID3 and PRISM is that PRISM concentrates on finding only relevant values of attributes, while ID3 is concerned with finding the attribute which is most relevant overall, even though some values of that attribute may be irrelevant. All other differences between the two algorithms stem form this: ID3 divides a training set into homogeneous subsets without reference to the class of this subset, whereas PRISM must identify subsets of a specific class. This has the disadvantage of slightly incresed computational effort, but the advantage of an output in the form of modular rules rather than a decision tree.

7.7 Implementation

Version:	0.9
Status:	Experimental
Language:	Common Lisp
Authors:	Lindsey Spratt (spratt@hawk.cs.ukans.edu), Spring 1990, modified by Stefan F. Keller (keller@ifi.unizh.ch), Summer 1991.

References

- Cendrowska, Jadzia (1987); PRISM: An algorithm for inducing modular rules, in: Int. Journal of Man-Machine Studies, Vol. 26, Nr.1,2,4, Vol.27, Nr.2,3,4.
- Cendrowska, Jadzia (1988); PRISM: An algorithm for inducing modular rules, in: B.R.Gaines & J.H.Boose (eds.), Knowledge Acquisition for Knowledge-Based Systems, Academic Press, 253-274.

7.8 Results on Running PRISM on the MONK's Test Sets

TEST PLATFORMS:
Mac: Macintosh Allegro Common Lisp 2.0b2, Macintosh IIci, 4MB memory
Sun: Franz Allegro Common Lisp 4.0.1, Sun sparc/320, 24MB memory

TEST SET 1:
No. of trainig-examples: 124
No. of test-examples: 432
No. of rules induced: 29
Covered test-examples: 86Mac run time: 80.14s, 85.10s, 80.43s, 81.10s, 80.05s
Sun run time: 23.30s, 22.80s, 23.50s, 23.12s, 23.08s
Average run time on Sun: 23.16s

TEST SET 2:
No. of trainig-examples: 169
No. of test-examples: 432
No. of rules induced: 73
Covered test-examples: 73Mac run time: (409.26s)
Sun run time: 121.50s, 122.50s, 120.75s, 122.18s, 121.00s
Average run time on Sun: 121.58s

TEST SET 3:
No. of trainig-examples: 122
No. of test-examples: 432
No. of rules induced: 26
Covered test-examples: 90Mac run time: (59.63s)
Sun run time: 16.77s, 17.00s, 16.63s, 16.60s, 17.30s
Average run time on Sun: 16.86s

7.8.1 Test Set 1 – Rules

```
(RULE-1
  (IF   ((jacket_color 1)))
  (THEN (class 1)))
(RULE-2
  (IF   ((head_shape 3) (body_shape 3)))
  (THEN (class 1)))
(RULE-3
  (IF   ((holding 1) (body_shape 2) (head_shape 2)))
  (THEN (class 1)))
(RULE-4
  (IF   ((body_shape 1) (head_shape 1)))
  (THEN (class 1)))
(RULE-5
  (IF   ((body_shape 2) (head_shape 2)))
  (THEN (class 1)))
(RULE-6
  (IF   ((head_shape 1) (jacket_color 4) (body_shape 3)))
  (THEN (class 0)))
(RULE-7
  (IF   ((jacket_color 2) (holding 2) (has_tie 2)))
  (THEN (class 0)))
(RULE-8
  (IF   ((jacket_color 3) (has_tie 1) (holding 3)))
  (THEN (class 0)))
(RULE-9
  (IF   ((jacket_color 3) (holding 2) (head_shape 1) (has_tie 2)))
  (THEN (class 0)))
(RULE-10
  (IF   ((jacket_color 2) (head_shape 1) (body_shape 3)))
  (THEN (class 0)))
(RULE-11
  (IF   ((jacket_color 4) (body_shape 1) (head_shape 2)))
  (THEN (class 0)))
(RULE-12
  (IF   ((jacket_color 3) (has_tie 1) (body_shape 3)))
  (THEN (class 0)))
(RULE-13
  (IF   ((jacket_color 3) (has_tie 1) (head_shape 2) (body_shape 1)))
  (THEN (class 0)))
(RULE-14
  (IF   ((jacket_color 2) (is_smiling 2) (holding 3) (body_shape 1)))
  (THEN (class 0)))
(RULE-15
  (IF   ((head_shape 1) (body_shape 2) (is_smiling 2)))
  (THEN (class 0)))
(RULE-16
  (IF   ((jacket_color 3) (is_smiling 2) (head_shape 2) (body_shape 3)))
  (THEN (class 0)))
```

```
(RULE-17
  (IF   ((jacket_color 2) (is_smiling 2) (head_shape 2) (body_shape 1)))
  (THEN (class 0)))
(RULE-18
  (IF   ((jacket_color 4) (head_shape 1) (is_smiling 1)))
  (THEN (class 0)))
(RULE-19
  (IF   ((jacket_color 2) (holding 2) (body_shape 3)))
  (THEN (class 0)))
(RULE-20
  (IF   ((jacket_color 3) (body_shape 2) (head_shape 1)))
  (THEN (class 0)))
(RULE-21
  (IF   ((jacket_color 2) (body_shape 2) (head_shape 1)))
  (THEN (class 0)))
(RULE-22
  (IF   ((jacket_color 4) (is_smiling 1) (body_shape 2) (head_shape 3)))
  (THEN (class 0)))
(RULE-23
  (IF   ((jacket_color 3) (head_shape 3) (body_shape 1)))
  (THEN (class 0)))
(RULE-24
  (IF   ((jacket_color 2) (head_shape 3) (body_shape 1)))
  (THEN (class 0)))
(RULE-25
  (IF   ((jacket_color 3) (holding 1) (head_shape 3) (body_shape 2)))
  (THEN (class 0)))
(RULE-26
  (IF   ((jacket_color 4) (holding 3) (has_tie 1) (head_shape 3)))
  (THEN (class 0)))
(RULE-27
  (IF   ((holding 3) (jacket_color 4) (is_smiling 1) (body_shape 3) (head_shape 2)))
  (THEN (class 0)))
(RULE-28
  (IF   ((jacket_color 3) (body_shape 1) (head_shape 2)))
  (THEN (class 0)))
(RULE-29
  (IF   ((jacket_color 2) (is_smiling 2) (holding 3) (has_tie 1)))
  (THEN (class 0)))
```

7.8.2 Test Set 2 – Rules

```
(RULE-1
  (IF   ((holding 1) (jacket_color 1) (has_tie 1)))
  (THEN (class 0)))
(RULE-2
  (IF   ((jacket_color 4) (body_shape 1) (has_tie 1)))
  (THEN (class 0)))
(RULE-3
  (IF   ((head_shape 1) (holding 1) (is_smiling 1)))
  (THEN (class 0)))
(RULE-4
  (IF   ((jacket_color 4) (body_shape 3) (is_smiling 2)))
  (THEN (class 0)))
(RULE-5
  (IF   ((jacket_color 3) (is_smiling 2) (holding 2) (has_tie 2)))
  (THEN (class 0)))
(RULE-6
  (IF   ((has_tie 1) (head_shape 1) (is_smiling 1)))
  (THEN (class 0)))
(RULE-7
  (IF   ((holding 1) (head_shape 1) (has_tie 1)))
  (THEN (class 0)))
(RULE-8
  (IF   ((head_shape 2) (has_tie 2) (body_shape 2) (is_smiling 2)))
  (THEN (class 0)))
(RULE-9
  (IF   ((jacket_color 1) (is_smiling 1) (body_shape 1)))
  (THEN (class 0)))
(RULE-10
  (IF   ((jacket_color 3) (is_smiling 2) (holding 3) (has_tie 2)))
  (THEN (class 0)))
(RULE-11
  (IF   ((holding 1) (jacket_color 1) (is_smiling 1)))
  (THEN (class 0)))
(RULE-12
  (IF   ((is_smiling 2) (jacket_color 2) (body_shape 2)))
  (THEN (class 0)))
(RULE-13
  (IF   ((jacket_color 3) (has_tie 1) (body_shape 1)))
  (THEN (class 0)))
(RULE-14
  (IF   ((jacket_color 1) (head_shape 1) (body_shape 1)))
  (THEN (class 0)))
(RULE-15
  (IF   ((head_shape 2) (holding 2) (jacket_color 4)))
  (THEN (class 0)))
(RULE-16
  (IF   ((jacket_color 3) (head_shape 3) (body_shape 3) (has_tie 1)))
```

```
  (THEN (class 0)))
(RULE-17
  (IF   ((head_shape 2) (holding 2) (body_shape 3) (jacket_color 2)))
  (THEN (class 0)))
(RULE-18
  (IF   ((holding 3) (is_smiling 2) (jacket_color 4) (has_tie 2)))
  (THEN (class 0)))
(RULE-19
  (IF   ((jacket_color 1) (is_smiling 1) (has_tie 1)))
  (THEN (class 0)))
(RULE-20
  (IF   ((jacket_color 3) (head_shape 2) (is_smiling 2) (holding 2)))
  (THEN (class 0)))
(RULE-21
  (IF   ((jacket_color 3) (head_shape 2) (has_tie 2) (holding 2)))
  (THEN (class 0)))
(RULE-22
  (IF   ((jacket_color 3) (head_shape 3) (body_shape 2) (has_tie 2) (holding 3)))
  (THEN (class 0)))
(RULE-23
  (IF   ((holding 1) (is_smiling 1) (has_tie 1)))
  (THEN (class 0)))
(RULE-24
  (IF   ((holding 3) (is_smiling 2) (head_shape 2) (jacket_color 2)))
  (THEN (class 0)))
(RULE-25
  (IF   ((jacket_color 1) (head_shape 1) (is_smiling 1)))
  (THEN (class 0)))
(RULE-26
  (IF   ((is_smiling 2) (holding 3) (body_shape 3) (jacket_color 3)))
  (THEN (class 0)))
(RULE-27
  (IF   ((head_shape 3) (body_shape 3) (jacket_color 2)))
  (THEN (class 0)))
(RULE-28
  (IF   ((body_shape 1) (jacket_color 1) (has_tie 1)))
  (THEN (class 0)))
(RULE-29
  (IF   ((jacket_color 3) (head_shape 3) (holding 2) (body_shape 3)))
  (THEN (class 0)))
(RULE-30
  (IF   ((holding 1) (body_shape 1) (is_smiling 1)))
  (THEN (class 0)))
(RULE-31
  (IF   ((body_shape 2) (jacket_color 3) (has_tie 2) (is_smiling 2)))
  (THEN (class 0)))
(RULE-32
  (IF   ((holding 3) (is_smiling 2) (jacket_color 2) (has_tie 2) (head_shape 3)))
  (THEN (class 0)))
(RULE-33
  (IF   ((body_shape 2) (holding 3) (jacket_color 1) (head_shape 1) (has_tie 1)))
```

```
  (THEN (class 0)))
 (RULE-34
  (IF   ((jacket_color 3) (holding 3) (head_shape 2) (has_tie 2) (body_shape 2)))
  (THEN (class 0)))
 (RULE-35
  (IF   ((jacket_color 2) (is_smiling 1) (body_shape 2)))
  (THEN (class 1)))
 (RULE-36
  (IF   ((jacket_color 2) (body_shape 1) (head_shape 3) (is_smiling 1)))
  (THEN (class 1)))
 (RULE-37
  (IF   ((holding 3) (body_shape 3) (jacket_color 1)))
  (THEN (class 1)))
 (RULE-38
  (IF   ((jacket_color 2) (body_shape 1) (is_smiling 2) (has_tie 1)))
  (THEN (class 1)))
 (RULE-39
  (IF   ((jacket_color 3) (is_smiling 1) (has_tie 2) (body_shape 1)))
  (THEN (class 1)))
 (RULE-40
  (IF   ((body_shape 2) (jacket_color 4) (is_smiling 1) (has_tie 2)))
  (THEN (class 1)))
 (RULE-41
  (IF   ((jacket_color 3) (body_shape 2) (head_shape 1) (has_tie 2) (is_smiling 1)))
  (THEN (class 1)))
 (RULE-42
  (IF   ((body_shape 3) (jacket_color 2) (holding 3) (head_shape 1)))
  (THEN (class 1)))
 (RULE-43
  (IF   ((head_shape 2) (has_tie 1) (body_shape 3) (jacket_color 4)))
  (THEN (class 1)))
 (RULE-44
  (IF   ((head_shape 2) (has_tie 1) (body_shape 3) (holding 1) (is_smiling 2)))
  (THEN (class 1)))
 (RULE-45
  (IF   ((holding 2) (jacket_color 1) (is_smiling 2)))
  (THEN (class 1)))
 (RULE-46
  (IF   ((jacket_color 3) (holding 1) (has_tie 2) (body_shape 1)))
  (THEN (class 1)))
 (RULE-47
  (IF   ((holding 3) (has_tie 1) (body_shape 2) (jacket_color 3) (is_smiling 2)))
  (THEN (class 1)))
 (RULE-48
  (IF   ((has_tie 2) (body_shape 1) (jacket_color 2) (holding 2)))
  (THEN (class 1)))
 (RULE-49
  (IF   ((has_tie 2) (body_shape 1) (jacket_color 2) (is_smiling 2) (holding 1)))
  (THEN (class 1)))
 (RULE-50
  (IF   ((holding 3) (jacket_color 1) (is_smiling 2) (head_shape 3)))
```

```
  (THEN (class 1)))
 (RULE-51
  (IF   ((jacket_color 3) (is_smiling 1) (head_shape 2) (holding 2) (has_tie 1)))
  (THEN (class 1)))
 (RULE-52
  (IF   ((has_tie 2) (head_shape 1) (jacket_color 2) (holding 2)))
  (THEN (class 1)))
 (RULE-53
  (IF   ((body_shape 2) (jacket_color 4) (has_tie 1) (head_shape 2)))
  (THEN (class 1)))
 (RULE-54
  (IF   ((has_tie 2) (head_shape 1) (jacket_color 4) (body_shape 1)))
  (THEN (class 1)))
 (RULE-55
  (IF   ((jacket_color 3) (holding 1) (has_tie 2) (is_smiling 1) (head_shape 3)))
  (THEN (class 1)))
 (RULE-56
  (IF   ((holding 3) (jacket_color 2) (is_smiling 1) (head_shape 2)))
  (THEN (class 1)))
 (RULE-57
  (IF   ((body_shape 2) (has_tie 1) (jacket_color 3) (holding 3) (head_shape 2)))
  (THEN (class 1)))
 (RULE-58
  (IF   ((has_tie 2) (head_shape 1) (holding 3) (jacket_color 2)))
  (THEN (class 1)))
 (RULE-59
  (IF   ((body_shape 2) (has_tie 1) (jacket_color 4) (is_smiling 2) (holding 2)))
  (THEN (class 1)))
 (RULE-60
  (IF   ((has_tie 2) (body_shape 3) (head_shape 1) (holding 3)))
  (THEN (class 1)))
 (RULE-61
  (IF   ((jacket_color 3) (head_shape 1) (has_tie 2) (body_shape 3)
                                         (is_smiling 1) (holding 2)))
  (THEN (class 1)))
 (RULE-62
  (IF   ((body_shape 2) (jacket_color 1) (is_smiling 2) (has_tie 2)))
  (THEN (class 1)))
 (RULE-63
  (IF   ((jacket_color 3) (holding 1) (has_tie 2) (body_shape 3) (is_smiling 2)))
  (THEN (class 1)))
 (RULE-64
  (IF   ((body_shape 2) (has_tie 1) (jacket_color 3) (is_smiling 2) (head_shape 1)))
  (THEN (class 1)))
 (RULE-65
  (IF   ((head_shape 3) (jacket_color 4) (holding 2) (has_tie 2)))
  (THEN (class 1)))
 (RULE-66
  (IF   ((jacket_color 1) (head_shape 2) (is_smiling 2) (has_tie 2)))
  (THEN (class 1)))
 (RULE-67
```

```
  (IF   ((body_shape 2) (head_shape 3) (is_smiling 1) (holding 2)))
  (THEN (class 1)))
(RULE-68
  (IF   ((body_shape 2) (has_tie 1) (holding 3) (is_smiling 2) (jacket_color 4)))
  (THEN (class 1)))
(RULE-69
  (IF   ((jacket_color 3) (holding 1) (has_tie 2) (is_smiling 1) (head_shape 2)))
  (THEN (class 1)))
(RULE-70
  (IF   ((head_shape 3) (jacket_color 3) (has_tie 1) (holding 3)))
  (THEN (class 1)))
(RULE-71
  (IF   ((head_shape 3) (jacket_color 4) (holding 1) (has_tie 2)))
  (THEN (class 1)))
(RULE-72
  (IF   ((body_shape 2) (has_tie 1) (is_smiling 2) (jacket_color 3) (holding 1)))
  (THEN (class 1)))
(RULE-73
  (IF   ((jacket_color 1) (holding 3) (head_shape 2) (body_shape 2)))
  (THEN (class 1)))
```

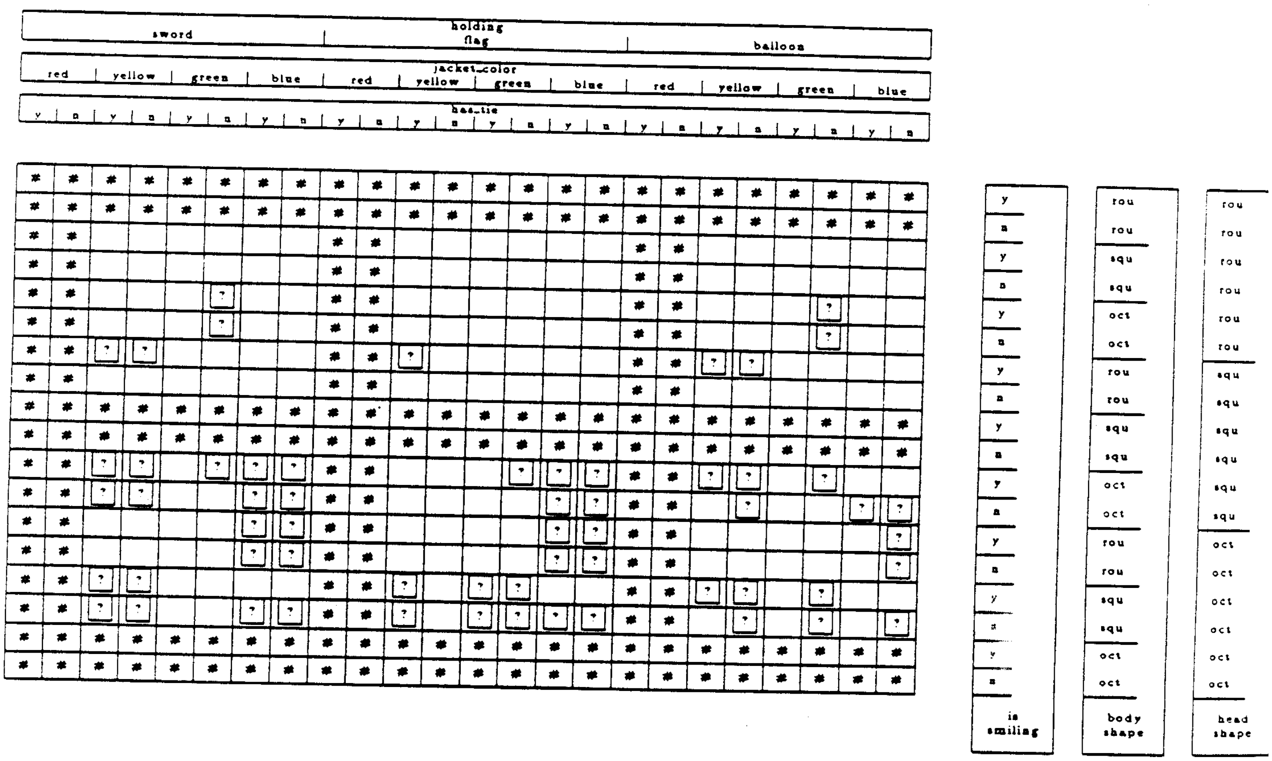
7.8.3 Test Set 3 – Rules

```
(RULE-1
  (IF   ((body_shape 2) (jacket_color 1)))
  (THEN (class 1)))
(RULE-2
  (IF   ((jacket_color 2) (body_shape 1)))
  (THEN (class 1)))
(RULE-3
  (IF   ((body_shape 2) (jacket_color 2) (head_shape 1)))
  (THEN (class 1)))
(RULE-4
  (IF   ((jacket_color 3) (holding 1) (body_shape 2)))
  (THEN (class 1)))
(RULE-5
  (IF   ((body_shape 1) (jacket_color 1)))
  (THEN (class 1)))
(RULE-6
  (IF   ((jacket_color 3) (body_shape 1) (has_tie 2)))
  (THEN (class 1)))
(RULE-7
  (IF   ((body_shape 2) (jacket_color 2) (has_tie 2)))
  (THEN (class 1)))
(RULE-8
  (IF   ((jacket_color 3) (holding 1) (body_shape 3)))
  (THEN (class 1)))
(RULE-9
  (IF   ((jacket_color 3) (body_shape 1) (is_smiling 2)))
  (THEN (class 1)))
(RULE-10
  (IF   ((jacket_color 3) (body_shape 2) (is_smiling 2)))
  (THEN (class 1)))
(RULE-11
  (IF   ((jacket_color 3) (head_shape 3) (is_smiling 1)))
  (THEN (class 1)))
(RULE-12
  (IF   ((body_shape 2) (head_shape 1) (has_tie 2) (is_smiling 1)))
  (THEN (class 1)))
(RULE-13
  (IF   ((head_shape 3) (holding 1) (is_smiling 1) (body_shape 3)))
  (THEN (class 1)))
(RULE-14
  (IF   ((jacket_color 4) (has_tie 2)))
  (THEN (class 0)))
(RULE-15
  (IF   ((jacket_color 4) (head_shape 1)))
  (THEN (class 0)))
(RULE-16
  (IF   ((body_shape 3) (is_smiling 2)))
  (THEN (class 0)))
```

```
(RULE-17
  (IF   ((jacket_color 4) (holding 3)))
  (THEN (class 0)))
(RULE-18
  (IF   ((body_shape 3) (holding 3)))
  (THEN (class 0)))
(RULE-19
  (IF   ((jacket_color 4) (body_shape 1)))
  (THEN (class 0)))
(RULE-20
  (IF   ((body_shape 3) (holding 2)))
  (THEN (class 0)))
(RULE-21
  (IF   ((jacket_color 4) (body_shape 2)))
  (THEN (class 0)))
(RULE-22
  (IF   ((body_shape 3) (head_shape 1)))
  (THEN (class 0)))
(RULE-23
  (IF   ((jacket_color 3) (is_smiling 1) (head_shape 1) (body_shape 1)))
  (THEN (class 0)))
(RULE-24
  (IF   ((jacket_color 3) (holding 3) (head_shape 2) (body_shape 2)))
  (THEN (class 0)))
(RULE-25
  (IF   ((holding 2) (has_tie 1) (is_smiling 1) (body_shape 2) (head_shape 2)))
  (THEN (class 0)))
(RULE-26
  (IF   ((jacket_color 3) (holding 2) (head_shape 1)))
  (THEN (class 0)))
```

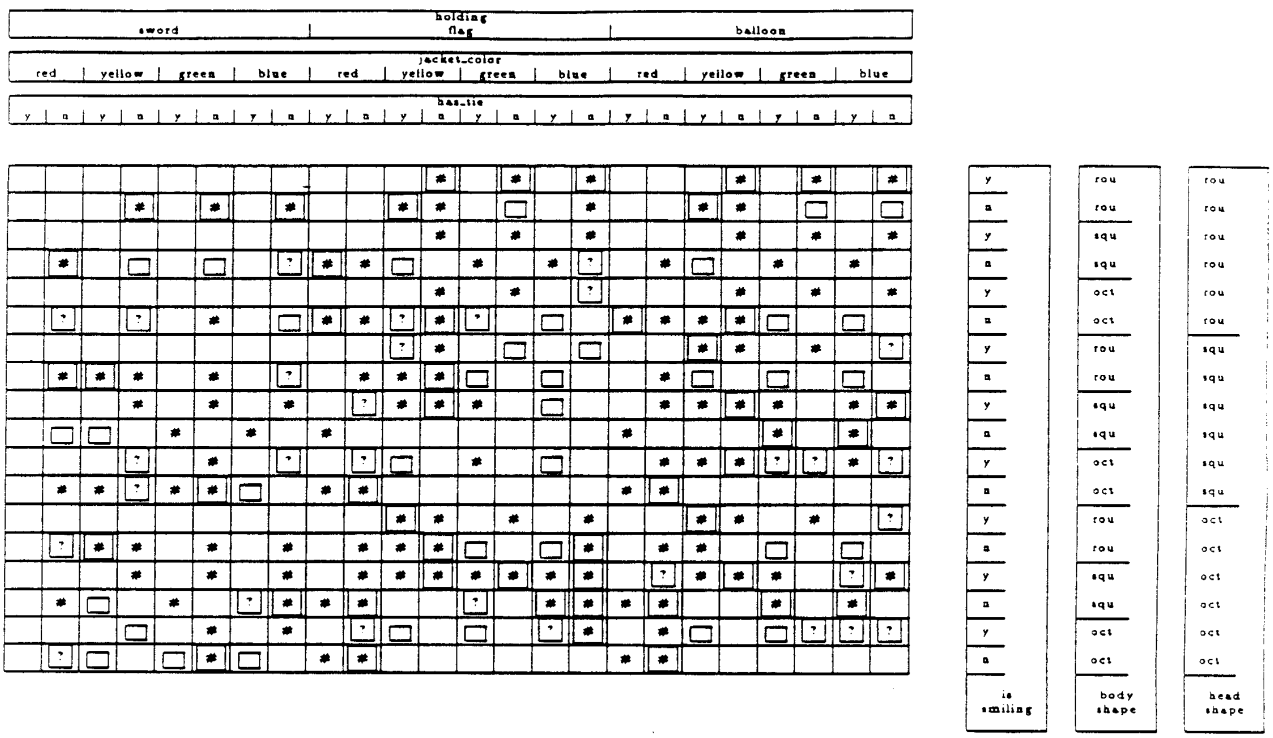
7.9 Classification diagrams

Result of PRISM on test set No. 1



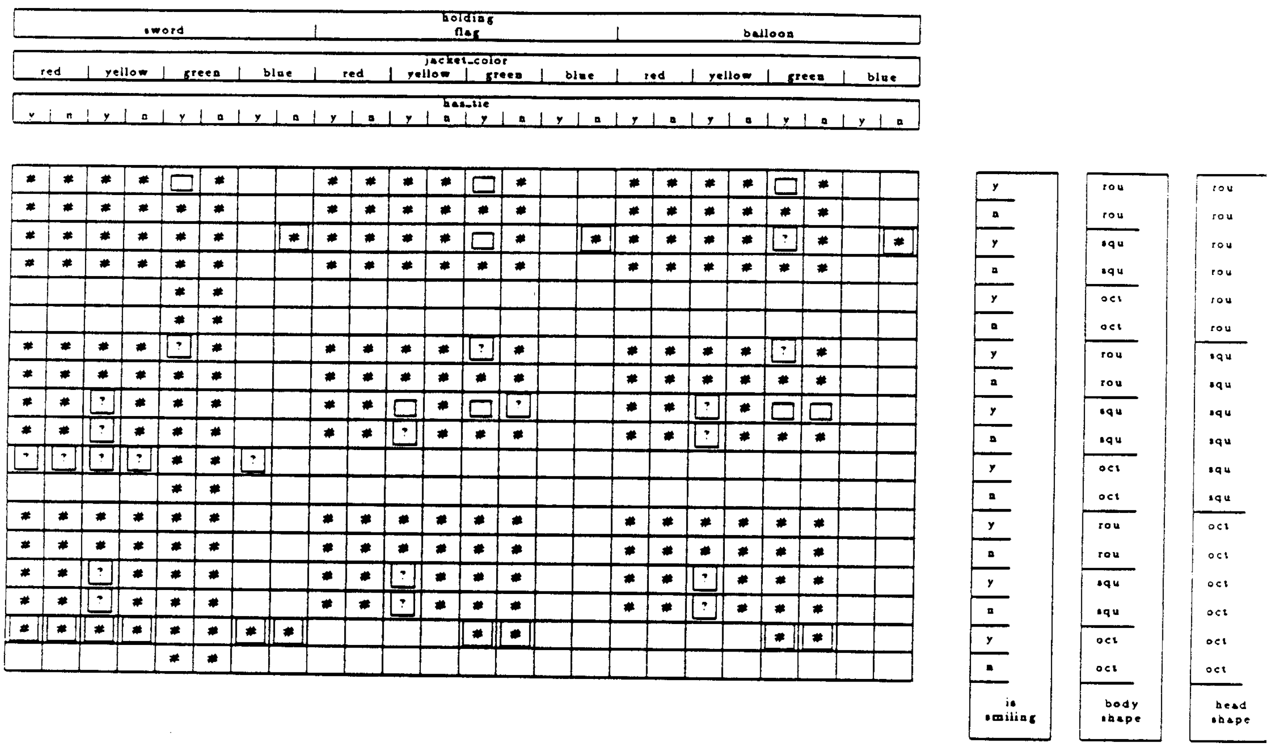
Accuracy: 86.342593

Result of PRISM on test set No. 2



Accuracy: 72.685185

Result of PRISM on test set No. 3



Accuracy: 90.277778

Chapter 8

Backpropagation on the MONK's problems

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8.1 Introduction

This paper briefly describes the results of the plain backpropagation algorithm [1] obtained on the three MONK's problems. Backpropagation is a function approximation algorithm for multilayer feed-forward perceptrons based on gradient descent. Conversely to many symbolic learning algorithms, backpropagation learns functions by nonlinear approximations. This technique has been successfully applied to a variety of real-world problems like speech recognition, bomb detection, stock market prediction etc.

Although multilayer networks represent continuous functions, they are quite often restricted to binary classification tasks as the MONK's problems. In all three cases we used the following architecture: There were 17 input units, all having either value 0 or 1, corresponding to which attribute-value was set. All input units had a connection to 2 hidden units, which itself had a connection to the output unit. An input was classified as class member if the output, which is naturally restricted to $]0,1[$, was $\geq .5$.

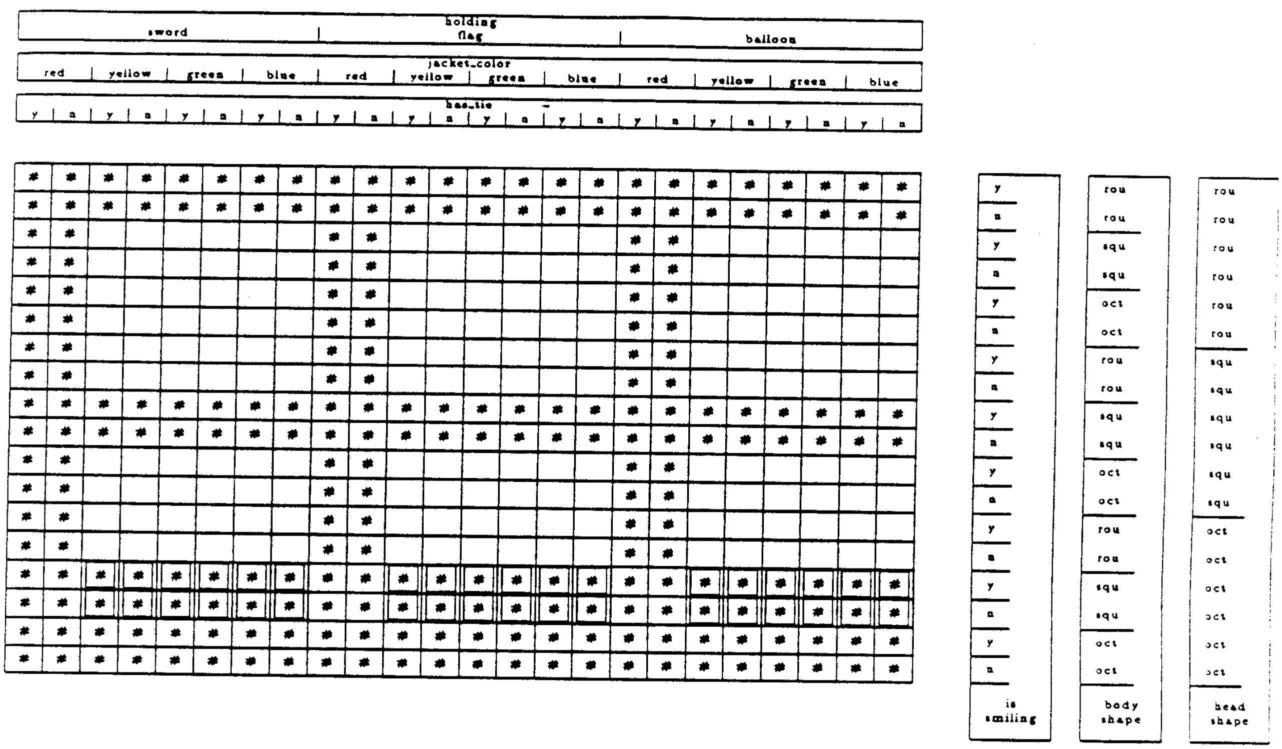
The results are shortly summarized by:

	accuracy
MONK's # 1	91.7%
MONK's # 2	100%
MONK's # 3	87.7%

The learned classes are visualized in the following three diagrams:

8.2 Classification diagrams

Results of BACKPROP on test set 1



Accuracy: 91.666667

Results of BACKPROP on test set 2

holding	sword								flag								balloon										
jacket_color	red		yellow		green		blue		red		yellow		green		blue		red		yellow		green		blue				
has_tie	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	is smiling	body shape	head shape
																									y	rou	rou
												#		#		#				#		#		#	n	rou	rou
												#		#		#				#		#		#	y	squ	rou
				#		#		#		#	#		#		#			#	#		#		#		n	squ	rou
												#		#		#				#		#		#	y	oct	rou
				#		#		#		#	#		#		#			#	#		#		#		n	oct	rou
												#		#		#				#		#		#	y	rou	squ
				#		#		#		#	#		#		#			#	#		#		#		n	rou	squ
				#		#		#		#	#		#		#			#	#		#		#		y	squ	squ
		#	#		#		#		#								#								n	squ	squ
				#		#		#		#	#		#		#			#	#		#		#		y	oct	squ
		#	#		#		#		#								#								n	oct	squ
												#		#		#				#		#		#	y	rou	oct
				#		#		#		#	#		#		#			#	#		#		#		n	rou	oct
				#		#		#		#	#		#		#			#	#		#		#		y	squ	oct
		#	#		#		#		#								#								n	squ	oct
				#		#		#		#	#		#		#			#	#		#		#		y	oct	oct
		#	#		#		#		#								#								n	oct	oct

Accuracy: 100.000000

Results of BACKPROP on test set 3

holding	sword								flag								balloon										
jacket_color	red		yellow		green		blue		red		yellow		green		blue		red		yellow		green		blue				
has_tie	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	is smiling	body shape	head shape
	#	#	#	#	□	□			#	#	□	#	□	□			#	#	#	#	□	#			y	rou	rou
	#	#	#	#	#	#			#	#	#	#	□	#			#	#	#	#	#	#			n	rou	rou
	□	#	#	#	#	#			□	#	#	#	□	□			#	#	#	#	□	#			y	squ	rou
	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#			n	squ	rou
					□	□																			y	oct	rou
					□	□																			n	oct	rou
	#	#	□	#	□	□			#	#	□	#	□	□			#	#	#	#	□	#			y	rou	squ
	#	#	#	#	#	#			#	#	□	#	□	□			#	#	#	#	□	#			n	rou	squ
	□	#	#	#	□	#			□	#	□	□	□	□			#	#	□	#	□	□			y	squ	squ
	#	#	#	#	#	#			□	#	#	#	#	#			#	#	#	#	#	#			n	squ	squ
					□	□																			y	oct	squ
					□	□																			n	oct	squ
	#	#	#	#	#	#			#	#	□	#	□	□			#	#	#	#	□	#			y	rou	oct
	#	#	#	#	#	#			#	#	#	#	#	#			#	#	#	#	#	#			n	rou	oct
	#	#	#	#	#	#			□	#	#	#	#	#			#	#	#	#	#	#			y	squ	oct
	#	#	#	#	#	#		#	#	#	#	#	#	#			#	#	#	#	#	#			n	squ	oct
					□	□																			y	oct	oct
				#	□	□																			n	oct	oct

Accuracy: 87.731481

8.3 Resulting weight matrices

MONKS's problem # 1: weights and biases			
	to-node		
from-node	hidden_1	hidden_2	output
input_1 (head_shape round)	-11.497804	-0.816155	
input_2 (head_shape square)	2.270476	0.850737	
input_3 (head_shape octagon)	8.328228	0.830791	
input_4 (body_shape round)	-11.950109	-4.841933	
input_5 (body_shape square)	13.642350	8.683802	
input_6 (body_shape octagon)	-1.446303	-3.217279	
input_7 (is_smiling yes)	0.145956	0.270281	
input_8 (is_smiling no)	-0.281478	0.252456	
input_9 (holding sword)	0.309568	0.016335	
input_10 (holding balloon)	0.024776	0.079897	
input_11 (holding flag)	-0.143636	0.032427	
input_12 (jacket_color red)	7.599377	-8.896360	
input_13 (jacket_color yellow)	-2.220987	3.177385	
input_14 (jacket_color green)	-2.884973	3.121367	
input_15 (jacket_color blue)	-1.911483	3.179718	
input_16 (has_tie yes)	0.186731	0.180876	
input_17 (has_tie no)	-0.142039	0.173498	
bias	0.340921	0.450262	
hidden_1			24.172497
hidden_2			-35.035843
bias			11.738854

MONKS's problem # 2: weights and biases

	to-node		
from-node	hidden_1	hidden_2	output
input_1 (head_shape round)	-4.230213	3.637149	
input_2 (head_shape square)	1.400753	-2.577242	
input_3 (head_shape octagon)	1.479862	-2.492254	
input_4 (body_shape round)	-4.363966	3.835199	
input_5 (body_shape square)	1.154510	-2.347489	
input_6 (body_shape octagon)	1.542958	-2.227530	
input_7 (is_smiling yes)	-3.396133	2.984736	
input_8 (is_smiling no)	1.868955	-2.994535	
input_9 (holding sword)	-4.041057	4.239548	
input_10 (holding balloon)	1.293933	-2.195403	
input_11 (holding flag)	1.160514	-2.272035	
input_12 (jacket_color red)	-4.462360	4.451742	
input_13 (jacket_color yellow)	0.749287	-1.869545	
input_14 (jacket_color green)	0.640353	-1.727654	
input_15 (jacket_color blue)	1.116349	-1.332642	
input_16 (has_tie yes)	-3.773187	3.290757	
input_17 (has_tie no)	1.786105	-3.296139	
bias	-1.075762	-0.274980	
hidden_1			-11.038625
hidden_2			-9.448544
bias			5.031395

MONKS's problem # 3: weights and biases			
	to-node		
from-node	hidden_1	hidden_2	output
input_1 (head_shape round)	0.289323	-1.699048	
input_2 (head_shape square)	0.362874	-8.404805	
input_3 (head_shape octagon)	2.376665	4.989420	
input_4 (body_shape round)	-8.085952	3.107904	
input_5 (body_shape square)	0.469876	12.290827	
input_6 (body_shape octagon)	10.788216	-19.197418	
input_7 (is_smiling yes)	0.998298	-8.935371	
input_8 (is_smiling no)	2.894973	4.401900	
input_9 (holding sword)	4.283379	4.033230	
input_10 (holding balloon)	1.615812	-5.180389	
input_11 (holding flag)	-2.107015	-2.652152	
input_12 (jacket_color red)	-8.990289	0.162749	
input_13 (jacket_color yellow)	-1.135560	12.240005	
input_14 (jacket_color green)	4.198663	5.653207	
input_15 (jacket_color blue)	10.960098	-21.990112	
input_16 (has_tie yes)	-0.391244	-4.312347	
input_17 (has_tie no)	-0.391244	-0.212025	
bias	3.979607	-4.687716	
hidden_1			-12.178342
hidden_2			20.635145
bias			9.146520

References

[1] Rumelhart, D. E. and McClelland, J.: *Parallel Distributed Processing. Vol. I + II*, MIT Press 1986

Chapter 9

The Cascade-Correlation Learning Algorithm on the MONK's Problems

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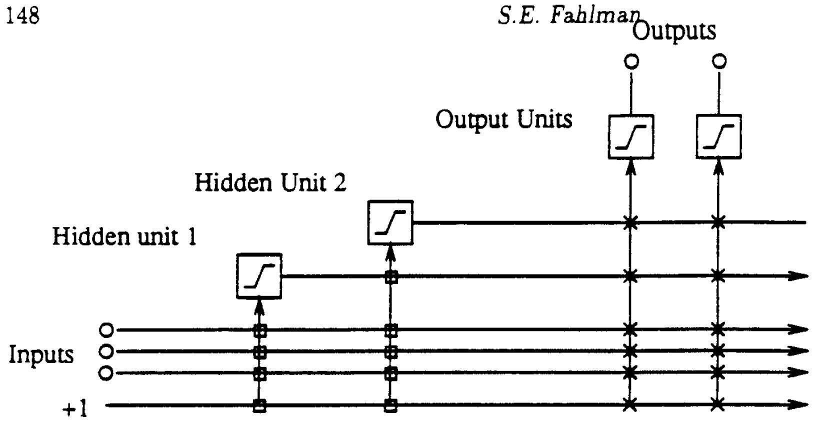


Figure 9.1: see text for details

9.1 The Cascade-Correlation algorithm

Cascade-Correlation [Fahlman, 1990] is a supervised neural network learning architecture that builds a near-minimal multi-layer network topology in the course of training. Initially the network contains only inputs, output units, and the connections between them. This single layer of connections is trained (using the Quickprop algorithm [Fahlman, 1988]) to minimize the error. When no further improvement is seen in the level of error, the network's performance is evaluated. If the error is small enough, we stop. Otherwise we add a new hidden unit to the network in an attempt to reduce the residual error.

To create a new hidden unit, we begin with a pool of *candidate units*, each of which receives weighted connections from the network's inputs and from any hidden units already present in the net. The outputs of these candidate units are not yet connected into the active network. Multiple passes through the training set are run, and each candidate unit adjusts its incoming weights to maximize the correlation between its output and the residual error in the active net. When the correlation scores stop improving, we choose the best candidate, freeze its incoming weights, and add it to the network. This process is called "tenure." After tenure, a unit becomes a permanent new feature detector in the net. We then re-train all the weights going to the output units, including those from the new hidden unit. This process of adding a new hidden unit and re-training the output layer is repeated until the error is negligible or we give up. Since the new hidden unit receives connections from the old ones, each hidden unit effectively adds a new layer to the net. (See figure 1.)

Cascade-correlation eliminates the need for the user to guess in advance the network's size, depth, and topology. A reasonably small (though not minimal) network is built automatically. Because a hidden-unit feature detector, once built, is never altered or cannibalized, the network can be trained incrementally. A large data set can be broken up into smaller "lessons," and feature-building will be cumulative.

Cascade-Correlation learns much faster than backprop for several reasons: First only a single layer of weights is being trained at any given time. There is never any need to propagate error information backwards through the connections, and we avoid the dramatic slowdown that is typical when training backprop nets with many layers. Second, this is a "greedy" algorithm: each new unit grabs as much of the remaining error as it can. In a standard backprop net, the all the hidden units are changing at once, competing for the various jobs that must be done—a slow and sometimes unreliable process.

9.2 Results

For all these problems I used the standard Common Lisp implementation of Cascade-Correlation on a Decstation 3100. This code is public-domain and is available to outside users via anonymous FTP. Contact sef@cs.cmu.edu for details.

I used the same parameters in all of these tests. Here is the printout of those parameters:

SigOff 0.10	WtRng 1.00	WtMul 1.00		
OMu 2.00	OEps 1.00	ODcy 0.0000	OPat 20	OChange 0.010
IMu 2.00	IEps 1.00	IDcy 0.0000	IPat 15	IChange 0.030
Utype :GAUSSIAN	Otype :SIGMOID	RawErr NIL	Pool 8	
(train 100 100 10)				

9.2.1 Monk #1:

After 95 epochs, 1 hidden unit: 0 Errors on training set. 0 Errors on test set.
Elapsed real time: 5.11 seconds

9.2.2 Monk #2:

After 82 epochs, 1 hidden unit: 0 Errors on training set. 0 Errors on test set.
Elapsed real time: 7.75 seconds

9.2.3 Monk #3:

After 259 epochs, 3 hidden units: 0 Errors on training set. 40 errors on test set (i.e. accuracy 95.4%).
Elapsed real time 12.27 seconds.

Training and test-set performance was tested after each output-training phase. The minimum test-set error was observed after the initial output-training phase, before any hidden units were added. (Not surprising, since with no noise this problem is linearly separable.) Using any sort of cross-validation system, this is where the algorithm would stop.

At that point, the results were as follows:

Training: 7 of 122 wrong:

Head: RND	Body: RND	Smile: Y	Holding: SWD	Jacket: GRN	Tie: Y	Output: T
Head: RND	Body: SQR	Smile: Y	Holding: BAL	Jacket: GRN	Tie: Y	Output: T
Head: SQR	Body: SQR	Smile: Y	Holding: BAL	Jacket: YEL	Tie: Y	Output: T
Head: SQR	Body: SQR	Smile: Y	Holding: FLG	Jacket: GRN	Tie: N	Output: T
Head: SQR	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: Y	Output: NIL
Head: OCT	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: N	Output: NIL
Head: OCT	Body: OCT	Smile: Y	Holding: SWD	Jacket: BLU	Tie: Y	Output: NIL

Test: 14 of 432 wrong:

Head: RND	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: Y	Output: NIL
Head: RND	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: N	Output: NIL
Head: RND	Body: OCT	Smile: N	Holding: SWD	Jacket: GRN	Tie: Y	Output: NIL
Head: RND	Body: OCT	Smile: N	Holding: SWD	Jacket: GRN	Tie: N	Output: NIL
Head: SQR	Body: SQR	Smile: Y	Holding: BAL	Jacket: GRN	Tie: Y	Output: NIL
Head: SQR	Body: SQR	Smile: Y	Holding: FLG	Jacket: GRN	Tie: Y	Output: NIL
Head: SQR	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: Y	Output: NIL
Head: SQR	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: N	Output: NIL
Head: SQR	Body: OCT	Smile: N	Holding: SWD	Jacket: GRN	Tie: Y	Output: NIL
Head: SQR	Body: OCT	Smile: N	Holding: SWD	Jacket: GRN	Tie: N	Output: NIL
Head: OCT	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: Y	Output: NIL
Head: OCT	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: N	Output: NIL
Head: OCT	Body: OCT	Smile: N	Holding: SWD	Jacket: GRN	Tie: Y	Output: NIL
Head: OCT	Body: OCT	Smile: N	Holding: SWD	Jacket: GRN	Tie: N	Output: NIL

So on the test set, performance is 96.7%

By turning up the OUTPUT-DECAY parameter to 0.1 (an odd thing to do, but sometimes useful when the training set is too small for good generalization), we can do a little better. After the initial output-training phase:

Training: 8 of 122 wrong:

Head: RND	Body: RND	Smile: Y	Holding: SWD	Jacket: GRN	Tie: Y	Output: T
Head: RND	Body: SQR	Smile: Y	Holding: BAL	Jacket: GRN	Tie: Y	Output: T
Head: SQR	Body: SQR	Smile: Y	Holding: BAL	Jacket: YEL	Tie: Y	Output: T
Head: SQR	Body: SQR	Smile: Y	Holding: FLG	Jacket: GRN	Tie: Y	Output: T
Head: SQR	Body: SQR	Smile: Y	Holding: FLG	Jacket: GRN	Tie: N	Output: T
Head: SQR	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: Y	Output: NIL
Head: OCT	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: N	Output: NIL
Head: OCT	Body: OCT	Smile: Y	Holding: SWD	Jacket: BLU	Tie: Y	Output: NIL

Test: 12 of 432 wrong:

Head: RND	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: Y	Output: NIL
Head: RND	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: N	Output: NIL
Head: RND	Body: OCT	Smile: N	Holding: SWD	Jacket: GRN	Tie: Y	Output: NIL
Head: RND	Body: OCT	Smile: N	Holding: SWD	Jacket: GRN	Tie: N	Output: NIL
Head: SQR	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: Y	Output: NIL
Head: SQR	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: N	Output: NIL
Head: SQR	Body: OCT	Smile: N	Holding: SWD	Jacket: GRN	Tie: Y	Output: NIL
Head: SQR	Body: OCT	Smile: N	Holding: SWD	Jacket: GRN	Tie: N	Output: NIL
Head: OCT	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: Y	Output: NIL
Head: OCT	Body: OCT	Smile: Y	Holding: SWD	Jacket: GRN	Tie: N	Output: NIL
Head: OCT	Body: OCT	Smile: N	Holding: SWD	Jacket: GRN	Tie: Y	Output: NIL
Head: OCT	Body: OCT	Smile: N	Holding: SWD	Jacket: GRN	Tie: N	Output: NIL

Score on test set: 97.2%

We can see here what the problem is: All the bad test-set cases are Green and holding a sword, so they should be true. But this positive value is not strong enough to offset the negative weight from Octagonal body.

In the training set, there are only two examples showing the green-sword combination overpowering an octagonal body, and that is apparently not enough to make the point. There are 11 cases showing that octagonal/sword should be negative and 8 cases showing that octagonal/green should be negative.

If we switch the training and test set, we see how easy it is to solve this problem in the absence of noise and small-sample fluctuations.

Switching the training and test set: After 16 epochs and 0 hidden units:

Training: 0 of 432 wrong. Test: 6 of 122 wrong.

Head: RND	Body: RND	Smile: Y	Holding: SWD	Jacket: GRN	Tie: Y	Output: T
Head: RND	Body: SQR	Smile: Y	Holding: BAL	Jacket: GRN	Tie: Y	Output: T
Head: SQR	Body: SQR	Smile: Y	Holding: BAL	Jacket: YEL	Tie: Y	Output: T
Head: SQR	Body: SQR	Smile: Y	Holding: FLG	Jacket: GRN	Tie: Y	Output: T
Head: SQR	Body: SQR	Smile: Y	Holding: FLG	Jacket: GRN	Tie: N	Output: T
Head: OCT	Body: OCT	Smile: Y	Holding: SWD	Jacket: BLU	Tie: Y	Output: NIL

These, I believe, are exactly the noise cases deliberately inserted in the original training set. Note that three of these noise cases are

Square/Square/Yes $\Longrightarrow$ NIL (when T is correct)

This explains the other two error cases observed in the first run of this problem. If we look at square/square/yes cases in the training set, NIL cases outnumber T cases, 5 to 3.

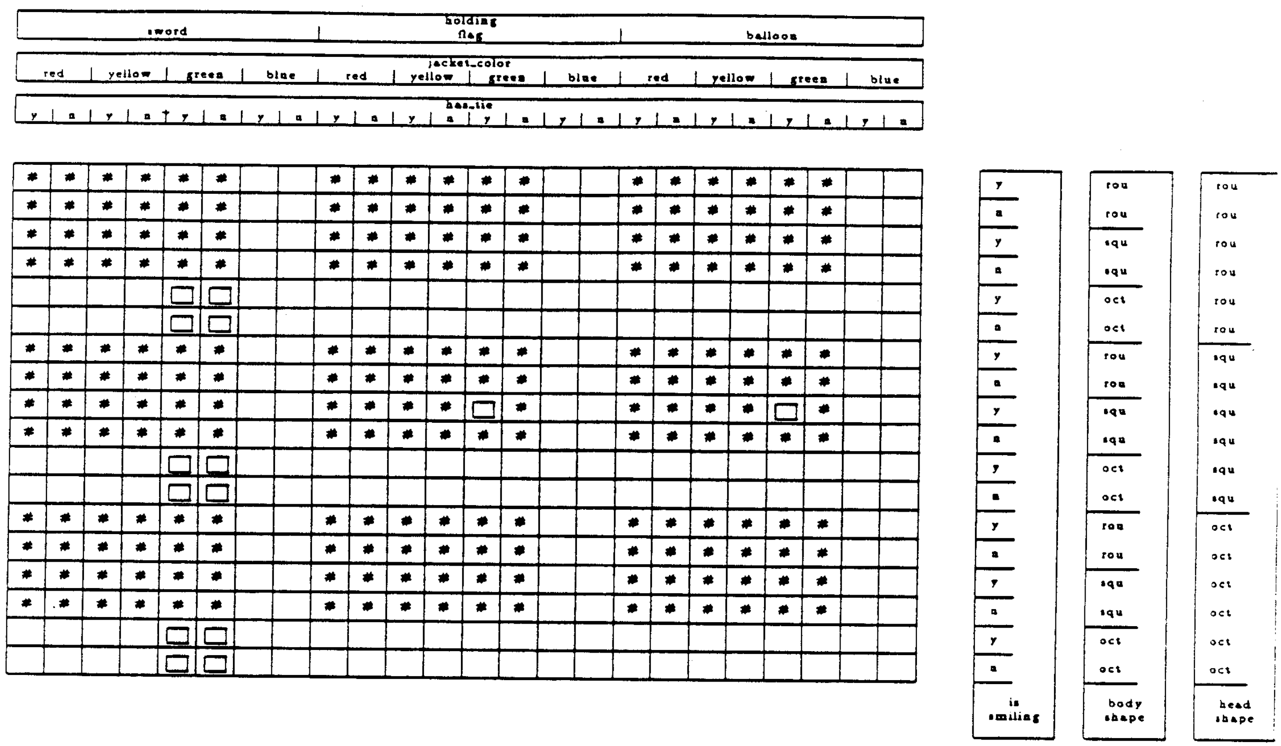
Bibliography

[Fahlman, 1988] Fahlman, S. E. (1988) "Faster-Learning Variations on Back-Propagation: An Empirical Study" in *Proceedings of the 1988 Connectionist Models Summer School*, Morgan Kaufmann.

[Fahlman, 1990] Fahlman, S. E. and C. Lebiere (1988) "The Cascade-Correlation Learning Architecture" in D. S. Touretzky (ed.), *Advances in Neural Information Processing Systems 2*, Morgan Kaufmann.

9.3 Classification diagrams

Training set #3 first run



Accuracy: 96.759259

Training set #3 second run

holding: sword								flag								balloon										
jacket_color: 1		2		3		4		1		2		3		4		red		yellow		green		blue				
has_tie: y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	is smiling	body shape	head shape
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				☐	☐																			n	oct	squ
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				☐	☐																			n	oct	oct

Accuracy: 97.222222